Space-Dependent Feedback Kinetics Calculation in the Analytic Function Expansion Nodal Method

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Abstract

Recently, Kim and Cho[1] developed and applied an AFEN kinetics method based on Galerkin weighting. We extend the method in this paper to treat space dependent cross sections (which may be induced from temperature feedback or burnup). It is easily incorporated into the method and enhances the accuracy significantly with marginal increase of computing time.

I. Introduction

The analytic function expansion nodal (AFEN) method[2,3] has been quite successful in static reactor analysis. Since AFEN does not use transverse integration, for kinetics problems, we need to develop a different formulation from those of the usual nodal methods. In the kinetics method with Galerking weighting[1], the time-dependent solution is decomposed into analytic part and polynomial correction part. The polynomial correction part is then determined by a Galerkin weighting scheme. The results say that linear polynomial correction is comparable to quadratic correction and we can use less number of nodes (e.g., one node per assembly) and larger time steps than those of the conventional nodal methods with acceptable accuracy. In this paper, we describe an AFEN kinetics formulation with Galerkin weighting considering space-dependent feedback. Linear variation of absorption cross sections is assumed to consider temperature feedback. The results on a test problem show that the correction is very effective in reducing the error.

II. Theory and Methodology

It starts from the two-group time-dependent diffusion equations and delayed neutron

precursor equations in the following :

$$[V]^{-1}\frac{\partial\vec{\phi}^n(\vec{r},t)}{\partial t} = \nabla\cdot[D^n(t)]\nabla\vec{\phi}^n(\vec{r},t) - [A^n(\vec{r},t)]\vec{\phi}^n(\vec{r},t)$$

$$+(1-\beta)\vec{\chi}_0[F^n(t)]^T\vec{\phi}^n(\vec{r},t) + \sum_{d=1}^{D}\lambda_d C_d^n(t)\vec{\chi}_d, \qquad (1)$$

$$\frac{\partial C_d^n(\vec{r},t)}{\partial t} = -\lambda_d C_d^n(\vec{r},t) + \beta_d[F^n(t)]^T\vec{\phi}^n(\vec{r},t), d = 1,\cdots,D,$$

where the notations are standard except the space dependent absorption cross section $[A^n(\vec{r},t)]$. A fully-implicit time approximation with analytic solution expression for the delayed neutron precursors assuming a linear variation of the reaction rate leads to

$$-\nabla\cdot[D^n(p+1)] + \vec{\phi}^n(\vec{r},p+1) + \left\{[A^n(\vec{r},p+1)] - \vec{\chi}_0(1-\beta)[F^n(p+1)]^T\right\}\vec{\phi}^n(\vec{r},p+1)$$

$$+[\gamma^n(p+1)]\vec{\phi}^n(\vec{r},p+1) = [\gamma^n(p)]\vec{\phi}^n(\vec{r},p) + \vec{S}(\vec{r},p), \qquad (2)$$

where

$$[\gamma^n(p+1)] = \left[\frac{1}{\Delta t}[V]^{-1} + \sum_{d=1}^{D}\vec{\chi}_d\beta_d\left\{\frac{1}{\Delta t\lambda_d}\left(1-e^{-\lambda_d\Delta t}\right)-1\right\}[F^n(p+1)]^T\right]$$

$$[\gamma^n(p)] = \left[\frac{1}{\Delta t}[V]^{-1} + \sum_{d=1}^{D}\vec{\chi}_d\beta_d\left\{\frac{1}{\Delta t\lambda_d}\left(1-e^{-\lambda_d\Delta t}\right)-1\right\}[F^n(p)]^T\right]$$

$$\vec{S}(\vec{r},p) = \sum_{d=1}^{D}\vec{\chi}_d\lambda_d e^{-\lambda_d\Delta t}C_d^n(\vec{r},p) - \sum_{d=1}^{D}\vec{\chi}_d\beta_d e^{-\lambda_d\Delta t}[F^n(p)]^T\vec{\phi}^n(\vec{r},p).$$

In this study, linear variation of absorption cross sections is assumed as follows :

$$[A^n(\vec{r},p+1)] = \begin{pmatrix} \Sigma_{a1}^n(\vec{r},p+1)+\Sigma_{1\to2}^n & 0 \\ -\Sigma_{1\to2}^n & \Sigma_{a2}^n(\vec{r},p+1) \end{pmatrix}$$

$$= \begin{pmatrix} \Sigma_{a1,0}^n(p+1)+\Sigma_{1\to2}^n & 0 \\ -\Sigma_{1\to2}^n & \Sigma_{a2,0}^n(p+1) \end{pmatrix}$$

$$+ \begin{pmatrix} c_x^{n,f}\dfrac{2x}{h_x^n}+c_y^{n,f}\dfrac{2y}{h_y^n}+c_z^{n,f}\dfrac{2z}{h_z^n} & 0 \\ 0 & c_x^{n,t}\dfrac{2x}{h_x^n}+c_y^{n,t}\dfrac{2y}{h_y^n}+c_z^{n,t}\dfrac{2z}{h_z^n} \end{pmatrix} \qquad (3)$$

$$= [A_0^n(p+1)] + [A_1^n(x,y,z,p+1)].$$

Eq. (2) is an inhomogeneous differential equation. We write the solution as a combination of analytic part $\vec{\phi}_a^n(\vec{r},p+1)$ and polynomial correction part $\vec{\phi}_c^n(\vec{r},p+1)$:

$$\vec{\phi}^n(\vec{r},p+1) = \vec{\phi}_a^n(\vec{r},p+1) + \vec{\phi}_c^n(\vec{r},p+1). \qquad (4)$$

We choose the analytic part of Eq. (4) to satisfy the steady state diffusion equation, and we

impose the following weighted residual equation on the polynomial part :

$$\int \omega(\vec{r})\{-\nabla \cdot [D^n(p+1)]\nabla + [A^n(\vec{r}, p+1)] - \chi_0[F^n(p+1)]^T\}\vec{\phi}^n(\vec{r}, p+1)dV$$

$$- \int \omega(\vec{r})\{[\gamma(p)](\vec{\phi}_a^n(\vec{r}, p) + \vec{\phi}_c^n(\vec{r}, p)) - (\vec{\chi}_0\beta[F^n(p+1)]^T + [\gamma(p+1)]) \quad (5)$$

$$\times (\vec{\phi}_a^n(\vec{r}, p+1) + \vec{\phi}_c^n(\vec{r}, p+1))\}dV - \int \omega(\vec{r})\vec{S}(\vec{r}, p)dV = \vec{0}.$$

Legendre polynomials are used both for the weighting function $\omega(\vec{r})$ and for the basis functions of $\vec{\phi}_p^n(\vec{r}, p+1)$. In contrast to the conventional nodal methods, in AFEN we know exactly all the terms in Eq.(5), except for the third integration term containing $\vec{S}(\vec{r}, p)$ which is small relative to the second integration term and includes previous time step variables. There is no approximation in $\vec{\phi}_a^n(\vec{r}, p)$ and $\vec{\phi}_a^n(\vec{r}, p+1)$ in the second term. Thus, we can use low order Legendre polynomials. In addition, when there is no transient, the second and third terms become zero, which makes the correction part go to zero and removes the null transient problem[4]. We use the coarse group rebalance (CGR) method[5] for its effective acceleration.

III. Numerical Results

To test the effect of space dependent cross sections, the 2-D LRA BWR problem[6] was investigated. This is a full-core BWR kinetics problem with two neutron energy groups and two delayed neutron precursor families. The problem has proven to be extremely difficult because it has severe flux tilts which occur during the transient and requires a highly accurate spatial neutronics model. Fig. 1 shows the geometrical configuration of the 2-D LRA BWR problem.
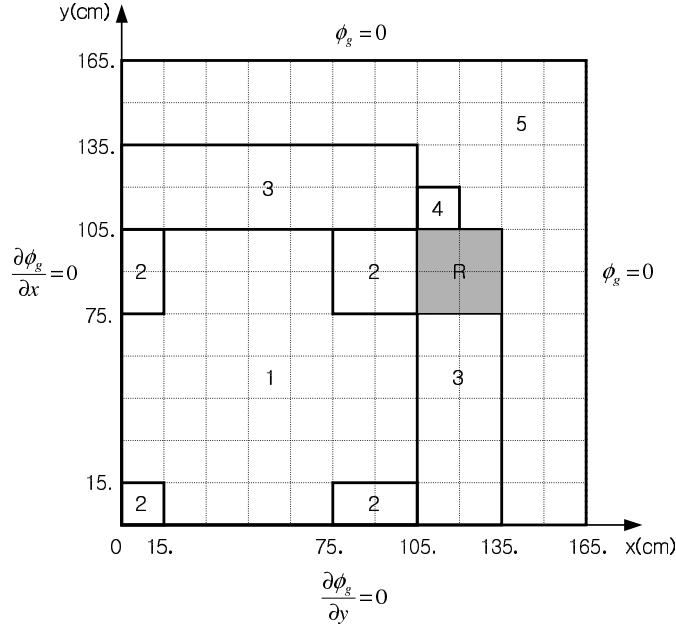

Figure 1: Core configuration of 2-D LRA BWR problem

In this problem, Smith[7] pointed out that temperature shape calculation is needed for coarse mesh models. Table 1 shows the results of the problem.

Table 1: Comparison and results of 2-D LRA BWR problem

	Case	$N_m^{(a)}$	k_{eff} at t=0	No. of time steps	P_{peak}(w/cc) (%error)	T_{peak}(s) (%error)	Computing time[b]
AFEN	$1^{(c),(d)}$	484	0.99637	2600	1.43475	5358	
	2	484	0.99637	2600	1.435(0.02)	5404(0.85)	
	$3^{(d)}$	121	0.99631	2600	1.43725(0.17)	5370(0.22)	551.61(481.16[e])
	4	121	0.99631	2600	1.4375(0.19)	5538(3.36)	508.20(461.92[e])
	$5^{(d)}$	121	0.99631	1000	1.4350(0.02)	5347(-0.20)	370.46(319.70[e])
	6	121	0.99631	1000	1.4355(0.03)	5514(2.91)	335.95(308.31[e])
CUBBOX[8]	7	121	0.99633	1200	1.421(-0.95)	5734(7.02)	
IQSBOX[8]	8	121	0.99631	522	1.445(0.71)	5451(1.74)	
Shober[6]	9	484	0.99636	2600	1.436(0.09)	5411(0.99)	
	10	121	0.99655	1000	1.426(-0.61)	5552(3.62)	
QUANDRY[7]	$11^{(f)}$	121	0.99641	329	1.429(-0.40)	5538(3.36)	
	12	121	0.99641	329	1.426(-0.61)	5699(6.36)	
CONQUEST[8]	13	121	0.99633	1000	1.437(0.16)	5505(2.74)	
SPANDEX[8]	14	121	0.99639	1579	1.441(0.44)	5490(2.46)	

(a) number of total nodes
(b) HP C180, source convergence criteria : 1.E-6
(c) reference
(d) spatial cross section gradient corrected
(e) update correction term per 5 outer iteration
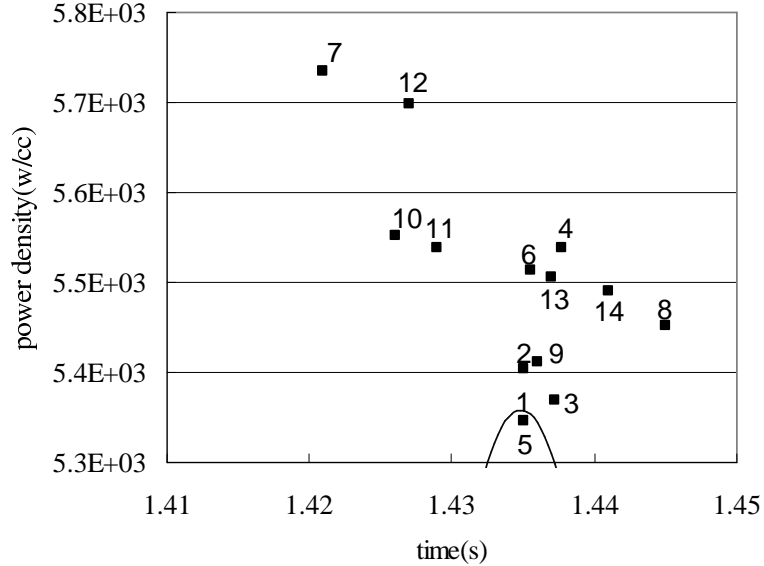(f) temperature shape calculated

Figure 2: Peak time and power of 2-D LRA BWR problem
(The numerals designate the cases in Table 1.)

Case 1 of fine node(484) and small time step(2600) with temperature feedback correction is used as reference. Case 9 of Shober[6](ANM, two step leakage approximation) has the same node size and time steps to those of case 2 in AFEN and they show nearly the same results. Cases 3 and 5 of AFEN improve the accuracy significantly by applying spatial cross section gradient than cases 4 and 6. In cases 1,3 and 5, with feedback correction, the errors in the peak power and time are less than 0.3%, which means that one node/assembly and 1000 time step calculation with space dependent feedback correction is comparable to the reference calculation. In fig. 2, although there are larger errors, cases 2,4 and 6 show higher accuracy than other nodal codes in comparison. Only the fine node small time step calculation of Shober has less than 1.0% error in the conventional nodal codes. Computing times of cases 3,4,5 and 6 show that no correction is about 10% faster than correction when we update at every nodal iteration. But the difference becomes less than 5% when we update per every 5 nodal iterations. Finally, we can say that it is very economical to use space dependent feedback with infrequent update of correction terms than using fine node calculation.

IV. Conclusions

We have extended the AFEN kinetics calculation method based on Galerkin weighting to treat the space dependent feedback. It is very easy to implement the correction in the Galerkin scheme (only the weighting of correction term is added). Similarly, all other types of space-dependent cross sections can be treated in the same procedure. The results show that the correction enhances the accuracy significantly and enables one node/assembly calculation in transient nodal calculation. The computing time increase is marginal, and it becomes negligible with infrequent update of the correction terms.

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