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MPC-Based Auto-tuned PID Controller for the Steam Generator Water Level

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ABSTRACT

In this work, proportional-integral-derivative (PID) control gains are automatically tuned by using a model predictive control (MPC) method. The MPC has received much attention as a powerful tool for the control of industrial process systems. An MPC-based PID controller can be derived from the second order linear model of a process. The steam generator is usually described by the well-known 4^{th} order linear model which consists of the mass capacity, reverse dynamics and mechanical oscillations terms. But the important terms in this linear model are the mass capacity and reverse dynamics terms, both of which can be described by a 2^{nd} order linear system. The proposed auto-tuned PID controller was applied to a linear model of steam generators. The parameters of a linear model for steam generators are very different according to the power levels. The proposed controller showed good performance for the water level deviation and sudden steam flow disturbances that are typical in the existing power plants by changing only the input-weighting factor according to the power level.

1. INTRODUCTION

The water level of a nuclear steam generator must be properly controlled in order to secure the sufficient cooling water for removing the primary heat and to prevent the damage of turbine blades. The inadequate and insufficient performance of the conventional controllers for the steam generator water level has often resulted in reactor trip (shutdown) and enforced nuclear plant operators to hang on manual operation at low power (mainly, at startup time of a nuclear power plant). Also, the non-minimum phase effects are significantly greater at low power, which makes more dangerous the use of a high gain of the control loop at a reduced power level. Even to a skilled operator, therefore, it is difficult to effectively cope with the swell and shrink phenomena of the water level, which is induced by the non-minimum phase effects. Also, the steam generator is highly complex, non-linear, and time-varying system. Particularly, its parameters change largely according to changes in operating conditions [1]. The steam generator with narrow stability margin cannot work satisfactorily with PI controller gains fixed over all power levels. Therefore, many advanced control methods that include adaptive controllers [1-2], optimal controllers [3-4], fuzzy logic controllers [5-8], and a model predictive controller [9] have been suggested to solve the water level control problem of nuclear steam generators. Also, the gain-scheduled PI controllers of which the gains are defined differently according to several fixed power levels have been implemented and used at some nuclear power plants.

The model predictive control methodology has received much attention as a powerful tool for the control of industrial process systems [10-15]. The basic concept of the model predictive control is to solve an optimization problem for a finite future at current time and to implement the first optimal control input as the current control input. That is, at the present time k the behavior of the process over a horizon N is considered and the process output to changes in the manipulated variable is predicted by using a mathematical design model. The moves of the manipulated variables are selected such that the predicted output has certain desirable characteristics and only the first computed change in the manipulated variable is implemented. The procedure is then repeated at each subsequent instant. This method has many advantages over the conventional infinite horizon control because it is possible to handle input and state (or output) constraints in a systematic manner during the design and implementation of the control. In particular, it is a suitable control strategy for time varying systems.

The purpose of this paper is to design an automatic controller for the steam generator water level without any manual operation from start-up to full load transient conditions. We are familiar with the conventional PID controller but have to solve the problems mentioned above. Therefore, in this work the PID gains are automatically tuned by applying an MPC methodology and using a reduced 2^{nd} order linear steam generator model at each power level. The proposed control method is applied to a linear model [1] of steam generators.

2. MODEL PREDICTIVE CONTROL METHOD

Model predictive control is a popular technique for the control of slow dynamical systems. At every time instant, model predictive control requires the on-line solution of an optimization problem to compute optimal control inputs over a fixed number of future time instants, known as the time horizon. The on-line optimization can be typically reduced to either a linear program or a quadratic program. The basic idea of model predictive control is to calculate a sequence of future control signals in such a way that it minimizes a multistage cost function defined over a prediction horizon. The associated performance index is the following quadratic function:

$$J = \frac{1}{2}\sum_{j=1}^{N}\left[\left(\hat{y}(t+j\,|\,t) - w(t+j)\right)^T Q\left(\hat{y}(t+j\,|\,t) - w(t+j)\right)\right] + \frac{1}{2}\sum_{j=1}^{M}\left[\left(\Delta u(t+j-1) - \Delta v(t+j-1)\right)^T R\left(\Delta u(t+j-1) - \Delta v(t+j-1)\right)\right], \tag{1}$$

where Q and R weight particular components of the water level error $\left(\hat{y} - w\right)$ and flowrate error (flowrate difference between the feedwater flowrate and the steam flowrate $= \Delta u - \Delta v$) at certain future time intervals, respectively, and w is a setpoint or reference sequence for the output signal. $\hat{y}(t+j\,|\,t)$ is an optimum j-step ahead prediction of the system output (steam generator water level) based on data up to time t; that is, the expected value of the output at time t if the past input and output and the future control sequence are known. N is the prediction horizon and M is the control horizon. The prediction horizon represents the limit of the instant in which it is desired for the output to follow the reference sequence. In order to obtain control inputs, the predicted outputs have to be first calculated as a function of past values of inputs and outputs and of future control signals. The control law is imposed by the use of the control horizon concept that after a certain interval $M < N$ there is no variation in the control signals, that is: $\Delta u(t+j-1) = 0$ for $j > M$.

The model predictive control method is to solve an optimization problem for a finite future at current time and to implement the first optimal control input as the current control input. The procedure is then repeated at each subsequent instant. Figure 1 shows this basic concept [12]. As it were, for any assumed set of present and future control moves, the future behavior of the process outputs can be predicted over a horizon N , and the M present and future control moves ($M \le N$) are computed to minimize a quadratic objective function. Although M control moves are calculated, only the first control move is implemented. At the next time step, new values of the measured output are obtained, the control horizon is shifted forward by one step, and the same calculations are repeated. The purpose of taking new measurements at each time step is to compensate for unmeasured disturbances and model inaccuracy, both of which cause the measured system output to be different from the one predicted by the model. The optimal control input that minimizes the above cost function will be derived from now on.

The process to be controlled is described by the following Controlled Auto-Regressive and Integrated Moving Average (CARIMA) model, which is widely used as a mathematical model of control design methods:

$$A(q^{-1})y(t) = B(q^{-1})u(t-1) + C(q^{-1})v(t-1) + \frac{1}{\Delta}D(q^{-1})x(t) , \tag{2}$$

where y is an output (water level), u is a control input (feedwater flowrate), v is a measurable disturbance (steam flowrate), x is a stochastic random noise sequence with zero mean value, q^{-1} is the backward shift operator, e.g., $q^{-1}y(t) = y(t-1)$, and Δ is defined as $\Delta = 1 - q^{-1}$. In Eq. (2), $A(q^{-1})$ and $D(q^{-1})$ are monic polynomials as a function of the backward shift operator q^{-1}, and $B(q^{-1})$ and $C(q^{-1})$ are polynomials. For example, the polynomial $A(q^{-1})$ is expressed as follows:

$$A(q^{-1}) = a_0 + a_1 q^{-1} + a_2 q^{-2} + \dots + a_{nA} q^{-nA} , \tag{3}$$

where a_0, a_1, $\dots$, a_{nA} are coefficients and nA is the order of the polynomial.

The process output at time $t + j$ can be predicted from the measurements of the output and input up to time step t . The optimal prediction is derived by solving a Diophantine equation, whose solution can be found by an efficient recursive algorithm. In this derivation, the most usual case when $D(q^{-1}) = 1$, will be considered. The j-step-ahead output prediction of a process is derived below.

Multiplying Eq. (2) by $\Delta E_j(q^{-1})$ from the left gives

$$y(t+j) - E_j(q^{-1})x(t+j) = F_j(q^{-1})y(t) + E_j(q^{-1})B(q^{-1})\Delta u(t+j-1) + E_j(q^{-1})C(q^{-1})\Delta v(t+j-1) , \tag{4}$$

where $E_j(q^{-1})$ and $F_j(q^{-1})$ are polynomials satisfying

$$1 = E_j(q^{-1})\tilde{A}(q^{-1}) + q^{-j}F_j(q^{-1}) , \tag{5}$$

$$E_j(q^{-1}) = e_{j,0} + e_{j,1}q^{-1} + \cdots + e_{j,j-1}q^{-(j-1)}\,, \tag{6}$$

$$F_j(q^{-1}) = f_{j,0} + f_{j,1}q^{-1} + f_{j,2}q^{-2} + \cdots + f_{j,nA}q^{-nA}\,, \tag{7}$$

$$\tilde{A}(q^{-1}) = A(q^{-1})\Delta\,. \tag{8}$$

Equation (5) is called the Diophantine equation and there exist unique polynomials $E_j(q^{-1})$ and $F_j(q^{-1})$ of order $j-1$ and nA, respectively such that $e_{j,0} = 1$. By taking the expectation operator and considering that $E\{x(t)\} = 0$, the optimal j-step-ahead prediction of $\hat{y}(t + j \mid t)$ satisfies

$$\hat{y}(t+j \mid t) = F_j(q^{-1})y(t) + E_j(q^{-1})B(q^{-1})\Delta u(t+j-1) + E_j(q^{-1})C(q^{-1})\Delta v(t+j-1)\,, \tag{9}$$

where $\hat{y}(t+j \mid t) = E\{y(t+j) | t\}$ that denotes an estimated value of the output at time step $t+j$ based on all the data up to time step t. The output prediction can easily be extended to the nonzero mean noise case by adding a term $\mathrm{E}_j(q^{-1})E\{x(t)\}$ to the output prediction $\hat{y}(t+j \mid t)$.

By making the matrix polynomial $E_j(q^{-1})B(q^{-1}) = G_j(q^{-1}) + q^{-j}G_{jp}(q^{-1})$ and $E_j(q^{-1})C(q^{-1}) = H_j(q^{-1}) + q^{-j}H_{jp}(q^{-1})$ with $d\left(G_j(q^{-1})\right) < j$ and $d\left(H_j(q^{-1})\right) < j$, the prediction equation can now be written as

$$\begin{aligned}\hat{y}(t+j \mid t) = {} & G_j(q^{-1})\Delta u(t+j-1) + H_j(q^{-1})\Delta v(t+j-1) \\ & + G_{jp}(q^{-1})\Delta u(t-1) + H_{jp}(q^{-1})\Delta v(t-1) + F_j(q^{-1})y(t)\,,\end{aligned} \tag{10}$$

where $d(\cdot)$ denotes the order of a polynomial. The last three terms of the right hand side of Eq. (10) consist of past values of the process input, measurable disturbance and output variables and correspond to the free response of the process if the control and measurable input signals are kept constant, while the first two terms of the right hand side consist of future values of the control input signal and the measurable disturbance, and can be interpreted as the forced response, that is: the response obtained when the initial conditions are zero $y(t-j) = 0, \Delta u(t-j) = 0$, $\Delta v(t-j) = 0$ for $j = 0, 1, \cdots$ [16]. Equation (10) can be rewritten as

$$\hat{y}(t+j \mid t) = G_j(q^{-1})\Delta u(t+j-1) + H_j(q^{-1})\Delta v(t+j-1) + f_j\,, \tag{11}$$

where

$$f_j = G_{jp}(q^{-1})\Delta u(t-1) + H_{jp}(q^{-1})\Delta v(t-1) + F_j(q^{-1})y(t)\,. \tag{12}$$

Then a set of N j-ahead output predictions can be expressed as

$$\hat{\mathbf{y}} = \mathbf{G}\Delta\mathbf{u} + \mathbf{H}\Delta\mathbf{v} + \mathbf{f}\,, \tag{13}$$

where

$$\hat{\mathbf{y}} = \begin{bmatrix}\hat{y}(t+1 \mid t) & \hat{y}(t+2 \mid t) & \cdots & \hat{y}(t+j \mid t) & \cdots & \hat{y}(t+N \mid t)\end{bmatrix}^T,$$

$$\mathbf{G} = \begin{bmatrix} g_0 & 0 & \cdots & 0 & \cdots & 0 \\ g_1 & g_0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ g_{j-1} & g_{j-2} & \cdots & g_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{N-1} & g_{N-2} & \cdots & \cdots & \cdots & g_0 \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} h_0 & 0 & \cdots & 0 & \cdots & 0 \\ h_1 & h_0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ h_{j-1} & h_{j-2} & \cdots & h_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ h_{N-1} & h_{N-2} & \cdots & \cdots & \cdots & h_0 \end{bmatrix}$$

$$\Delta\mathbf{u} = \begin{bmatrix}\Delta u(t) & \Delta u(t+1) & \cdots & \Delta u(t+j) & \cdots & \Delta u(t+N-1)\end{bmatrix}^T,$$

$$\Delta\mathbf{v} = \begin{bmatrix}\Delta v(t) & \Delta v(t+1) & \cdots & \Delta v(t+j) & \cdots & \Delta v(t+N-1)\end{bmatrix}^T,$$

$$\mathbf{f} = \begin{bmatrix}f_1 & f_2 & \cdots & f_j & \cdots & f_N\end{bmatrix}^T,$$

$$G_j(q^{-1}) = \sum_{i=0}^{j-1} g_i q^{-i} \ ,$$

$$H_j(q^{-1}) = \sum_{i=0}^{j-1} h_i q^{-i} \ .$$

If all initial conditions are zero, the free response $\mathbf{f}$ is zero. If a unit step is applied to the first input at time t; that is, $\Delta\mathbf{u} = [1\,0 \ldots 0]^T$, the expected output sequence $[\hat{y}(t+1)\ \hat{y}(t+2) \ldots \ \hat{y}(t+N)]^T$ is equal to the first column of the matrix $\mathbf{G}$. That is, the first column of the matrix $\mathbf{G}$ can be calculated as the step response of the plant when a unit step is applied to the first control signal. The matrix $\mathbf{H}$ can be calculated in the same way.

The computation of the control input involves the inversion of an $N \times N$ matrix $\mathbf{G}$ that requires a substantial amount of computation. If the control signal is kept constant after the first M control moves (that is, $\Delta u(t+j) = 0$ for $j > M$), this leads to the inversion of an $M \times M$ matrix that reduces the amount of computation. If so, the set of predictions affecting the cost function can be expressed as

$$\hat{\mathbf{y}} = \mathbf{G}_s \Delta\mathbf{u}_s + \mathbf{H}\Delta\mathbf{v} + \mathbf{f} \ , \tag{14}$$

where

$$\mathbf{G}_s = \begin{bmatrix} g_0 & 0 & \ldots & 0 \\ g_1 & g_0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_{N-1} & g_{N-2} & \ldots & g_{N-M} \end{bmatrix},$$

$$\Delta\mathbf{u}_s = [\Delta u(t) \quad \Delta u(t+1) \quad \ldots \quad \Delta u(t+M-1)]^T .$$

The cost function of Eq. (10) can be rewritten as the following matrix-vector form:

$$J = \frac{1}{2}(\mathbf{G}_s\Delta\mathbf{u}_s + \mathbf{H}\Delta\mathbf{v} + \mathbf{f} - \mathbf{w})^T \tilde{\mathbf{Q}} (\mathbf{G}_s\Delta\mathbf{u}_s + \mathbf{H}\Delta\mathbf{v} + \mathbf{f} - \mathbf{w}) + \frac{1}{2}(\Delta\mathbf{u}_s - \Delta\mathbf{v}_s)^T \tilde{\mathbf{R}} (\Delta\mathbf{u}_s - \Delta\mathbf{v}_s) , \tag{15}$$

where $\mathbf{w} = [w(t+1\,|\,t) \quad w(t+2\,|\,t) \quad \ldots \quad w(t+N\,|\,t)]^T$, $\Delta\mathbf{v}_s = [\Delta v(t) \quad \Delta v(t+1) \quad \ldots \quad \Delta v(t+M-1)]^T$, $\tilde{\mathbf{Q}} = diag(Q, \ldots, Q)$ is a diagonal matrix, and $\tilde{\mathbf{R}} = diag(R, \ldots, R)$. Usually $\tilde{\mathbf{Q}} = \mathbf{I}_{N\times N}$ and $\tilde{\mathbf{R}} = \mu \times \mathbf{I}_{M\times M}$ are used and μ is called an input-weighting factor.

The optimal control input can be expressed as

$$\Delta\mathbf{u}_s = \left(\mathbf{G}_s^T \tilde{\mathbf{Q}} \mathbf{G}_s + \tilde{\mathbf{R}}\right)^{-1} \left[\mathbf{G}_s^T \tilde{\mathbf{Q}} (\mathbf{w} - \mathbf{f} - \mathbf{H}\Delta\mathbf{v}) + \tilde{\mathbf{R}}\Delta\mathbf{v}_s\right]. \tag{16}$$

Since only $\Delta u(t)$ is needed at time step t, only the first row of the matrices $\left(\mathbf{G}_s^T \tilde{\mathbf{Q}} \mathbf{G}_s + \tilde{\mathbf{R}}\right)^{-1} \mathbf{G}_s^T \tilde{\mathbf{Q}}$ and $\left(\mathbf{G}_s^T \tilde{\mathbf{Q}} \mathbf{G}_s + \tilde{\mathbf{R}}\right)^{-1} \tilde{\mathbf{R}}$ has to be computed.

In order to obtain the control input from Eq. (16), it is necessary to calculate the matrices $\mathbf{G}_s$ and $\mathbf{H}$, and the vector $\mathbf{f}$. These matrix and vector can be calculated recursively. From now on, the derivation will be described. By taking into account a new Diophantine equation corresponding to the prediction for $\hat{y}(t+j+1\,|\,t)$, Eq. (5) can also be rewritten as follows:

$$1 = E_{j+1}(q^{-1})\tilde{A}(q^{-1}) + q^{-(j+1)} F_{j+1}(q^{-1}) \ . \tag{17}$$

Subtracting Eq. (5) from Eq. (17) gives

$$0 = \left[E_{j+1}(q^{-1}) - E_j(q^{-1})\right]\tilde{A}(q^{-1}) + q^{-j}\left[q^{-1}F_{j+1}(q^{-1}) - F_j(q^{-1})\right] \ . \tag{18}$$

Since the matrix $E_{j+1}(q^{-1}) - E_j(q^{-1})$ is of order j, the matrix can be written as

$$E_{j+1}(q^{-1}) - E_j(q^{-1}) = \tilde{P}(q^{-1}) + p_j q^{-j} \ , \tag{19}$$

where $\tilde{P}(q^{-1})$ is a polynomial of order smaller than or equal to $j-1$. By substituting Eq. (19) into Eq. (18)

$$0 = \tilde{P}(q^{-1})\,\tilde{A}(q^{-1}) + q^{-j}\left[p_j \tilde{A}(q^{-1}) + q^{-1}F_{j+1}(q^{-1}) - F_j(q^{-1})\right] \ . \tag{20}$$

Since $\tilde{A}(q^{-1})$ is monic, it is easy to see that $\tilde{P}(q^{-1}) = 0$. Therefore, from Eq. (19) the polynomial $E_{j+1}(q^{-1})$ can be calculated recursively by

$$E_{j+1}(q^{-1}) = E_j(q^{-1}) + p_j q^{-j} . \tag{21}$$

The following expressions can easily be obtained from Eq. (20):

$$p_j = f_{j,0} , \tag{22}$$

$$f_{j+1,i} = f_{j,i+1} - p_j \tilde{a}_{i+1} \text{ for } i = 0, \ldots, d(F_{j+1}) . \tag{23}$$

Also, it can easily be seen that the initial conditions for the recursion equation are given by

$$E_1 = 1 , \tag{24}$$

$$F_1 = q\left(1 - \tilde{A}(q^{-1})\right). \tag{25}$$

The free response vector $\mathbf{f}$ can be computed by the following recursive relationship:

$$f_{j+1} = q\left(1 - \tilde{A}(q^{-1})\right) f_j + B(q^{-1})\Delta u(t+j) + C(q^{-1})\Delta v(t+j) \text{, with } f_0 = y(t) \text{ and } \Delta u(t+j) = 0 \text{ for } j \geq 0 . \tag{26}$$

At every time instant, the model predictive controller solves *on-line* an optimization problem by using Eqs. (16), (23) and (26) to compute optimal control inputs.

3. AUTO-TUNING OF PID CONTROLLERS USING MPC METHOD

If a controlled process is a 2nd-order linear system, Eq. (16) can be written by

$$\begin{aligned}\Delta\mathbf{u}_s &= \left(\mathbf{G}_s^T \tilde{\mathbf{Q}} \mathbf{G}_s + \tilde{\mathbf{R}}\right)^{-1} \left[\mathbf{G}_s^T \tilde{\mathbf{Q}} (\mathbf{w} - \mathbf{H}\Delta\mathbf{v}) + \tilde{\mathbf{R}} \Delta\mathbf{v}_s\right] \\ &\quad - \left(\mathbf{G}_s^T \tilde{\mathbf{Q}} \mathbf{G}_s + \tilde{\mathbf{R}}\right)^{-1} \mathbf{G}_s^T \tilde{\mathbf{Q}} \begin{bmatrix} f_{1,0} y(t) + f_{1,1} y(t-1) + f_{1,2} y(t-2) \\ f_{2,0} y(t) + f_{2,1} y(t-1) + f_{2,2} y(t-2) \\ \vdots \\ f_{N,0} y(t) + f_{N,1} y(t-1) + f_{N,2} y(t-2) \end{bmatrix}. \end{aligned} \tag{27}$$

The first optimal control input is

$$\begin{aligned}\Delta u(t) &= \mathbf{k}_1^T (\mathbf{w} - \mathbf{H}\Delta\mathbf{v}) + \mathbf{k}_2^T \Delta\mathbf{v}_s - \left[\sum_{i=1}^{N} k_i f_{i,0}\right] y(t) - \left[\sum_{i=1}^{N} k_i f_{i,1}\right] y(t) - \left[\sum_{i=1}^{N} k_i f_{i,2}\right] y(t) \\ &= \mathbf{k}_1^T (\mathbf{w} - \mathbf{H}\Delta\mathbf{v}) + \mathbf{k}_2^T \Delta\mathbf{v}_s - \alpha_0 w(t) - \alpha_1 w(t-1) - \alpha_2 w(t-2) + \alpha_0 e(t) + \alpha_1 e(t-1) + \alpha_2 e(t-2), \end{aligned} \tag{28}$$

where

$\mathbf{k}_1^T$ = the first row of a matrix $\left(\mathbf{G}_s^T \tilde{\mathbf{Q}} \mathbf{G}_s + \tilde{\mathbf{R}}\right)^{-1} \mathbf{G}_s^T \tilde{\mathbf{Q}} = [k_1 \ k_2 \ \ldots \ k_N]$,

$\mathbf{k}_2^T$ = the first row of a matrix $\left(\mathbf{G}_s^T \tilde{\mathbf{Q}} \mathbf{G}_s + \tilde{\mathbf{R}}\right)^{-1} \tilde{\mathbf{R}}$,

$\alpha_0 = \sum_{i=1}^{N} k_i f_{i,0}$,

$\alpha_1 = \sum_{i=1}^{N} k_i f_{i,1}$,

$\alpha_2 = \sum_{i=1}^{N} k_i f_{i,2}$,

$e(t) = w(t) - y(t)$.

In Eq. (28), all the terms of the right hand side except the last three terms are feed-forward terms and the last three terms are a standard PID block. Then the control input can be rewritten by

$$\Delta u(t) = \Delta u_{ff}(t) + \Delta u_{pid}(t) , \tag{29}$$

where

$$\Delta u_{ff}(t) = \mathbf{k}_1^T(\mathbf{w} - \mathbf{H}\Delta\mathbf{v}) + \mathbf{k}_2^T \Delta\mathbf{v}_s - a_0 w(t) - a_1 w(t-1) - a_2 w(t-2),$$

$$\Delta u_{pid}(t) = a_0 e(t) + a_1 e(t-1) + a_2 e(t-2).$$

A standard PID controller can be expressed as

$$u_{pid}(t) = K_p e(t) + K_i \sum_{i=0}^{t} e(i) + K_d \Delta e(t), \tag{30}$$

where K_p, K_i, and K_d are the proportional, integral, and derivative control gains, respectively. From Eq. (30), the control input change is as follows:

$$\Delta u_{pid}(t) = (K_p + K_i + K_d)e(t) - (K_p + 2K_d)e(t-1) + K_d e(t-2). \tag{31}$$

The control gains can be automatically tuned from Eqs. (29) and (31) as follows:

$$K_p = -a_1 - 2a_2, \quad K_i = a_0 + a_1 + a_2, \quad K_d = a_2. \tag{32}$$

Figure 2 shows the structure of the proposed MPC based auto-tuned PID controller. In this figure, it is shown that the changes of the water level setpoint and steam flowrate drive the control actions.

4. APPLICATION TO THE STEAM GENERATOR WATER LEVEL CONTROL

Numerical simulations are performed to study the performance of the proposed algorithm. The dynamics of a steam generator is described in terms of input (feedwater flowrate; u), output (water level; y) and measurable disturbance (steam flowrate; v). Based on the step response of the steam generator water level for step changes of the feedwater flowrate and the steam flowrate, Irving [1] derived the following 4-th order Laplace transfer function for steam generators:

$$y(s) = \frac{G_1}{s}[u(s) - v(s)] - \frac{G_2}{1 + \tau_2 s}[u(s) - v(s)] + \frac{G_3 s}{\tau_1^{-2} + 4\pi^2 T^{-2} + 2\tau_1^{-1}s + s^2}u(s), \tag{33}$$

where s is a Laplace variable. This plant has a single input (feedwater) and a single output (water level). The parameter values of a steam generator at several power levels are given in Table 1 and the parameters are very different according to the power levels. Since $(G_2 - G_1\tau_2)$ is greater than zero, Eq. (33) has a positive zero that represents a non-minimum phase effect. An unstable zero lowers the control gain to preserve stability. As the load decreases, the zero moves to the right, stability being more critical and the water level of the steam generator being more difficult to control. The third term of the right hand side in Eq. (33) is extremely small in affecting the water level response [17]. Therefore, the 4-th linear model can be reduced well without making a great difference by a second order linear model and the proposed control method can be designed by using this reduced nuclear steam generator model. In these numerical simulations, the sampling time is chosen to be 5 sec.

Figures 3 and 4 show the performances of this proposed controller. The setpoint of the water level was suddenly changed at 100 sec and the steam flowrate (measurable disturbance) was changed at 3000 sec. In these figures, all values represent the difference from the corresponding steady state values. Therefore, all values are zeros at the steady state. The magnitude of the disturbance at 3000 sec is 5 percent rated steam flowrate [$71.75kg/\sec = 0.05 \times 1435\, kg/\sec$ (rated steam flowrate)]. Q was chosen as 1 and the input-weighting factor R was chosen as 50000 for all power levels. The prediction and control horizons are 60 and 20, respectively. If the changes of the future water level setpoint and steam flowrate are known, these changes are considered in advance over the prediction and control horizons (refer to the responses around time steps 100 and 3000 sec of Figs. 3 and 4). The proposed controller show good performance for the water level deviation and sudden steam flow disturbances that are typical in the existing power plants.

Also, if the input-weighting factor changes according to the power level, the proposed controller is expected to have better performance. Figures 5 and 6 show the performances of this proposed controller for the same situations as the computer simulation of Figs. 3 and 4 except that the varying input-weighting factor is used. The input-weighting factor was differently chosen according to the power level in order to have good performances and ensure the stability. As the input-weighting factor usually increases, its relative stability does so. The proposed control algorithm tracks better the setpoint and steam flowrate changes. Figure 7 shows the input-weighting factor versus power level. As the power level increases, the factor decreases exponentially. The swell and shrink phenomena are larger at low power levels than at high power levels. The water level tracks its setpoint faster at high powers than at low powers.

5. CONCLUSIONS

In order to overcome the drawbacks of the conventional PID controller with fixed control gains which are familiar to us, in this work the MPC based auto-tuned PID controller was developed to control the water level of nuclear steam generators. The proposed controller was applied and verified to a linear model for nuclear steam generators. The parameters of the linear model for a steam generator are very different according to the power levels. Although the proposed controller was designed for the reduced linear steam generator model at each corresponding power level, the controller showed good performance by changing only the input-weighting factor according to the power level in case the water level setpoint and steam flowrate (measurable disturbance) change suddenly. As the power level increases, the input-weighting factor decreases exponentially and the input-weighting factor can be easily selected according to power level change.

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Table 1. Parameters of a steam generator linear model at several powers.

Power level (%)	G_1	G_2	G_3	t_1 (sec)	t_2 (sec)	T (sec)	V_0 (kg/sec)
5	0.058	9.630	0.181	41.9	48.4	119.6	57.4
15	0.058	4.46	0.226	26.3	21.5	60.5	180.8
30	0.058	1.83	0.310	43.4	4.5	17.7	381.7
50	0.058	1.05	0.215	34.8	3.6	14.2	660.0
100	0.058	0.47	0.105	28.6	3.4	11.7	1435.0

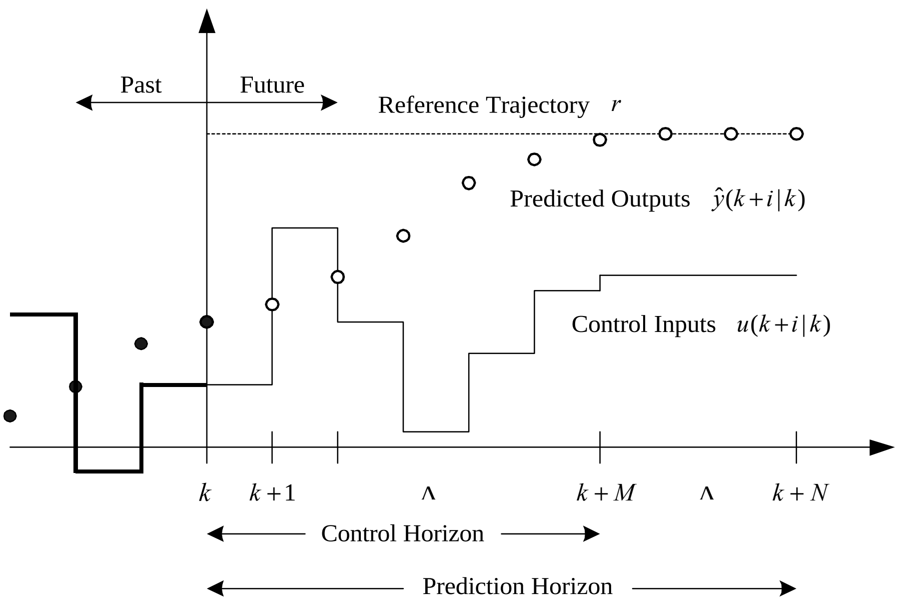


Fig. 1. Model predictive control method.

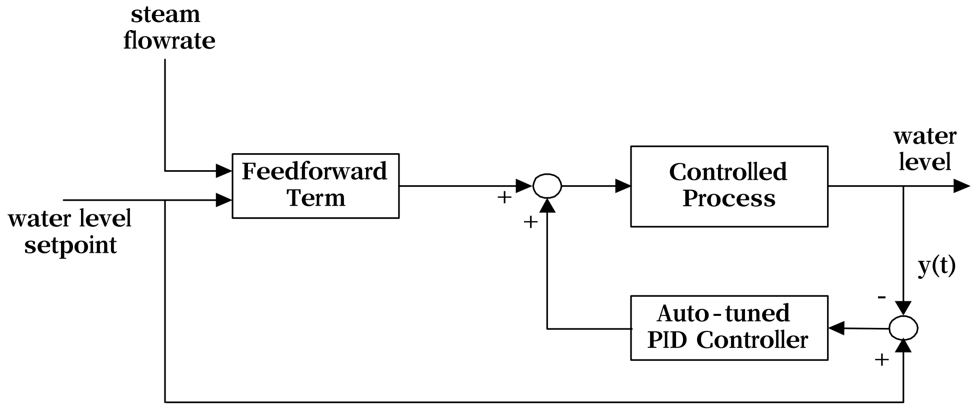
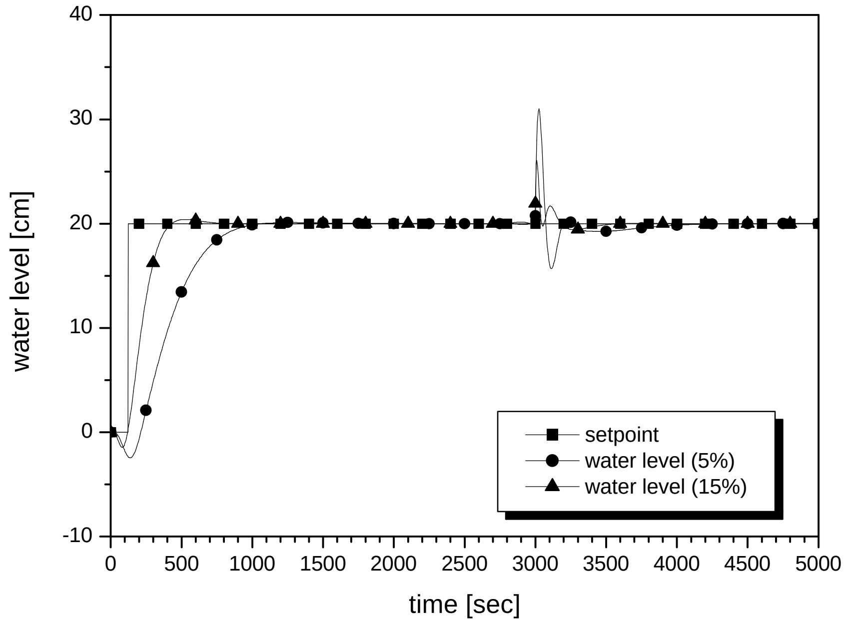
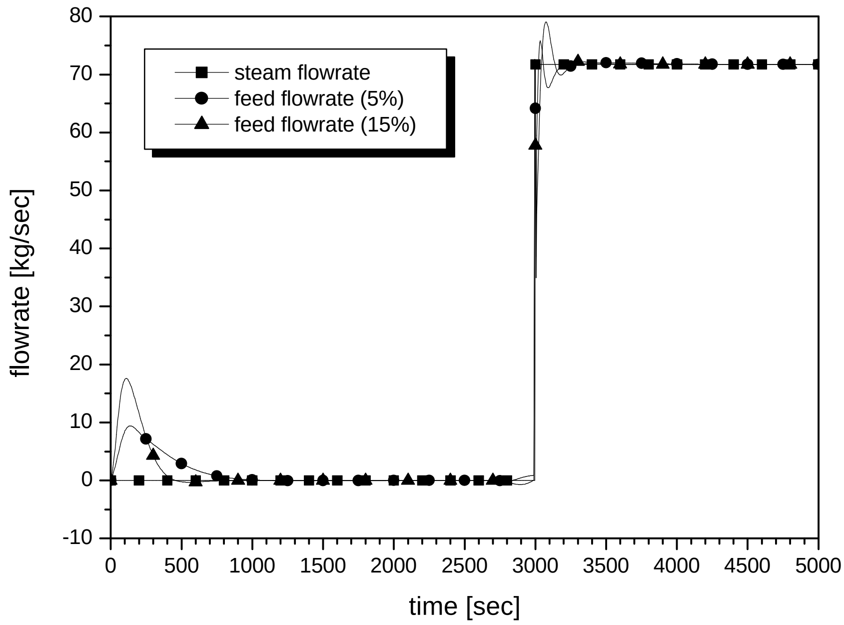


Fig. 2. Structure of the proposed MPC based auto-tuned PID controller.

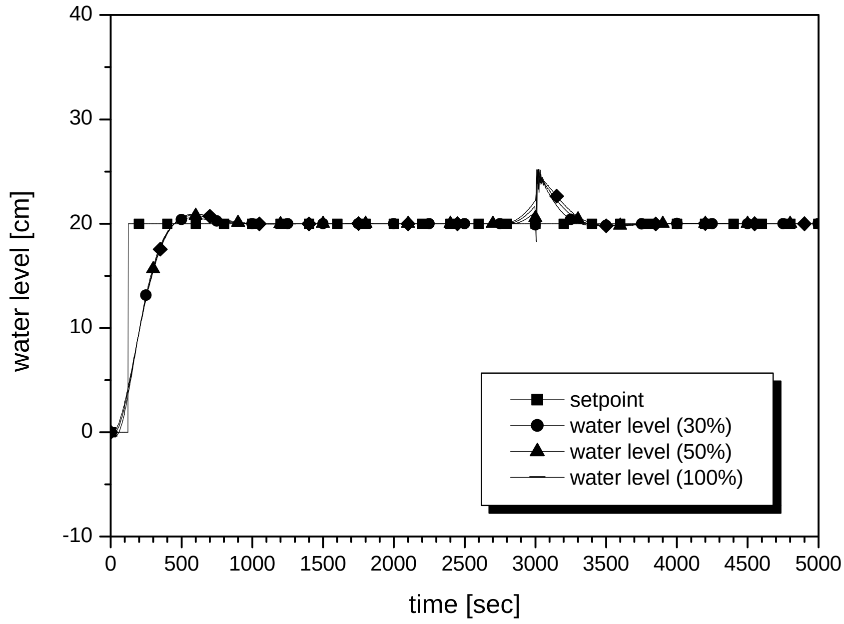


(a) water level

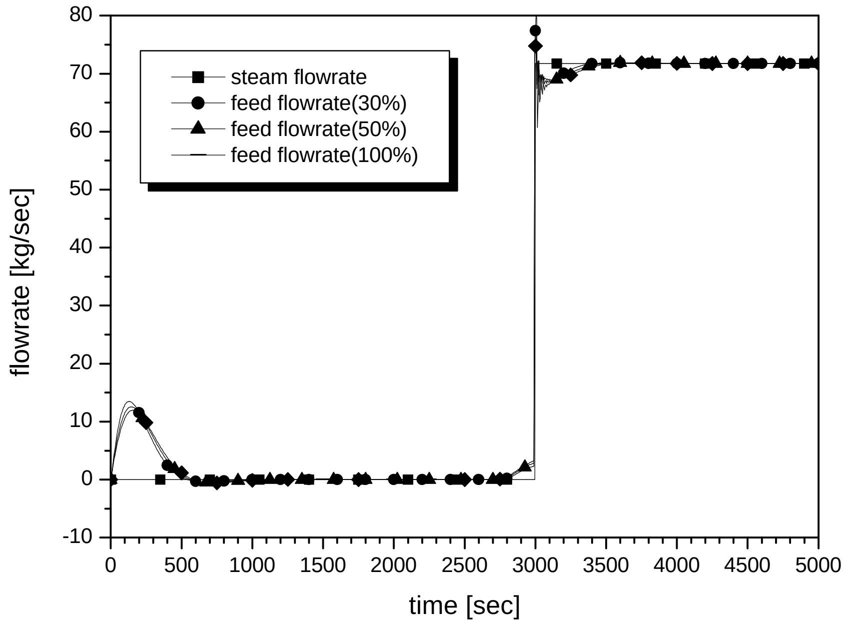


(b) flowrate

Fig. 3. Performance of the proposed controller (low powers).

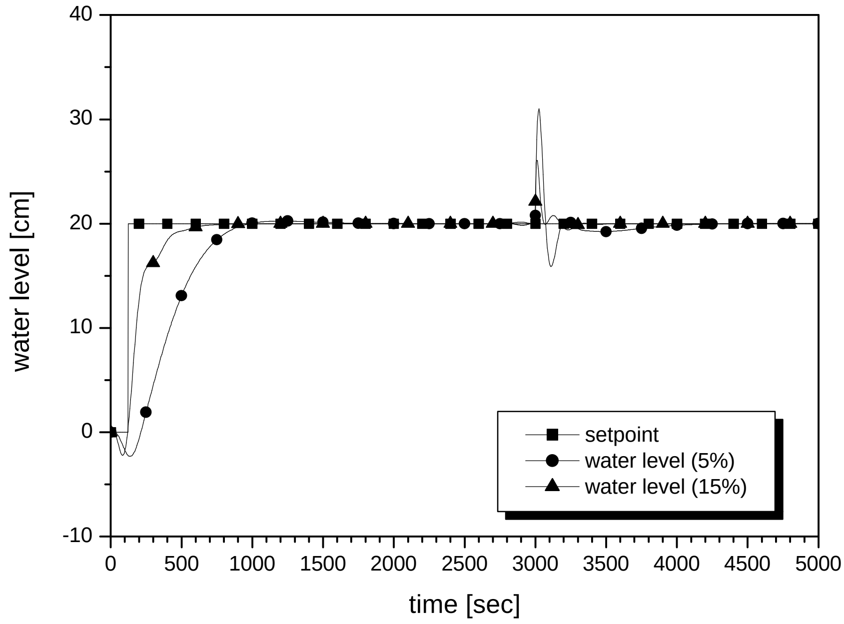


(a) water level

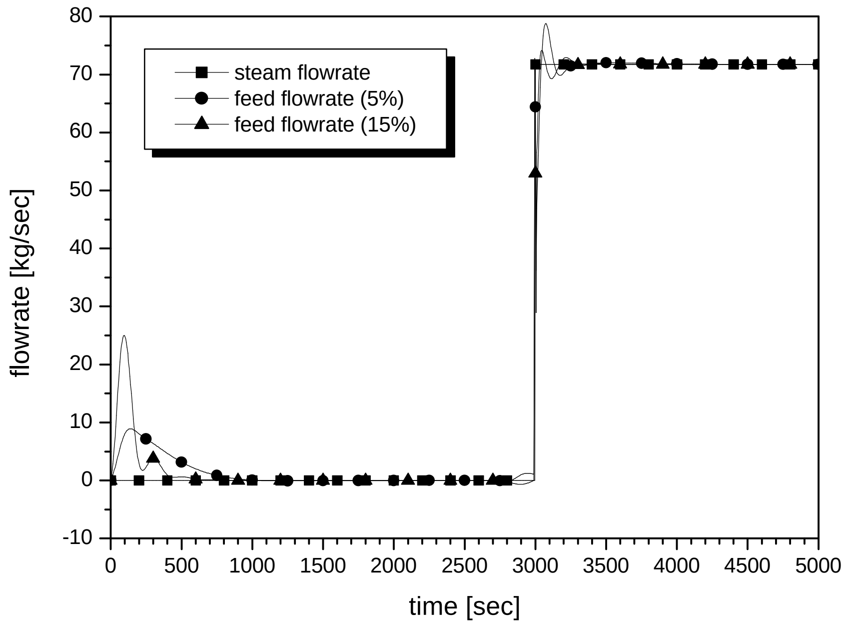


(b) flowrate

Fig. 4. Performance of the proposed controller (high powers).

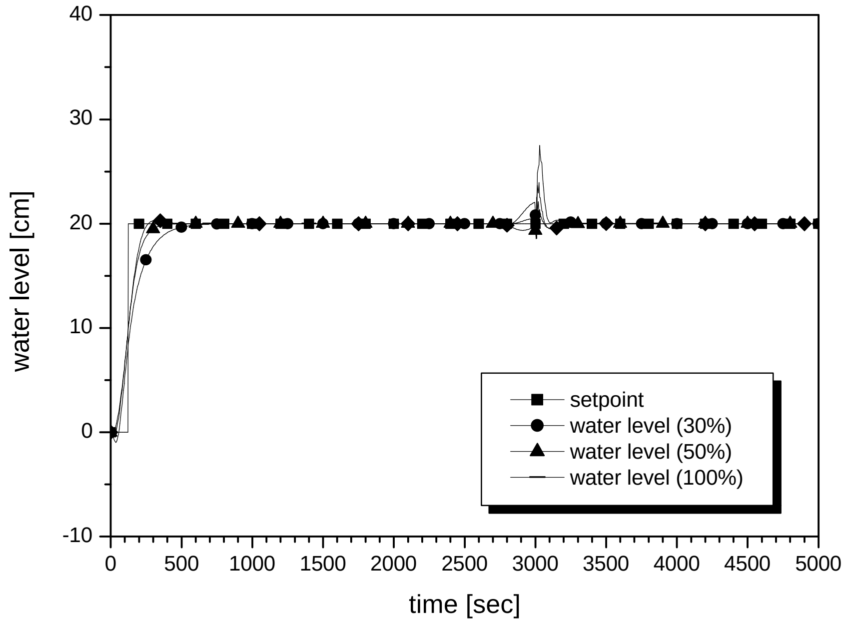


(a) water level

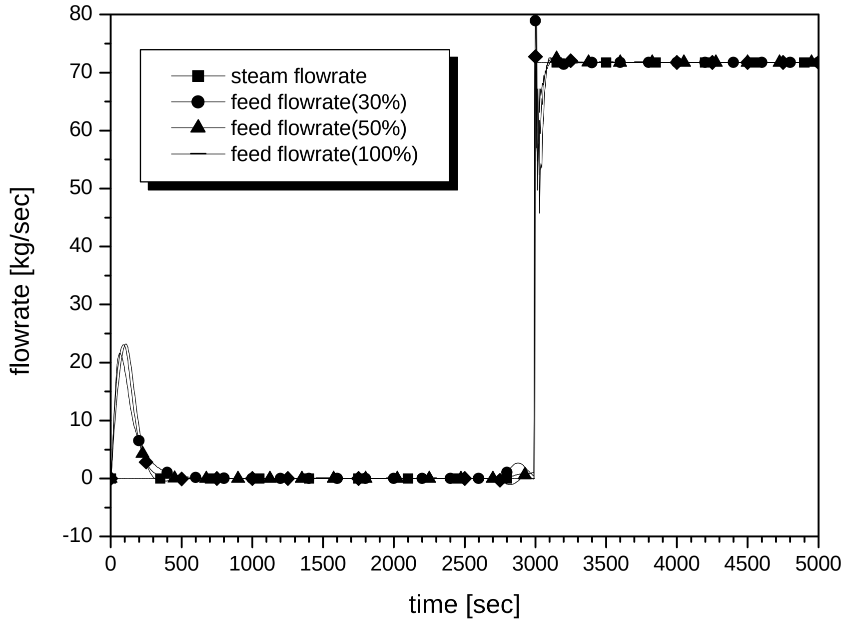


(b) flowrate

Fig. 5. Performance of the proposed controller with varying input-weighting factor (low powers).



(a) water level



(b) flowrate

Fig. 6. Performance of the proposed controller with varying input-weighting factor (high powers).

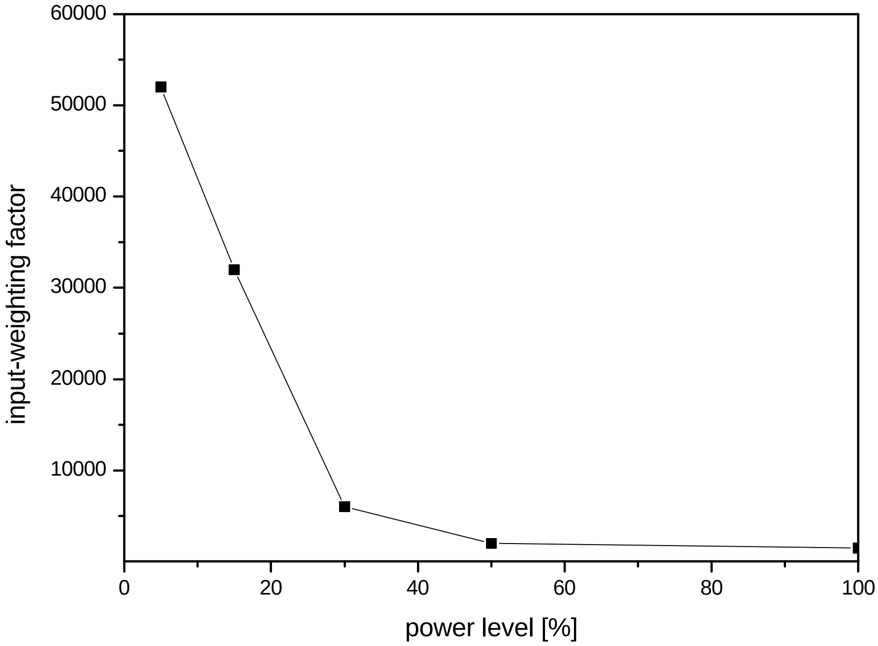


Fig. 7. Input-weighting factor versus power level.