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Applicability of Point Reactor Kinetic Equations for Subcriticality Monitoring

Yoichiro Shimazu, Masashi Tsuji

Hokkaido University
Kita-13, Nishi-8, Kita-ku
Sapporo, 060-8682, Japan

Abstract

Conventional digital reactivity meter is based on a simple principle to solve inverse point reactor kinetic equations and thus the system can be installed even on a PC. It is desirable to use a simple system as much as possible. From this point of view, feasibility was studied for a conventional digital reactivity meter to be used as a subcriticality monitor. There are a few difficulties to be solved for a digital reactivity meter to be used for subcriticality monitoring. For example, the applicability of the point reactor kinetic equations must be verified for the system where neutron distribution is dependent on the subcriticality. From this point of view, a numerical investigation was made for the applicability of the point reactor kinetic equations for the subcriticality monitoring.

1. Introduction

The criticality accident broke out in Tokai-mura in 1999 has shown impressively that certain subcriticality monitoring device is essential in nuclear facilities. In order to develop such monitors there have been many studies using several methods such as Feynman- α [1] and neutron noise analysis [2] [3].
With the recent development of computer technology, these methods can be used in an on-line real-time monitor. However, these methods require analysis of certain length of time sequence data to estimate the subcriticality, thus the monitor using these methods is not a continuous monitor in that sense.

On the other hand a digital reactivity meter can continuously give the real-time reactivity[4]. However it has not been used as a subcriticality monitor. The reasons are supposed to be as follows. The most important issue is the applicability of the point reactor kinetic equations for the analysis of subcritical systems. It is not well known about the applicability of the inverse point reactor kinetic equations to the system where the neutron flux distribution is dependent on the subcriticality. The other issues are thought to be as follows:

1) In the environment where subcriticality is measured, it is normally expected that the neutron flux level is quite low. Therefore the fluctuation of the neutron signal is quite large, which makes the calculation of reactivity using the inverse kinetic equations difficult.
2) In order to solve the inverse kinetic equations for a subcritical system with neutron source, it is inevitable to know the accurate strength of the neutron source of the system. But this is usually quite difficult to be done.

When these difficulties are solved a digital reactivity meter can be used as a subcriticality monitor. From this point of view, in this paper, we investigated the applicability of the point reactor kinetic equations to subcritical systems by numerical analysis.

2. Applicability of the inverse kinetic equations

The point reactor kinetic equations are derived on the assumption that the neutron flux distribution is not dependent on the reactivity. As the reactivity meter solves the inverse kinetic equations, it cannot take into account of the effect of the neutron flux distribution change in the calculation of the reactivity. On the other hand, it is known that the flux distribution in the subcritical system with the neutron source is dependent on the system subcriticality. However the sensitivity of the effect on the calculated reactivity is not well known from the subcriticality monitoring point of view.

For a critical reactor, the theoretical analysis has been given, for example, in reference (5). It is shown that if the reactor is initially critical and operating in the fundamental mode and becomes super critical or subcritical as the result of a uniform change in the system, the reactor will remain in the fundamental mode. Thus the assumption for the point reactor kinetic equations holds. It means that the neutron flux for the calculation of the reactivity can be taken at any position in the reactor system. It is not true for the subcritical systems with neutron source where the neutron flux is composed of not only the fundamental mode but also of the modes due to the neutron source. In order to calculate reactivity correctly, we have to obtain the information of neutron flux, which is dependent only on the subcriticality as much as possible.

From this point of view, we studied the relationship of the neutron flux distribution and the subcriticality using a simple one dimensional reactor model. For a small system, we used the C35 critical assembly of Research Reactor Institute, Kyoto University[6)] as the reference system. The core size is 28cm wide, 36cm long and 57cm high with water reflector. Transverse bucking treats the neutron leakage for the transverse direction. Two group nuclear constants are those given for the critical assembly as listed in Table 1. The neutron source is assumed to be one distributed homogeneously in the reactor or a plane at the center of the reactor. The subcriticality is adjusted by changing the absorption cross-section. For a larger system, we used the same nuclear constants but changed the core size to be 6m by 6m by 6m with water reflector. The other assumptions are the same as above except using $\nu\Sigma_{2f}$ of 0.10.

Table 1 Nuclear constants of C35 critical assembly

Zone	Fast group			Thermal group			Fast fission factor	Reflector saving (cm)
	D_1 (cm)	Σ_{1a} (cm^{-1})	$\Sigma_{1\to 2}$ (cm^{-1})	D_2 (cm)	Σ_{2a} (cm^{-1})	$\nu\Sigma_{2f}$ (cm^{-1})		
Core	1.54	0.00286	0.0212	0.237	0.0850	0.149	1.092	8.2
Reflector	1.41	0.0	0.0476	0.117	0.0191	0.0	---	---

The results for a small system for the homogeneous source are shown in Figs. 1 and 2 for the normalized fast group neutron flux and the normalized thermal group neutron flux for shallow subcriticality, respectively. Note that the normalized flux means that the flux is multiplied by the reactivity. The strength of the neutron source is fixed as a constant. The system with a plane source and a larger system with homogeneous source, which case will be expected in a spent fuel pit, are also discussed.

From these results, the following facts are found.

(1) For the system with homogeneous neutron source, the normalized thermal flux distributions stays almost constant in the reactor region. The normalized fast flux varies with the reactivity but the variation could be tolerable for subcriticality monitoring as shown in Fig. 1.

(2) For the system with a plane source at the center, the normalized fast flux near the reactor peripheral stays constant. The normalized thermal flux varies with the reactivity but it has a stationary point in the reactor region.

(3) For the larger system with homogeneous source, the normalized flux near the center of the system stays almost constant for both fast and thermal groups.

When the shape function changes with subcriticality, for example, which can be seen in Fig.1, the reactivity calculated by the reactivity meter is not valid. The reason is as follows. As explained later, the neutron source strength is estimated at the initial condition based on the assumption that the shape function is constant, which assumption is not valid when the neutron distribution has been changed. However when the normalized flux increases with reactivity, the reactivity meter gives safer estimation of the reactivity for the subcriticality monitoring because the higher flux gives higher reactivity.

Based on these facts we discuss the applicability of the reactivity meter for monitoring the subcriticality.

For the point reactor model, the neutron flux $\Phi(r,\Omega,E,t)$ is expressed by a product of an amplitude factor $P(t)$, which is dependent on time only, and a shape factor $f(r,\Omega,E)$; thus

$$\Phi(r,\Omega,E,t) = P(t)\ f(r,\Omega,E) \tag{1}$$

The point reactor model is obtained when the time dependence of the shape function is taken to be independent on time. The amplitude function $P(t)$ is determined by the reactivity of the system, which the reactivity meter uses to calculate the reactivity. In other words when the shape function at the detector position is independent on time, the reactivity meter can calculate correct reactivity using the detector signal.

In the point reactor kinetic equations for the subcritical system, the amplitude is inversely proportional to the reactivity. Hence when the product of steady state flux and reactivity, $\boldsymbol{r}$, stays constant, then the reactivity calculated using this neutron flux is valid. In other words, the reactivity meter based on the point reactor kinetic equations can be used for the reactivity calculation when the neutron detector is positioned where the product of the steady state flux and the reactivity stays constant.

The other issues can be solved in the following manner.

1) Fluctuation of Neutron Flux Level

The fluctuation of the neutron flux level is random around the average. Such fluctuation can be filtered with a simple averaging or using a filter of first order delay. The former filtering can be done automatically in the nuclear instrumentation when the signal is obtained as the integral of the raw signal during some time interval. It is pointed out that when we use a filter of first order delay, the response of the reactivity is also delayed. But the delay is of the order less than 5 times of the time constant of the filter. The time constant of a few seconds is good enough so that the reactivity response delays only 10 to 20 seconds.

2) Unknown Neutron Source Strength

The point reactor kinetic equations are expressed by one point reactor model as follows. For the simplicity one group delayed neutron model is used in the discussion. In the actual application, the model of six-delayed-neutron-group is used for the reactivity calculation in the reactivity meter.

$$\frac{dn}{dt} = \frac{\boldsymbol{r} - \boldsymbol{b}}{\ell} n + \boldsymbol{l} C + S\ , \tag{2}$$

$$\frac{dC}{dt} = \frac{\boldsymbol{b}}{\ell} n - \boldsymbol{l} C\ . \tag{3}$$

When the source strength is not available but the stable initial neutron signal is available, then we can solve the equations in the following manner. The stable neutron signal and the corresponding reactivity $\boldsymbol{r}_0$ have the following relation.

$$n_0 = -\frac{\ell S}{\boldsymbol{r}_0}\ , \tag{4}$$

When we can estimate the initial system reactivity, r_0, with a reasonable accuracy, we can use this information in place of the actual neutron source strength. The estimation of the initial reactivity with a reasonable accuracy is not so difficult using the state-of-the-arts nuclear calculation codes. It is known that even the estimation has an error, the effect of which is not so fatal for subcriticality monitoring.

3. Conclusion

We have shown that a reactivity meter based on the point reactor kinetic equations can be used as a subcriticality monitoring when the neutron detector is placed in an appropriate position. The concept requires an estimation of the initial reactivity but does not require information of accurate neutron source strength. The reactivity meter can monitor the subcriticality reasonably on the real-time basis. With the actual value of the real-time subcriticality, the monitoring criteria and also the monitoring system can be easily established.

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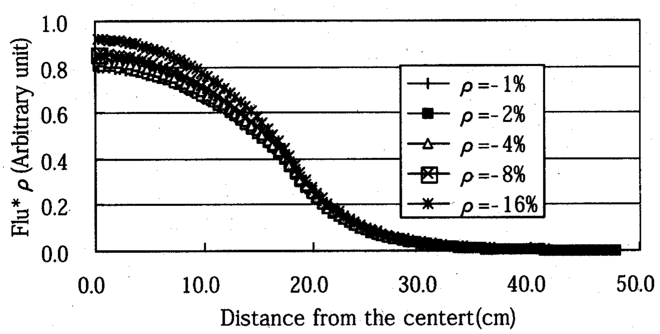

Fig. 1 Normalized fast group neutron flux distribution with homogeneous neutron source

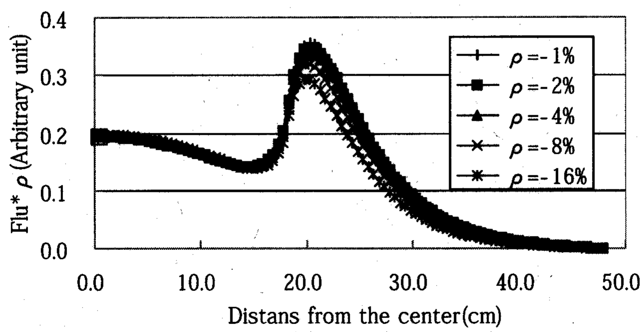

Fig. 2 Normalized thermal group neutron flux distribution with homogeneous neutron source