

CCFL Model Based on the Hyperbolic Two-Fluid Equations and Non-uniform Flow Interface Shape

가

Abstract

The maximum flow rates of gas and liquid phases which flow in opposite-directions (counter-current flow) are limited by a phenomenon known as a Counter-Current Flow Limitation (CCFL or Flooding). The mass and momentum conservation equations for each phase were established to build a first-order hyperbolic partial derivative equations system. A new CCFL model is developed based on the characteristic equation of the hyperbolic PDE system. The present model has its application to the case in which a non-uniform flow is developed around a square or sharp-edged entrance of liquid phase. The model is able to be used to predict the operating-limit of components in which mass and heat transfer are taking place between liquid and gas phases.

1.

(liquid phase) (gas phase)
(counter-current flow) (two-phase flow)가
가 . 가 가
(annular flow pattern)

(liquid film)

가 가 . 가 가
(wave) (wave height)가
(solitary wave) 가

(chaotic flow pattern)

가 (co-current flow) .
(counter-current flow limitation, CCFL)

(flooding)

가 가

50

가

가

(Emergency core cooling system, ECCS)

가

(1) (wave dynamics) :

. Cetinbudaklar & Jameson¹⁾,

Richter²⁾, Shearer & Davidson³⁾, Wallis & Makkenchery⁴⁾

(2) (drop dynamics) :

(drag force)

. Lee et al.

⁵⁾, McCarthy & Lee⁶⁾

(3) (liquid film dynamics) :

(net flow) 0

Maron & Dukler⁷⁾, Taitel et al.⁸⁾, Wallis⁹⁾,

Bharathan et al.¹⁰⁾

(1), (2)

가

$$j_g^{*1/2} + m j_f^{*1/2} = C^2 \tag{1}$$

$$K_g^{1/2} + m K_f^{1/2} = C^2 \tag{2}$$

$$j_k^* = j_k \sqrt{\frac{\mathbf{r}_k}{gD(\mathbf{r}_f - \mathbf{r}_g)}}, \quad (3)$$

$$K_k^* = j_k \sqrt{\frac{\mathbf{r}_k^2}{g\mathbf{S}(\mathbf{r}_f - \mathbf{r}_g)}}, \quad (4)$$

$g, D, \mathbf{s}, \mathbf{r}, j$ 가 , (), , ,
 g, f, k , , .
 (1), (2) m C 가
 ,
 .
 가 .
 가

2. CCFL

singular points

$$\underline{A} \frac{\partial X}{\partial t} + \underline{B} \frac{\partial X}{\partial z} = \underline{C} \quad (5)$$

(wave front) 가 가 , \mathbf{x} , .

$$\mathbf{x} = z + \underline{I}t \quad (6)$$

\underline{I} 가
 (t, z) \mathbf{x} (5) .

$$\underline{(\underline{A}\underline{I} + \underline{B})} \frac{\partial X}{\partial \mathbf{x}} = \underline{C} \quad (7)$$

(7) x_i .

$$\frac{\partial x_i}{\partial \mathbf{x}} = \frac{N_i}{\Delta} \quad (8)$$

$$x_i : \underline{X} \quad i$$

$$\Delta = |\underline{AI} + \underline{B}|,$$

$$N_i : (A_{ij}\mathbf{I} + B_{ij}) \quad i \quad C_j$$

$$\Delta \neq 0, \quad x_i \quad \text{regular points}, \quad \Delta = 0, \quad (8)$$

singular points 가 . (5)

(characteristic equation)

, \mathbf{I} , 가

가 ^{11), Lax¹²⁾} (5)

well-posed

well-posed (stability)

\mathbf{I} 가

가

, \mathbf{I} ,

가

가

singular point

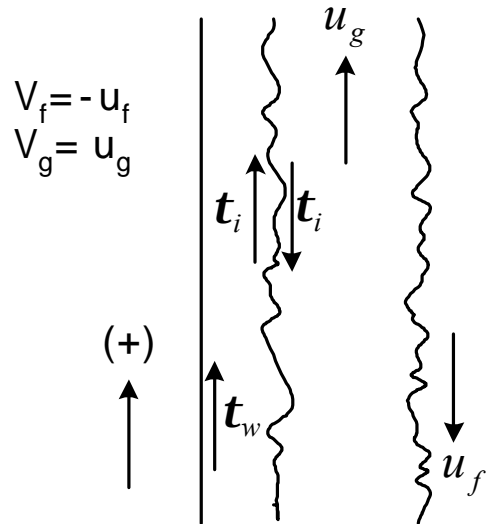


Fig. 1 Schematic of a vertical annular flow system.

Fig. 1

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$$\frac{\partial}{\partial t}(\mathbf{a}_g \mathbf{r}_g) + \frac{\partial}{\partial z}(\mathbf{a}_g \mathbf{r}_g V_g) = \Gamma \quad (9)$$

$$\frac{\partial}{\partial t}(\mathbf{a}_f \mathbf{r}_f) + \frac{\partial}{\partial z}(\mathbf{a}_f \mathbf{r}_f V_f) = -\Gamma \quad (10)$$

Fig. 1 u_g $-u_f$, Γ , \mathbf{a}_g , \mathbf{a}_f
 (),
 , $\mathbf{a}_g + \mathbf{a}_f = 1$. 가
 가 . $\partial \Delta P / \partial z =$
 $(\partial \Delta P / \partial \mathbf{a}_g) \partial \mathbf{a}_g / \partial z$. $\Delta P = P_f - P_g$. 가

$$\begin{aligned} & \mathbf{a}_g \mathbf{r}_g \frac{\partial V_g}{\partial t} + \mathbf{a}_g \mathbf{r}_g V_g \frac{\partial V_g}{\partial z} + \mathbf{a}_g \frac{\partial P_g}{\partial z} \\ & = \Gamma(V_{gi} - V_g) - 4 \frac{\sqrt{\mathbf{a}_g}}{D} \mathbf{t}_i - \mathbf{a}_g \mathbf{r}_g g \end{aligned} \quad (11)$$

$$\begin{aligned} & \mathbf{a}_f \mathbf{r}_f \frac{\partial V_f}{\partial t} + \mathbf{a}_f \mathbf{r}_f V_f \frac{\partial V_f}{\partial z} + \mathbf{a}_f \frac{\partial P_f}{\partial z} \\ & = -\Gamma(V_{fi} - V_f) + 4 \frac{\sqrt{\mathbf{a}_g}}{D} \mathbf{t}_i - \frac{4}{D} \mathbf{t}_w - \mathbf{a}_f \mathbf{r}_f g \end{aligned} \quad (12)$$

$$P_f = P_g + \Delta P, \quad \Delta P = P_f - P_g, \quad \frac{\partial P_f}{\partial z} = \frac{\partial P_g}{\partial z} + \frac{\partial \Delta P}{\partial \mathbf{a}_g} \frac{\partial \mathbf{a}_g}{\partial z}$$

$$\mathbf{t}_i = \frac{f_i}{2} \mathbf{r}_g |V_g| V_g, \quad \mathbf{t}_w = \frac{f_w}{2} \mathbf{r}_f |V_f| V_f, \quad \Gamma = \frac{Q_w}{h_{fg}}$$

(9) - (12) 4

(5)

\underline{X} , \underline{A} , \underline{B} ,

\underline{C}

$$\underline{X} = (P_g, \mathbf{a}_g, V_g, V_f)^T \quad (13)$$

$$\underline{A} = \begin{pmatrix} \mathbf{a}_g C_g^{-2} & \mathbf{r}_g & 0 & 0 \\ \mathbf{a}_f C_f^{-2} & -\mathbf{r}_f & 0 & 0 \\ 0 & 0 & \mathbf{a}_g \mathbf{r}_g & 0 \\ 0 & 0 & 0 & \mathbf{a}_f \mathbf{r}_f \end{pmatrix} \quad (14)$$

$$\underline{B} = \begin{pmatrix} \mathbf{a}_g V_g C_g^{-2} & \mathbf{r}_g V_g & \mathbf{a}_g \mathbf{r}_g & 0 \\ \mathbf{a}_f V_f C_f^{-2} & -\mathbf{r}_f V_f & 0 & \mathbf{a}_f \mathbf{r}_f \\ \mathbf{a}_g & 0 & \mathbf{a}_g \mathbf{r}_g V_g & 0 \\ \mathbf{a}_f & \mathbf{a}_g \frac{\partial \Delta P}{\partial \mathbf{a}_g} & 0 & \mathbf{a}_f \mathbf{r}_f V_f \end{pmatrix} \quad (15)$$

$$\underline{C} = \begin{pmatrix} \Gamma \\ -\Gamma \\ \Gamma(V_{gi} - V_g) - 4 \frac{\mathbf{a}_g^{1/2}}{D} \mathbf{t}_i - \mathbf{a}_g \mathbf{r}_g g \\ -\Gamma(V_{fi} - V_f) + 4 \frac{\mathbf{a}_g^{1/2}}{D} \mathbf{t}_i - \frac{4}{D} \mathbf{t}_w - \mathbf{a}_f \mathbf{r}_f g \end{pmatrix} \quad (16)$$

$$C_g^{-2} = \frac{\partial \mathbf{r}_g}{\partial P_g}, \quad C_f^{-2} = \frac{\partial \mathbf{r}_f}{\partial P_f}$$

$$\mathbf{x} = \mathbf{z} + \mathbf{l}t$$

가

, C_g^{-2}

C_f^{-2}

, \underline{X}

$$\frac{\partial \mathbf{a}_g}{\partial \mathbf{x}} = \frac{N(\mathbf{a}_g)}{\Delta} \quad (17)$$

$$\Delta = -\mathbf{a}_f \mathbf{r}_g (\mathbf{l} + V_g)^2 - \mathbf{a}_g \mathbf{r}_f (\mathbf{l} + V_f)^2 + \mathbf{a}_g \mathbf{a}_f \frac{\partial \Delta P}{\partial \mathbf{a}_g} \quad (18)$$

$$N(\mathbf{a}_g) = 4 \frac{\sqrt{\mathbf{a}_g}}{D} \mathbf{t}_i - \frac{4}{D} \mathbf{a}_g \mathbf{t}_w - \mathbf{a}_g \mathbf{a}_f \Delta \mathbf{r} \mathbf{g} - \Gamma [\mathbf{a}_f (\mathbf{I} + V_g) - \mathbf{a}_g (\mathbf{I} + V_f)] \quad (19)$$

(18)

$$\Delta(\mathbf{I}) = 0$$

$$\mathbf{l} = p \pm \sqrt{(p^2 - q)} \quad (20)$$

$$p = \frac{\mathbf{a}_f \mathbf{r}_g V_g + \mathbf{a}_g \mathbf{r}_f V_f}{\mathbf{a}_f \mathbf{r}_g + \mathbf{a}_g \mathbf{r}_f} \quad (21)$$

$$q = \frac{\mathbf{a}_f \mathbf{r}_g V_g^2 + \mathbf{a}_g \mathbf{r}_f V_f^2 + \mathbf{a}_g \mathbf{a}_f \frac{\partial \Delta P}{\partial \mathbf{a}_g}}{\mathbf{a}_f \mathbf{r}_g + \mathbf{a}_g \mathbf{r}_f} \quad (22)$$

(neutral stability condition) (20)

"0"

$$(V_g - V_f)^2 = - \frac{\partial \Delta P}{\partial \mathbf{a}_g} \frac{\mathbf{a}_f \mathbf{r}_g + \mathbf{a}_g \mathbf{r}_f}{\mathbf{r}_g \mathbf{r}_f} \quad (23)$$

(23) ()가

(23)

(3)

$$\frac{j_g^*}{\mathbf{a}_g} + N_r^{1/2} \frac{j_f^*}{\mathbf{a}_f} = N_{ip} \sqrt{\mathbf{a}_g + \mathbf{a}_f N_r} \quad (24)$$

$$N_r = \mathbf{r}_g / \mathbf{r}_f \quad (25)$$

$$N_{ip} = \left(- \frac{\partial \Delta P / \partial \mathbf{a}_g}{g D \Delta \mathbf{r}} \right)^{1/2} \quad (26)$$

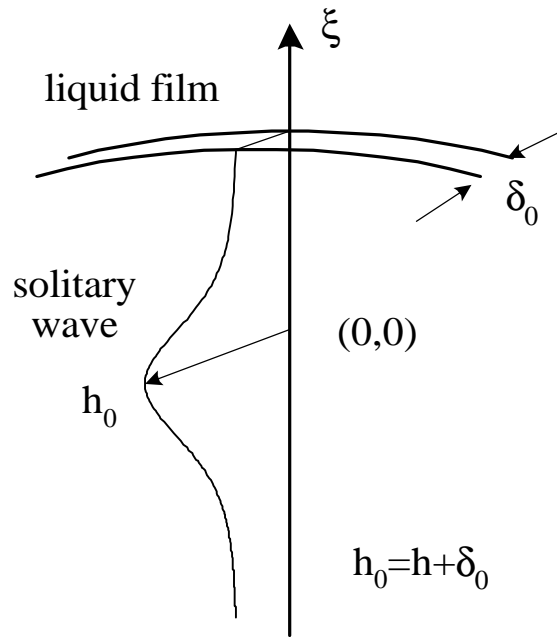


Fig. 2 Simplified solitary wave to obtain its radius of curvature.

(26)

Fig. 2 Korteweg-de Vries (solitary wave)¹³⁾ 가

$$d(x) = h \operatorname{sech}^2(kx) + d_0 \quad (27)$$

d , d_0 , h_0 , k , $(h + d_0)$, (wavenumber, $2p / l_w$)
 가 가 Bernoulli 가 가
 가 가

$$\Delta P = -s \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad \text{at } x = 0 \quad (28)$$

$$R_1 = R - (h + d_0) \quad (29)$$

$$R_2 = \left(\frac{d^2 d(x)}{dx^2} \Big|_{x=0} \right)^{-1} = -\frac{1}{2hk^2} \quad (30)$$

$$R, \quad 1/R_1, \quad 1/R_2, \quad 1/R_2 \text{ 가 } 1/R_1 \quad (28)$$

$$\Delta P = 2s k^2 (h_0 - d_0) \quad (31)$$

$$(31) \quad \mathbf{a}_g, \quad \mathbf{a}_g = (1 - 2h_0/D)^2 \text{ 가 } .$$

$$\frac{\partial \Delta P}{\partial \mathbf{a}_g} = -\frac{D}{2} s k^2 \mathbf{a}_g^{-1/2} \quad (32)$$

$$(32) \quad \mathbf{k} = 2\mathbf{p} / l_w \quad (26)$$

$$N_{ip} = \frac{1}{\sqrt{2}} \sqrt{\frac{s}{g\Delta r}} \frac{2\mathbf{p}}{l_w} \mathbf{a}_g^{-1/4} \quad (33)$$

가 Helmholtz instability 가
가 Helmholtz instability

$$, \quad l_w = 2\mathbf{p} \sqrt{s / g\Delta r} \quad (33) \quad (24)$$

$$j_g^* = \frac{1}{\sqrt{2}} \mathbf{a}_g^{5/4} - \frac{\mathbf{a}_g}{\mathbf{a}_f} \sqrt{\frac{\mathbf{r}_g}{\mathbf{r}_f}} j_f^* \quad (34)$$

\mathbf{a}_g

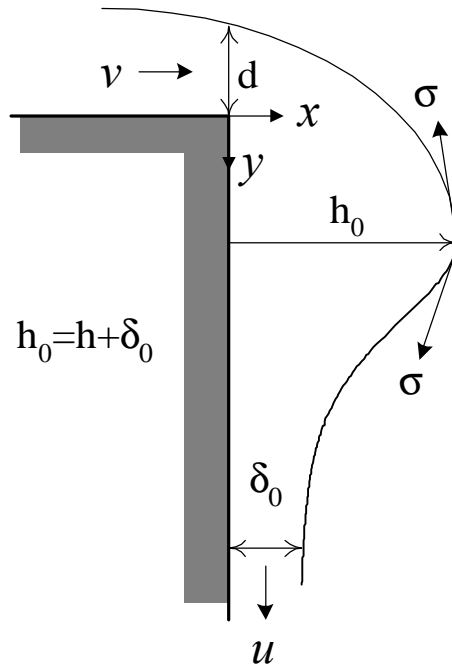


Fig. 3 Schematic of a non-uniform flow around a rectangular liquid entrance.

Fig. 3

. Fig. 3

(nun-uniform flow)

. x-
가

$$\frac{1}{2} \mathbf{r}_f v^2 = -\mathbf{s} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (35)$$

(27) Korteweg-de Vries

가 .

$1/R_1$ $1/R_2$

$1/R_2$ 가 $1/R_1$

(35) $1/R_1$

.

$$\frac{1}{2} \mathbf{r}_f v^2 = 2\mathbf{s} \mathbf{k}^2 h \quad (36)$$

Fig. 3

(27)

(stream line)

(36) (k)

Helmholtz 가 Helmholtz
가 .

$$\mathbf{k}^2 = C_1 \frac{g\Delta\mathbf{r}}{\mathbf{s}} \quad (37)$$

(37) (36)

$$h = \frac{\mathbf{r}_f v^2}{4C_1 g\Delta\mathbf{r}} \quad (38)$$

, 3 (v) (d) , $v = \sqrt{gd}$ 가 ,
(q_f) , $q_f = v \cdot d$ 가 ,
가 .

$$v = (gq_f)^{1/3} \quad (39)$$

(3), (39) $Q_f = \mathbf{p} D q_f$ (38)

$$h = \frac{\mathbf{r}_f}{4C_1 g\Delta\mathbf{r}} \left(\frac{gD}{4} \right)^{2/3} \left(\frac{gD\Delta\mathbf{r}}{\mathbf{r}_f} \right)^{1/3} j_f^{*2/3} \quad (40)$$

, $\mathbf{a}_f \cong 4h/D$ 가 가 .

$$\mathbf{a}_f = C_2 \cdot j_f^{*2/3} \quad (41)$$

3.

Fig. 4

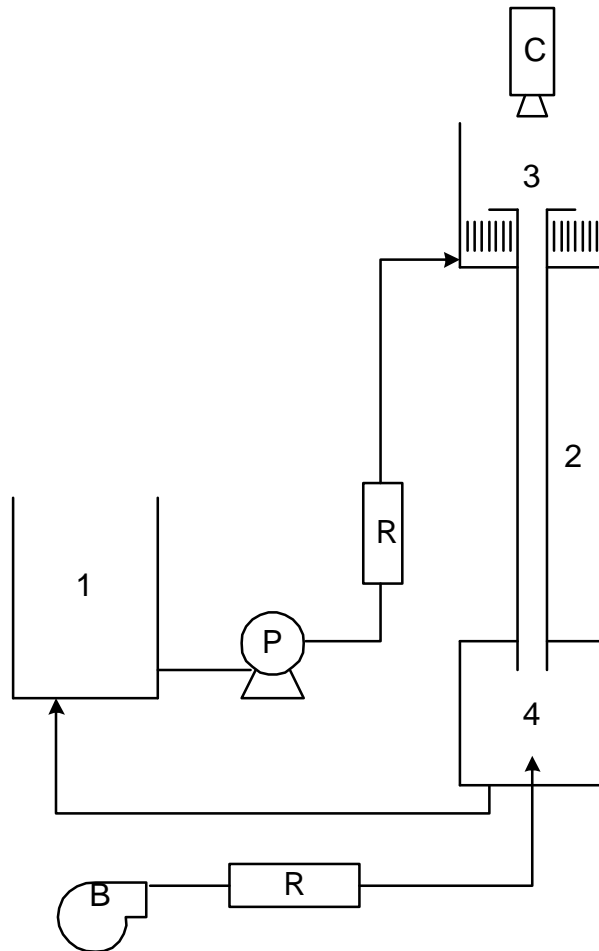
100cm, 3 cm

, 가

7 가

가

가 가
Rotameter
CCD PC



1. Water reservoir, 2. Test section, 3. Upper plenum, 4. Lower plenum
B. Air blower, C. CCD camera, P. pump, R. flow meter

Fig. 4 Schematic diagram of experimental apparatus.

4.

(41)

2,3,4

5cm

CCD

가 , PC (41)

$$a_f = 1.166 \cdot j_f^{*2/3}, \quad j_f^{*2/3} < 0.8 \quad (42)$$

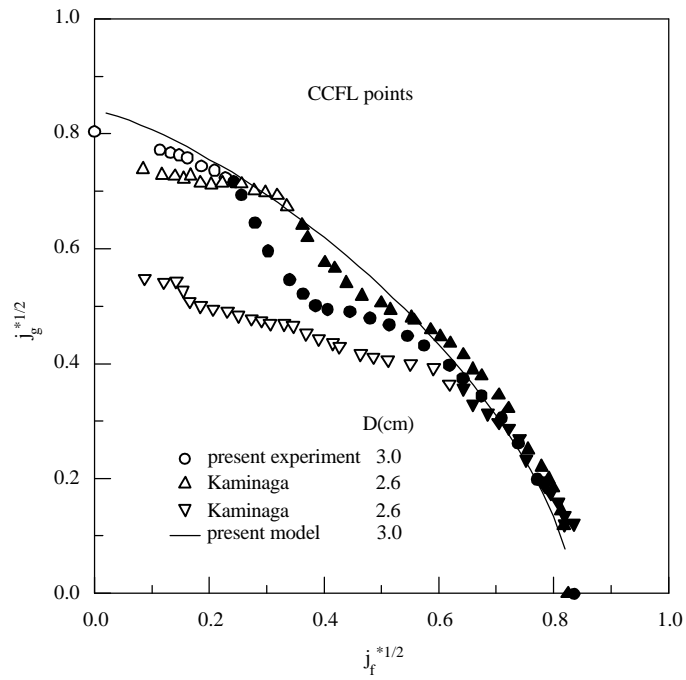


Fig. 5 Comparisons among present model and experimental measurements.

14) Kaminaga Fig. 5 Kaminaga 3cm 3cm Kaminaga 2.6cm 가 Fig. 5 (●, ▲, ▼) (○, □, □) 가 Kaminaga (□) 가 Fig. 3 가 (42) 가

가

Jeong & No¹⁵⁾

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5.

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