Analysis of Core Meltdown Energetics in KALIMER

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Abstract

Core disruptive accidents have been investigated at Korea Atomic Energy Research Institute(KAERI) as part of the work to demonstrate the inherent and ultimate safety of conceptual design of the Korea Advanced Liquid Metal Reactor(KALIMER), a 150 Mwe pool-type sodium cooled prototype fast reactor that uses U-Pu-Zr metallic fuel. In this study, a simple method was first developed using a modified Bethe-Tait method to simulate kinetics and hydraulic behavior of a homogeneous spherical core over the period of the super-prompt critical power excursion induced by the ramp reactivity insertion due to the fuel compaction. A set of test calculations was made and compared fairly well with the results of more detailed analysis by Hicks and Menzies for core meltdown energetics of the oxide fuelled fast reactor.

Calculations of the energy release during excursions in the sodium-voided core of the KALIMER were subsequently performed using the simple method for various reactivity insertion rates up to 200 $/s. Given the value of Doppler constants of the KALIMER(in the range of -0.001 to -0.02 at around the melting temperature of the fuel), power excursions were terminated without an energetic disassembly even for the extremely large reactivity insertion rates of 100$/s, which are considered the upper limit of ramp rates due to fuel compaction ; either the core does not reach the boiling temperature of the fuel , or the central core in small scale dispersal at low pressure.

1. Introduction

In early safety studies of small uranium metal reactors like EBR-II [1] and the Fermi Reactor[2], a sequence of super-prompt critical accident caused by fuel slumping in the sodium voided core, which is eventually terminated by disassembly of the core, was assumed to set the upper-bound design limits of containment systems. The analytic method used in the evaluation of this type of super-prompt critical core disruptive accident(CDA) in a fast reactor was originally developed by Bethe and Tait [3], further elaborated by Jankus [4].

Various studies afterwards have indicated that such a rate of reactivity insertion with coherent slumping of the whole core would be impossible. Moreover, molten fuel would move down through the lower structure, spreading widely into the lower plenum. It was shown that the most severe class of events that have the potential to develop into core disruptive accidents for the KALIMER are the unprotected transient overpower(UTOP) and the unprotected loss of flow(ULOF), which are extremely unlikely to occur and to be arrested should they occur[5]. It is estimated that,

depending upon the coherence of fuel slumping, a few tens of dollars per second of reactivity insertion rate would be theoretically possible in the steady-state core when fuel slumping starts. In this study, upper limit of reactivity insertion rate was set to be 100$/s to test the structural strength of the reactor system.

In an effort to evaluate the inherent safety of a conceptual design of KALIMER for core meltdown accidents, a simple analytic method has been developed using Bethe-Tait method. The two main modifications which have been made to the original method are the use of a more realistic equation of state of the fuel as well as the inclusion of the Doppler reactivity effect. The equations of state of the pressure-energy density relationship were derived for the saturated- vapor as well as the solid liquid of metallic uranium fuel, and implemented into the formulations of the disassembly reactivity. Mathematical formulations were then developed, in the framework of the modified Bethe-Tait method , in a form relevant to utilize the improved equations of state as well as to consider the Doppler effects, for a scoping analysis of the super-prompt-critical power excursions driven by a specified rate of reactivity insertion.

To test the accuracy of calculations with the simple method developed, a number of calculations were carried out and compared with a more detailed analysis results given in the work by Hicks and Menzies for oxide fuelled fast reactor[6]. Comparisons were made for the energy releases as a function of reactivity insertion rate and Doppler feedback, resulting in good agreement in most cases. Considering the uncertainties inherent in this kind of simple method, the extent of agreement was remarkably good. Calculations of the energy release during excursions in the sodium-voided core of the KALIMER were subsequently performed using the simple method for various reactivity insertion rates up to 200 $/s.

2. Method of Analysis

Basic Approach

It is assumed that the power excursion begins with the reactor prompt critical at time zero and the energy density generated during the excursion is governed by the reactor kinetics equation with no delayed neutrons and the source,

$$\frac{d^2Q}{dt^2} = \frac{k-1-\boldsymbol{b}}{\lambda}\frac{dQ}{dt} \tag{1}$$

where $Q(t)$ is the time dependence of the energy generation density. The other quantities in Eq.(1) is expressed in standard notation; k for multiplication constant, λ for prompt neutron lifetime, and $\boldsymbol{b}$ delayed neutron fraction. The neutron multiplication constant as a function of time may be expressed in the form

$$k(t) = k_0 + k_I(t) + k_d(t) + k_D(t) \tag{2}$$

where k_0 is the initial multiplication constant, $k_I(t)$ is the reactivity insertion responsible for initiating the excursion, $k_d(t)$ is the reactivity feedback resulting from material displacement during disassembly process, and $k_D(t)$ is the feedback from Doppler effect[7,8,9].

The rate of reactivity insertion initiating the excursion is assumed constant and $k_I(t)$ may be written as;

$$k_I(t) = [\frac{dk}{dt}]t = \boldsymbol{a}t \tag{3}$$

Applying the first-order perturbation theory to the one group diffusion equation, we obtain for a spherical reactor,

$$\dot{k_d} = \frac{48q^2 F / r_c}{4p\Sigma_{tr}n\Sigma_f b^7 [1-(6q/5)+(3q^2/7)]} \int p\,dV \qquad (4)$$

where Σ_{tr}, $n\Sigma_f$ are the transport and fission cross-sections, b is the core radius, F is fraction of fission in the core, r_c and p are density and pressure of the core, respectively. It was assumed that the flux can be approximated by a parabola in the core, $\Phi = 1 - q(r^2/b^2)$. Thus $\dot{k_d}$ is proportional to the pressure integrated over the volume of the core. The pressure-energy relations for the core during the power excursion are among the key parameters to be provided for the core disassembly process.

Meanwhile, the time rate of change of reactivity due to the Doppler effect can be expressed as

$$\frac{dk_D}{dt} = -(\frac{dk_D}{dT})_{T_0} (\frac{T_0}{T})^n \frac{dT}{dt} \qquad (5)$$

where $(dk_D/dT)_{T_0}$ is the Doppler temperature coefficient at temperature T_0. The Doppler effect is assumed to decrease in magnitude inversely as the nth power of the temperature T, measured from absolute zero.

Initial energy content $Q(0)$, initial power level $\dot{Q}(0)$, and $k(0)$ are the initial conditions to be specified for a set of the coupled equations in the above to have a unique solution. Starting with the initial conditions, the above equations can be numerically integrated using the Runge-Kutta method on a digital computer. The integration is continued until the reactor power falls below a preset value[13,14].

Reactivity Insertion and Initial Conditions

In the case that a ramp insertion of reactivity initiates the accident, an equivalent step insertion is frequently used in Bethe-Tait analysis. For the purposes of determining the equivalent step insertion, it is convenient to divide the power excursion into two phases. During the first phase, reactivity is added at an assumed constant rate and the power rises until the time t_1, when the total energy generated becomes sufficiently large to produce pressures that bring about significant material movement. Once the core begins to disassemble it goes very rapidly, and it is found that one can safely neglect any further addition of reactivity afterward. An asymptotic representation of the time t_1 as a function of Q is given as[9],

$$t_1 \approx \sqrt{\frac{\lambda}{a}}\sqrt{\ln X + \ln(\ln X)} \qquad (6)$$

where

$$X \equiv \frac{aQ^2(t_1)}{\lambda}[\dot{Q}(0)]^{-2} \qquad (7)$$

Total reactivity inserted by the ramp prior to the large pressure is then given by

$$k_1(t_1) = at_1 = \sqrt{a\lambda}\sqrt{\ln X + \ln(\ln X)} \qquad (8)$$

It is assumed in our study that t_1 comes when the fuel boiling occurs at the peak

power location of the core. For the initial conditions of Q and $\dot{Q}$, the core is assumed initially at prompt critical in the molten state. The initial energy content of the core, $Q(0)$, is taken to be the internal energy to heat uranium from room temperature to the melting point. A simple formula for the power at the prompt critical state, $\dot{Q}(0)$, brought by introducing reactivity at the constant rate of a dollars per second to an initially delayed critical reactor of the power level, $\dot{Q}_{ss}$, may be derived by solving the one-group point kinetics equations without reactivity feedback[10] ; $\dot{Q}(0) = \dot{Q}_{ss}(\dfrac{pb}{2a\lambda})^{1/2}$.

Another initial condition to specify is the multiplication constant at prompt critical, which is by definition of the prompt critical state, $k(0) = k_0 = 1 + b$. Since we are assuming in this study that the step reactivity, equivalent to the total reactivity inserted by the ramp during the excursion, is initially introduced, the initial multiplication constant is defined as

$$k(0) = k_0 + k_I(t_1) = 1 + b + at_1 \tag{9}$$

We may interpret that the total amount of reactivity insertion, $k_I(t_1)$, is the reactivity beyond prompt critical , which drives the power excursion along to its termination. Since the net reactivity is initially at its maximum and reduced with the negative reactivity feedback from the Doppler effect and /or core disassembly during the excursion, $k_I(t_1)$ is termed k_{max} in the following for clarity as well as for convenience.

3. Benchmark Analysis

Background

We made a series of simulations for the cases calculated by Hicks and Menzies, as a means to check the extent of accuracy or conservatism of our method, particularly the assumption of step reactivity insertion equivalent to ramp rate. Hicks and Menzies investigated various aspects of the course of events during and following a hypothetical meltdown accident in a sodium-cooled PuO_2 /UO_2 fuelled fast reactor. Calculations of the energy release during a super-prompt critical excursion were made for a spherically symmetric sodium-voided core using the PHOENIX program, which retains the essential features of the Bethe-Tait Model. An extensive set of density-dependent equations of state for temperature and energy density as well as pressure and energy density was developed for the fuel assumed to be UO_2. The Doppler constant was estimated to be –0.24 % . In view of the uncertainties in Doppler estimates, the energy release was calculated for a series of values in the range from 0 to –0.5%. Other reactor parameters used in the calculations of energy release were[6]:

Prompt-neutron lifetime $=$ 10^{-7} sec
Delayed-neutron fraction $=$ 0.0033
Initial peak power density $=$ 100 W/g
Equivalent spherical core radius $=$ 65 cm
Actual core mass $=$ 4 tonnes of oxide fuel

For our benchmark calculation, typical values of oxide-fuelled core were assumed for the reactor parameters not available in the report by Hicks and Menzies, such as power distribution, material worth and reactor composition. As for the pressure-energy density relationship at constant volume, the one developed by Hicks and Menzies for the saturated vapor plus the single-phase region with the specific reduced volume of about

0.7 was curve-fitted to a fourth-order polynomial series ;

$$p = \sum_{i=0}^{4} B_i E^{i} \tag{10}$$

where

$B_0 = -8.725 \times 10^{-2}$, $\quad B_1 = 1.047 \times 10^{-1}$, $\quad B_2 = -1.530 \times 10^{-3}$,
$B_3 = -3.625 \times 10^{-2}$, $\quad B_4 = 1.107 \times 10^{-2}$,

and pressure and energy are in the units of Kbar and KJ/g. The curve, fitted by Eq.(10), starts from the boiling point of uranium oxide Q_b, which is assumed to be 1.5KJ/g.

The results of these calculations are summarized for several reactivity insertion rates in Table 1, which compares the peak values of the energy density at the core center with those given by Hicks and Menzies. It may be noted from the fourth column of the table that our method, using the values of t_1 and k_{max} given in Equations(6), (7), and (8), consistently predicts higher values of energy release than those by Hicks and Menzies, This trend is particularly pronounced for the cases of higher reactivity insertion rates, where more than two times of energy releases are predicted.

Table 1. Comparison of Energy Densities at Center of Oxide-Fuelled Core

Ramp Rate ($/s)	Doppler Constant (a_D)	Energy Density at Core Center(KJ/g)		
		Hicks & Menzies	Our Methods	
			k_{max} Evaluated by Eq.(8)	k_{max} Adjusted to Time of Boiling
	0.0	3.52	7.25	3.47
75	-0.001	1.81	4.10	1.63
	-0.002	1.71	2.42	1.57
	0.0	4.11	8.82	3.96
150	-0.001	1.99	5.30	1.76
	-0.002	1.90	3.55	1.70
	0.0	4.85	10.84	4.38
300	-0.001	2.82	6.98	1.99
	-0.002	2.39	4.69	1.88

The trend of overestimates of our method for energy release mostly comes from the conservatism put into estimating the amounts of step reactivity equivalent to the ramp rates. It was observed from our calculations that there exist significant differences between the asymptotic values of t_1 , time of fuel boiling at the center of the core obtained per Equations (6) in the above, and the actual values of time of boiling, t_b , which resulted from our analyses of the excursions(using k_{max} computed from t_1 by Eq.(8) as an initial condition for each case). The values of t_1 was much larger than t_b , in the range of about 50 times depending on the ramp rates and Doppler constants assumed for the study. Being converted into the inserted reactivity by Eq. (8), this results in the much overestimated values of k_{max} , which comes to drive power excursions much more severely in our calculations. We subsequently adjusted the input values of t_1 such that we get the values of t_b as t_1 close to each other for each case of excursions. Table 2 shows a comparison of the values obtained using Equations (6), (7) and (8) and adjusted values of them. We can see that the time of boiling and, hence, amounts of inserted reactivity were reduced by an order of magnitude from the

initial estimates by asymptotic solutions of them. The adjusted values of boiling time are in the range of a few tenth millisecond agreeing with the general trends of super-prompt critical accidents in oxide fuelled cores.

Table 2.Parameters for Determining Reactivity Insertions

a ($/s)	Values Given by Eqs(6),(8)		Adjusted Values	
	t_1 (msec)	k_{max} ($)	t_1 (msec)	k_{max} ($)
75	2.62	0.196	0.35	0.0263
150	1.93	0.290	0.24	0.036
300	1.42	0.426	0.18	0.054

   In the last column of Table 1 listed are the results for the values of the energy density at the center of the core resulted from using the adjusted values of step reactivity. As shown, the results are in good agreement with those of Hicks and Menzies being within about 10 % in cases of ramp rates of 75 and 150 $/s, which are in the range of our design-basis ramp rates. For the cases of higher ramp rates, our method with adjusted values of reactivity tends to give lower values of energy density, amounting as much as 40 % less for 300 $/s-case, than those of Hicks and Menzies. Such an agreement appears fairly acceptable, considering the uncertainties involved in these kinds of hypothetical accidents, including the high-temperature material properties, equations of state and reactor parameters, among others. Even the results of energy density obtained using the asymptotic values of step reactivity equivalent to assumed ramp rates render values two times higher than those of more detailed analyses by Hicks and Menzies and may be useful for the conservative estimate for the accident energetics.

4. Analysis of Core Meltdown Accident in KALIMER
Reactor Model

   The KALIMER core system is designed to generate 392MWt of power. The reference core utilizes a heterogeneous core configuration with driver fuel and internal blanket zones alternately loaded in the radial direction. The core consists of 48 driver fuel assemblies, 18 internal blankets, 6 control rods, 1 ultimate shutdown system(USS) assembly, 6 gas expansion modules (GEMs), and is surrounded by layers of radial blankets, reflectors, shield assemblies, and in-vessel storage of fuel assemblies, in an annular configuration. There are no upper or lower axial blankets surrounding the core. The reference core has an active core height of 120 cm and a radial

Table 3. Reactor Parameters of KALIMER

Reactor Parameters	U-Pu-Zr Core
$\Sigma_{tr} \, n\Sigma_f$	1.030×10^{-3}
q	0.6
λ(sec)	2×10^{-7}
b	0.035
b (cm)	77.8
Volume Fraction(%)	
Fuel Slug	29.75
Coolant	42.91
Structure	27.34
Core Density	
r_{hyd} (g/cm^3)	6.64
r_c (g/cm^3)	4.70
Fuel Loading(MT)	9.23

equivalent diameter(including control rods) of 172 cm[11].

The driver fuel assembly includes 271 fuel pins. The fuel pins are made of sealed HT-9 tubing containing metal fuel slug of U-Pu-10%Zr in columns. The driver fuel and blanket have smeared densities of 75% and 85%, respectively. The power fractions for the driver fuel, inner blanket and radial blanket at the beginning of the equilibrium cycle (BOEC) are 0.773, 0.093 and 0.121, respectively. The power fractions of the internal blankets significantly increase with burnup and, consequently, the location of the peak linear power shifts from the inner driver fuel zone to the innermost internal blanket region. The peaking factor is close to 1.5, which provides a basis for using the power-shape factor q of 0.6 in this study. The peak linear power is 286.5 W/cm, which is equivalent to a specific power of about 60 W per gram of fuel. Table3 lists the KALIMER reactor parameters used in this study for the base cases.

Thermal Properties and Equations of State

The core is assumed to be initially at prompt critical in molten state. Initial energy content of the core, Q_0, is therefore taken to be 0.25 KJ/g , the internal energy to heat uranium from room temperature to the melting point(1,400 K). The boiling temperature of the core is set to be around 4,500 K and the corresponding energy to be 0.8 KJ/g. The threshold energy for the transition to the solid liquid phase of the core is set to be 1.44 KJ/g. The specific heat of metallic fuel is assumed to be close to 0.2 J/g-K just above the melting point of 1,400 K and assumed to stay constant beyond. The basis for a equation of state for saturated-vapor pressure for the core is that given for uranium[12] ;

$$\log p\,(atm) = 5.702 - (23,300\,/T\,) \tag{12}$$

,where T is the fuel temperature in K. This pressure-temperature relation was converted to that of pressure and energy density, which was then curve-fitted to a fourth-order polynomial[13,14]. The equation of state in the single-phase region is given as

$$p = 5,939(E - 1.44) \tag{13}$$

where pressure is measured in Mpa and the energy in KJ/g.

The fuel temperature (Doppler) coefficients are evaluated for sodium-flooded/voided cases. It is estimated to vary as $0.11\,T^{-1.49}$ for the sodium-voided case, whereas it varies as $0.10\,T^{-1.43}$ in the case of the sodium-flooded core. The Doppler coefficient does not show any substantial change with burnup. Taking into account some uncertainty with the correlation for Doppler coefficients, –0.002 is taken as the best-estimate value of the Doppler constant for the subsequent analyses for the sake of conservatism.

Analysis Results

Table 4 shows the results for the peak values of energy generation density, temperature and pressure for reactivity insertion rates in the range up to 200 dollars per second, with three different values of Doppler constants considered; a_D = 0(no Doppler effect), –0.001 and –0.002.

We can see from the table the significant influences of the Doppler effect on the power excursion. The effects are more pronounced with the excursions initiated by low rates of reactivity insertion. It should be noted that the peak pressure rises dramatically

drop down as the Doppler constant increase. For the Doppler constant of -0.002 taken as the reference value for KALIMER in this study , the power excursions are terminated even before the core reaches the assumed energy density of the boiling point(0.8KJ/g) for reactivity insertion rates up to 200 $/s, resulting in shutdown of the reactor without any significant energy release.

Table 4. Calculation of Energy, Temperature and Pressures at Core Center for Various Doppler Constants and Reactivity Insertion Rates

Ramp Rate ($/s)	Doppler Constant (a_D)	Peak Values at Core Center		
		Energy Density(KJ/g)	Temperature (K)	Pressure (bar)
50	0.0	1.46	8,130	4,100
	-0.001	0.56	3,180	1
	-0.002	0.44	2,450	1
100	0.0	1.68	8,550	14,100
	-0.001	0.92	4,750	70
	-0.002	0.68	3,550	1
150	0.0	1.79	9,100	20,800
	-0.001	1.06	5,450	215
	-0.002	0.74	3,850	1
200	0.0	1.89	9,600	28,000
	-0.001	1.16	5,950	470
	-0.002	0.78	4,050	1

Even with the Doppler constant assumed to be –0.001, the energy densities at the peak location of the core go over the boiling point a bit but stays well below the threshold value of the solid liquid region, which is assumed 1.44KJ/g in this study. For the reference case of 100/s ramp rate, for instance, we get the peak value of energy density of 0.92 KJ/g. Hence only the peak spot of the core would boil, whereas most area of the core would be in the pre-boiling liquid state. As the fuel vapor generated there fill some of the voids left out of sodium coolant, the pressure gradually rises, while the power ,after reaching the maximum early on, continues to be in decline under the influence of Doppler feedback. The core dispersion would be then with the fuel of low energy density driven by much lower pressure acting for much longer times, the extent of which depends on the specific reactivity insertion rates. The results were not quite sensitive to such parameters of large uncertainties as equation of state and specific heat of the core, and other reactor parameters.

5. Conclusions

A simple method was developed in the framework of the Modified Bethe-Tait method utilizing the improved equations of state as well as considering the Doppler effects, for a scoping analysis of the super-prompt-critical power excursions driven by a specified rate of reactivity insertion. To test the accuracy of calculations with the method developed, comparisons were made for the energy releases as a function of reactivity insertion rate and Doppler feedback with a more detailed analysis results given in the work by Hicks and Menzies for oxide fuelled fast reactor. Agreements were within about 10 % in cases of ramp rates of 75 and 150 $/s, which are in the range of

our design-basis ramp rates of 100$/s..

Calculations of the energy release during excursions in the sodium-voided core of the KALIMER were subsequently performed using the simple method for various reactivity insertion rates up to 200 $/s. Given the best-estimate value of Doppler constants , power excursions were terminated without an energetic disassembly even for the extremely large reactivity insertion rates of 100$/s.

Acknowledgements

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