Experimental Characterization of MHD Pressure Drop of Liquid Sodium Flow under Uniform Magnetic Field

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Abstract

Magnetic field has many effects on the hydraulic pressure drop of fluids with high electrical conductivity. The theoretical solution about MHD pressure drop is sought for the uniform current density model with simplified physical geometry. Using the MHD equation in the rectangular duct of the sodium liquid flow under a transverse magnetic field, the electrical potential is sought in terms of the duct geometry and the electrical parameters of the liquid metal and duct material. By the product of the induced current inside the liquid metal and transverse magnetic field, the pressure gradient is found as a function of the duct size and the electrical conductivity of the liquid metal. The theoretically predicted pressure drop is compared with experimental results on the change of flow velocity and magnetic flux density.

I. Introduction

Liquid metal sodium with a high electrical conductivity is used as a coolant in Liquid Metal Reactors of the KALIMER kind.\textsuperscript{[1]} In the KALIMER, the pressure drop of the liquid sodium flow can be strongly affected by the external magnetic field. Various works have been carried out for the circulation systems that use Li or NaK.\textsuperscript{[2-5]} Practically, pressure drop is predicted to be caused by the duct system of non–magnetic stainless steel in addition to liquid sodium itself. Generally, a pressure drop negatively affects the hydraulic system. Electrically conducting fluid can experience a pressure drop by the magnetic field induced by devices such as an EM flowmeter or an EM pump in the Liquid Metal Reactor. Then, a pressure drop caused by the magnetic field becomes a function of duct geometry and electrical parameters. In the present study, theoretical calculations are worked out with the uniform current
model on the rectangular flow section. It is analyzed under the reasonable assumption that the pressure drop by wall friction is negligible compared to that by MHD action in the strong magnetic field. In this kind of flowing with a Hartmann number $\gg 1$ the viscous term in the Navier–Stokes equation is negligibly small in comparison with electromagnetic force term and thus pressure drop ($\nabla p$) becomes $\nabla p = J \times B$ for inviscid flow where $J$ is the uniform current density under the D. C. magnetic flux density ($B$). In the present study, the MHD pressure drop is theoretically analyzed and its experimental characterization is carried out.

II. Formulation

Fig.1 for the present experimental device shows a cross-section of the rectangular duct section with a transverse uniform magnetic field, and Fig.2 is an illustrative diagram for the analysis based on the uniform current density model. Along the upper wall, current and voltage are given as follows.$^{[4]}$

\[ I(x) = J x = \frac{Jx}{a} \]  
\[ V(x) = V_0 - \frac{1}{2} IR_s \left( \frac{x}{a} \right)^2 \]

In the sodium flowing, voltage is obtained from Ohm’s law.

\[ V(x) = u(x) B b - IR_i \]  

Combining Eqs. (2) and (3) gives velocity distribution.

\[ u(x) = u_0 - k_u (x/a)^2 \]

where $u_0 = (V_0 + IR_i)/B$ and $k_u = IR_s/2Bb$

And the average velocity over the flow channel is given by integration from 0 to a.

\[ U = u_0 - k_u/3 \]

Therefore,

\[ UBb = (V_0 + IR_i - IR_s/6) \]  

Along the side walls, applying Kirchhoff’s law shows the following relation.

\[ V_0 = \frac{IR_d}{2} + IR_w \]  \hspace{1cm} (7)

By eliminating \( V_0 \) from Eqs. (6) and (7), the current density is obtained as follows:

\[ J = \frac{UB_b}{\alpha(R_w + R_{e}^3 + R_d^3)} \]  \hspace{1cm} (8)

Therefore, pressure gradient is calculated by Lorentz’s product:\textsuperscript{[2-5]}

\[ -\frac{dp}{dz} = JB = K_p\sigma UB^2 \]  \hspace{1cm} (9)

\[ K_p = C/(1 + a/3b + C) \]  \hspace{1cm} (10)

(Ref. \( K_p = C’/(C’ + 1) \) in circular geometry)

where \( C = \sigma_w a_w/\sigma_\alpha \) and \( C’ = \sigma_w(R_0^2 - R_d^2)/\sigma_\alpha(R_0^2 + R_d^2) \). In Eq. (10), the pressure gradient is known to be proportional to the velocity and squared magnetic flux density. Practically, in Miyazaki’s NaK blowdown experiment\textsuperscript{[2-5]}, \( K_p \) has experimental values of 0.0819 and 0.04 each in the rectangular (aspect ratio = 0.451) and circular geometry (inner radius = 22.65mm and outer radius = 24.3mm) where the range of the operating magnetic flux density is 0.3 – 1.75T and that of velocity is 2–15m/sec. He also shows good agreement with the theoretical prediction.

**III. Experiment and Results**

In Fig. 3, the differential manometer for this experiment is shown. Its measurable pressure range is 600kPa in maximum and it can be available till 600°C. Its diaphragm is made of the stainless steel mesh and special oil compatible with a chemically reactive sodium. Fig. 4 shows the MHD experimental loop with the differential manometer (No. 12 in the Fig. 4). Firstly, the loop and storage tank (No. 1 in the Fig.4) are pre-heated up to 150°C for the melting of the solid sodium. After pre-heating, the loop is full of the liquid sodium by the argon gas blow and the liquid sodium is circulated by the pumping of an EM pump (No. 6 in the Fig. 4)
retaining the constant temperature of 150°C. Then, pressure difference between inlet and outlet of MHD test section (No. 10 in the Fig. 4) is measured on the change of the sodium flow velocity and magnetic field.

Fig. 5 and Fig. 6 show the comparison of the predicted and measured pressure drop. In Fig. 5, the pressure difference is shown to be proportional to the square of the magnetic field. In Fig. 6, the pressure difference is proportional to the sodium flow velocity. In Fig. 5 and Fig. 6, it is thought that the measurement is consistent with the theoretical prediction in a tendency although there are numerical errors between the prediction and measurement. On the other hand, pressure coefficient, $k_p$, $K_p$, is calculated to be approximately 0.1 from the experiment and it is 33% smaller compared with the theoretically predicted one, $K_p$ of 0.15.

IV. Conclusion and future plan

The pressure drop in the rectangular channel with a uniform magnetic field has been found as a function of duct geometry and magnetohydrodynamic parameters using the uniform current model. Theoretical prediction has been carried out for some variables, and it was shown that pressure drop was given as the value of the developed electromagnetic force multiplied by the coefficient, $K_p$. Experimental characterization has been performed on the change of the sodium velocity and magnetic field, and compared with the theoretical prediction. It was shown that the MHD pressure drop, $-\Delta p = \int K_p \rho U B^2 dz$, was significantly reduced because of small $k_p$.

- Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$J$</td>
<td>Current Density</td>
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<tr>
<td>$I$</td>
<td>Total current per unit length (=Ja)</td>
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<tr>
<td>$V_0$</td>
<td>Half of inter-upper wall voltage</td>
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<td>$B$</td>
<td>Magnetic flux density</td>
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<td>$p$</td>
<td>Pressure</td>
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<tr>
<td>$u$</td>
<td>Fluid velocity</td>
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<td>$U$</td>
<td>Mean velocity</td>
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<tr>
<td>$a$</td>
<td>Half length of upper wall</td>
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<tr>
<td>$b$</td>
<td>Half length of side wall</td>
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\( t_w \) : Thickness of duct wall
\( \sigma_f \) : Electrical conductivity of the sodium
\( \sigma_w \) : Electrical conductivity of the duct material
\( R_e \) : Resistance of upper wall \((=a/\sigma_w t_w)\)
\( R_w \) : Resistance of side wall \((= b/\sigma_w t_w)\)
\( R_i \) : Resistance of fluid \((= b/\sigma_f d)\)

V. Acknowledgements

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VI. References

Fig 1. Cross-sectional view of test section

Fig 2. Illustrative diagram for theoretical analysis
Fig 3. Schematic diagram of a differential manometer for liquid sodium

Fig 4. Experimental loop for the measurement of MHD pressure drop
Fig. 5. Pressure drop on the change of magnetic field at the different flow velocities

Fig. 6. Pressure drop on the change of flow velocity at the different magnetic fields