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A Study on Radial Buckling Correction for Improved ECP Estimation Using APOLLO

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Abstract

The accurate Estimation of Critical Position (ECP) is required to reduce radio waste and plant outage time in nuclear plant. The key factor for the calculation of ECP is the best prediction of xenon worth at the time of startup. Also the control rod worth should be considered since control rods are usually inserted into the core in order to achieve the criticality of nuclear reactor. Current methodology for the calculation of ECP is based on the results from 3D diffusion codes for the reference core status, like HFP ARO equilibrium xenon condition. However due to the lack of information about the operational history, it is very difficult to predict the ECP correctly in the presence of xenon. Thus, in this study the methodology to predict the ECP using the one-dimensional diffusion code considering the operational history is presented and also correction factors are introduced to match with the result of 3D code.

1. Introduction

The Estimated Critical Position (ECP) of nuclear power plant is the location of leading control banks with the boron concentration in Reactor Coolant System (RCS) specified or the boron concentration in RCS with the leading control bank position specified for the initial criticality of the reactor core. Several factors should be in consideration for the calculation of ECP, some of them are power operation history, coolant temperature, control rod position and xenon worth, etc. This study introduces the methodology for ECP calculation using one dimensional diffusion code, APOLLO[1], which runs in personal computer for the application in plant site. Originally, APOLLO

code had been available only on workstation environment, thus OASIS(pc-version APOLLO) was developed for the operating system based on PC.

The ECP calculation is usually required a few hours after the reactor trip when the xenon transient is in progress. At this stage, the xenon transient results in radial power shifts outward from the center of the core for most of core designs. This increases the neutron leakage. In conventional out-in loading pattern designs, this increases the power in region of the core of higher reactivity(core periphery). The net effect is the cancellation of the neutron leakage in core peripheral region because the fuel assemblies with higher reactivity are resided in the region. In low leakage loading patterns, the xenon transients results in the power shifts to a region of lower reactivity(core periphery). This results in a reactivity decrease in core peripheral region and therefore the cancellation of the effect does not occur.

Since APOLLO is a one-dimensional code, APOLLO can not take this radial leakage effect into account. After all, APOLLO shows big reactivity difference in comparison with 3D code(e.g. ANC) during HZP peak xenon transient in existence of significant amount of xenon. In order to address this problem, a radial buckling correction algorithm was developed and applied for the calculation of ECP in this study.

2. ECP Calculation with APOLLO

2.1 APOLLO Radial Buckling Methodology or Generalized Equivalence Theory

APOLLO is a one-dimensional, two-group steady state neutron diffusion theory program. The code can handle a maximum of 250-mesh intervals and has an axial slab geometry. Space dependent feedback effects due to xenon, samarium, rod position, boron, fuel temperature , and water density are accounted for.

The macroscopic and microscopic cross sections for APOLLO code are provided by ANC[2] which is a 3D nodal code. In APOLLO, the reactor is represented by a one-dimensional diffusion equation. Such a one-dimensional representation does not represent any true physical system either and there is no a priori reason that such a one-dimensional equation with volume-flux weighted cross section can reproduce the reaction rates of three-dimensional calculation. In addition, APOLLO has two more major approximations; the finite difference approximation and the approximate cross

section representation. These approximations can sometimes introduce significant errors in the computed reaction rates and reactivity. The one-dimensional finite difference equation with approximate cross sections is no more accurate than the homogeneous representation in the nodal method. Nevertheless, if we can force this approximate core representation to reproduce nodewise reaction rates and nodal surface boundary conditions, then this approximate core representation is equivalent to the three dimensional core representation as far as the axial integral parameters are concerned. The methodology employed by APOLLO, the generalized equivalence theory is described in the following.

The three dimensional diffusion equation can be written as

$$D_g(x,y,z)\nabla^2 f_g(x,y,z) - \Sigma_g(x,y,z) f_g(x,y,z) + S_g(x,y,z) = 0 \qquad (1)$$

To develop an equivalent one-dimensional equation, let us integrate this equation over the x-y plane:

$$\iint D_g(x,y,z)\nabla^2 f_g(x,y,z)dxdy - \iint \Sigma_g(x,y,z) f_g(x,y,z)dxdy + \iint S_g(x,y,z)dxdy = 0 \qquad (2)$$

Let

$$L_g(z) = -\iint D_g(x,y,z)(d^2/dx^2 + d^2/dy^2) f_g(x,y,z)dxdy = \int J(x,y,z)ds \qquad (3)$$
$$= Radial\ Leakage$$

$$\int_{\Delta z_N} dz \iint dxdy f_g(x,y,z) = f_g(N) \qquad (4)$$

$$\int_{\Delta z_N} dz \iint dxdy S_g(x,y,z) = S_g(N) = Source\ Term \qquad (5)$$

$$\Sigma_g(N) = \frac{\int_{\Delta z_N} dz \iint dxdy \Sigma_g(x,y,z) f_g(x,y,z)}{\int_{\Delta z_N} dz \iint dxdy f_g(x,y,z)} = Removal\ and/or\ Absorption\ Term \qquad (6)$$

where N = axial node index

Then,

$$-D_g(N)\frac{d^2 f_{gN}(z)}{dz^2}+\Sigma_g(N)f_{gN}(z)+L_g(z)=S_g(z) \tag{7}$$

As mentioned previously, there is no a priori reason that the solution to (7) would reproduce the same axial flux predicted by the 3D equation, Equation (1). There are two additional approximations in APOLLO:

a. Approximate cross section representation, $\overline{D}(N),\ \overline{\Sigma}(N)$

b. Finite Difference Approximation

The equation solved by APOLLO has the following form:

$$\frac{\overline{\Gamma}_g(N-1)\overline{\Gamma}_g(N)}{\overline{\Gamma}_g(N-1)+\overline{\Gamma}_g(N)}(f_g(N)-f_g(N-1))-\frac{\overline{\Gamma}_g(N)\overline{\Gamma}_g(N+1)}{\overline{\Gamma}_g(N)+\overline{\Gamma}_g(N+1)}(f_g(N+1)-f_g(N)) \tag{8}$$

$$+\quad \overline{\Sigma}_g(N)f_g(N)+L_g(N)=S_g(N)$$

Where

$$\overline{\Gamma}_g(N)=\frac{2\overline{D}_g(N)}{\Delta z(N)} \tag{9}$$

In all likelihood, the solution to Equation (8) would not agree with the solution to the 3D equation, i.e.

$$f_g(N)\neq\int_{\Delta z_N}dz\iint dxdy\,f_g(x,y,z) \tag{10}$$

To assure the equality of Equation (10), a new parameter needs to be introduced in Equation (8). In transport theory, this new parameter is called artificial source; in nodal theory, the discontinuity factor, in APOLLO application, this new parameter is called two group radial buckling. Introducing this buckling term into Equation (8) gives

$$\frac{\overline{\Gamma}_g(N-1)\overline{\Gamma}_g(N)}{\overline{\Gamma}_g(N-1)+\overline{\Gamma}_g(N)}(f_g(N)-f_g(N-1))-\frac{\overline{\Gamma}_g(N)\overline{\Gamma}_g(N+1)}{\overline{\Gamma}_g(N)+\overline{\Gamma}_g(N+1)}(f_g(N+1)-f_g(N)) \tag{11}$$

$$+\overline{\Sigma}_g(N)f_g(N)+\overline{D}_g(N)B_g^2(N)f_g(N)=S_g(N)$$

Plugging the 3D flux into this equation, we can obtain the buckling for each node,

B_g^2. The B_g^2 compensates for the three approximations in APOLLO: the 1D representation, the finite difference, and the cross section. In contrast, the discontinuity factor compensates for only the homogenization. Because of this expanded role for the buckling factor, this methodology is called the generalized equivalence theory.

This methodology assures that APOLLO will predict the same axial flux distribution as 3D ANC for the reference condition. The reference conditions are HFP ARO equilibrium xenon condition and HZP ARO no xenon conditions for each burnup steps for the reload application.

2.2. Radial Leakage Effect at HZP with xenon

Since APOLLO radial buckling is tuned for the HFP ARO equilibrium xenon and HZP ARO no xenon case with burnup, there exists an inherent modeling error in simulation of HZP transient with xenon using APOLLO, this error is reduced to zero when xenon decays out, though. The typical critical boron difference between APOLLO and ANC during HZP xenon transient with HFP equilibrium xenon is shown in Figure 1 at BOL, MOL and EOL burnup for YGN unit 1 cycle 12 design[3]. The figure shows that this error is maximized at around 9 or 10 hours after shutdown which is dependent upon initial xenon number density before the transient and thereafter the error decreases. Also the change of APOLLO xenon number density is shown in comparison with the critical boron difference between APOLLO and ANC for BOL xenon transient simulation in Figure 2. As it was described earlier this error was caused by a radial shift of power distribution due to the existence of xenon in low leakage loading patterns. Since APOLLO can not consider this radial redistribution of power which causes the radial leakage increase, APOLLO overestimates the reactivity of reactor core during the transient. It is clear that this critical boron difference will give rise to significant error for the prediction of ECP using APOLLO.

3. Radial Buckling Correction for improved ECP Estimation

As shown in Figure 2, the critical boron difference between APOLLO and ANC during HZP transient is similar to the APOLLO xenon density change. Now this can be

used for the correction of APOLLO simulation of HZP transient with xenon for the better prediction of ECP. The radial buckling of APOLLO code is calculated burnup and power dependently at each case, and this is actually the leakage term in Equation (8), thus if we increase this leakage, then the reactivity prediction by APOLLO would be decreased accordingly. In order to achieve this, time dependent buckling correction factor was introduced. Since the radial leakage error is strictly dependent on the xenon number density, the correction factor will also have the same shape function with the xenon number density change but the coefficient will be adjusted appropriately.

For the derivation of buckling correction factor, zero power rate equation and the solution for iodine and xenon number densities are given as follows:

$$\frac{dI}{dt} = -\lambda_I I$$

$$\frac{dX}{dt} = \lambda_I I - \lambda_X X \tag{12}$$

$$I = I_0 \exp(-\lambda_I t)$$

$$X = X_0 \exp(-\lambda_X t) + \frac{\lambda_I I_0}{\lambda_I - \lambda_X}[\exp(-\lambda_X t) - \exp(-\lambda_I t)]$$

Using the simple arithmetic, the time at which peak xenon occurs is given as below

$$t_{\max} = \frac{1}{(\lambda_X - \lambda_I)} \ln[\frac{\lambda_X}{\lambda_I} + \frac{\lambda_X X_0}{\lambda_I I_0}(1 - \frac{\lambda_X}{\lambda_I})] \tag{13}$$

And the peak xenon density is given by

$$X_{\max} = X(t_{\max}) \tag{14}$$

Now, assume a correction factor with the form of

$$C(t) = A\exp(-\lambda_X t) + B\frac{\lambda_I}{(\lambda_I - \lambda_X)}(\exp(-\lambda_X t) - \exp(-\lambda_I t)) \tag{15}$$

With the following constraints

$$C(0) = C_0$$
$$C(t_{\max}) = C_m \qquad (16)$$

Where, C_0 is the critical boron difference at the time of initial transient and C_m is that of at the time of peak xenon transient between ANC and APOLLO.

Using these constraints, A and B are determined as follows:

$$A = C_0 \qquad (17a)$$

$$B = \frac{(l_I - l_X)}{l_I} \frac{(C_m - C_0 \exp(-l_X t_{\max}))}{(\exp(-l_X t_{\max}) - \exp(-l_I t_{\max}))} \qquad (17b)$$

Now the buckling correction, CF(t), will be calculated using C(t) as follows:

$$CF(t) = 1 + C(t) \times \Delta B^2 \Big/ \Delta ppm \qquad (18)$$

where, ΔB^2 and Δppm should be determined cycle specifically and as a function of burnup which represents the change (Δppm) of boron concentration with the radial buckling increase of ($1+\Delta B^2$). These parameters can be calculated using pc-version APOLLO(OASIS).

In addition to this buckling correction factor, power correction factor which is a function of initial xenon density is also multiplied to the buckling correction factor. In case of no xenon, the power correction factor is zero, thus, there will be no buckling correction at all. If the reactor was in equilibrium at half of its rated power, only half of the buckling correction will be applied for the transient calculation.

4. Calculation Results

Radial buckling correction algorithm was incorporated into the OASIS code and comparison of critical boron change was made between APOLLO, OASIS and ANC for YGN unit 1 cycle 12 design. The test cases are HZP ARO xenon transient with HFP equilibrium xenon density. The buckling correction parameters, C_0, C_m, ΔB^2 and Δppm were calculated on burnup basis using pc-version APOLLO (OASIS) and are summarized in Table 1. Figure 3 through 5 shows the change of the critical boron during the HZP xenon transient for BOL, MOL and EOL. The results show that the buckling correction algorithm works good, that is, OASIS shows good agreement with ANC in predicting critical boron of reactor core during xenon transient at

HZP with xenon.

Also, ECP calculation was performed for Kori Unit 2 cycle 14[4]. The calculation condition is summarized in Table 2 and buckling correction parameters are summarized in Table 3 for Kori Unit 2 Cycle 14. In Table 3, Values of C_0 and C_m are smaller than those of Table 1 which are from the 3-loop core (YGN Unit 1) rather than 2-loop core, while Δppm is larger. It means that Kori Unit 2 design is a loading pattern having more neutron leakage and also more sensitive for leakage.

Initially the measured critical boron showed a difference of 41 ppm with predicted critical boron of ANC at HFP ARO equilibrium xenon condition. This large discrepancy is mainly attributed to B-10 depletion[6] phenomena rather than modeling error. This should be considered for the calculation of ECP in real application. Thus the final ECP calculated with bias was compared with that of measured. The results are summarized in Table 4. The result of OASIS shows small deviation(2 ppm) with ANC in predicting critical boron concentration before the transient begins and this seems to be caused by an iteration error. This error was also considered in the calculation of final ECP. Consequently, ANC and OASIS predicted 5 ppm away from the measured ECP while APOLLO showed 33ppm difference.

5. Conclusion

Radial buckling correction algorithm for leakage consideration was suggested in this study and it was incorporated into one-dimensional computer code, pc-version APOLLO(OASIS). For the on-site application, ECP was calculated for Kori Unit 2 cycle 14 and showed good agreement with the measured result. ECP calculation using one-dimensional code can give the accuracy of 3D code now considering operational history if the appropriate buckling correction parameters are given, which can be calculated by OASIS.

Figure 1. Difference in Critical Boron between ANC and APOLLO during HZP Xe Depletion for Y1C12

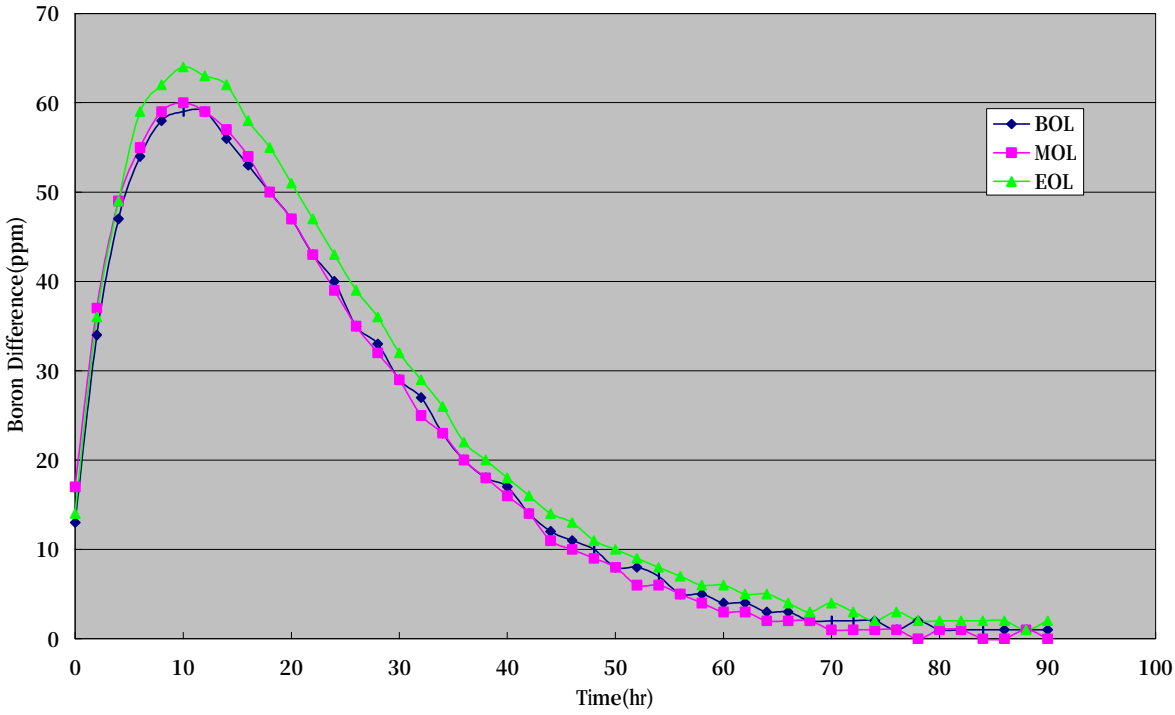


Figure 2. APOLLO Xenon Denisty Change and Critical Boron Difference (APOLLO-ANC)

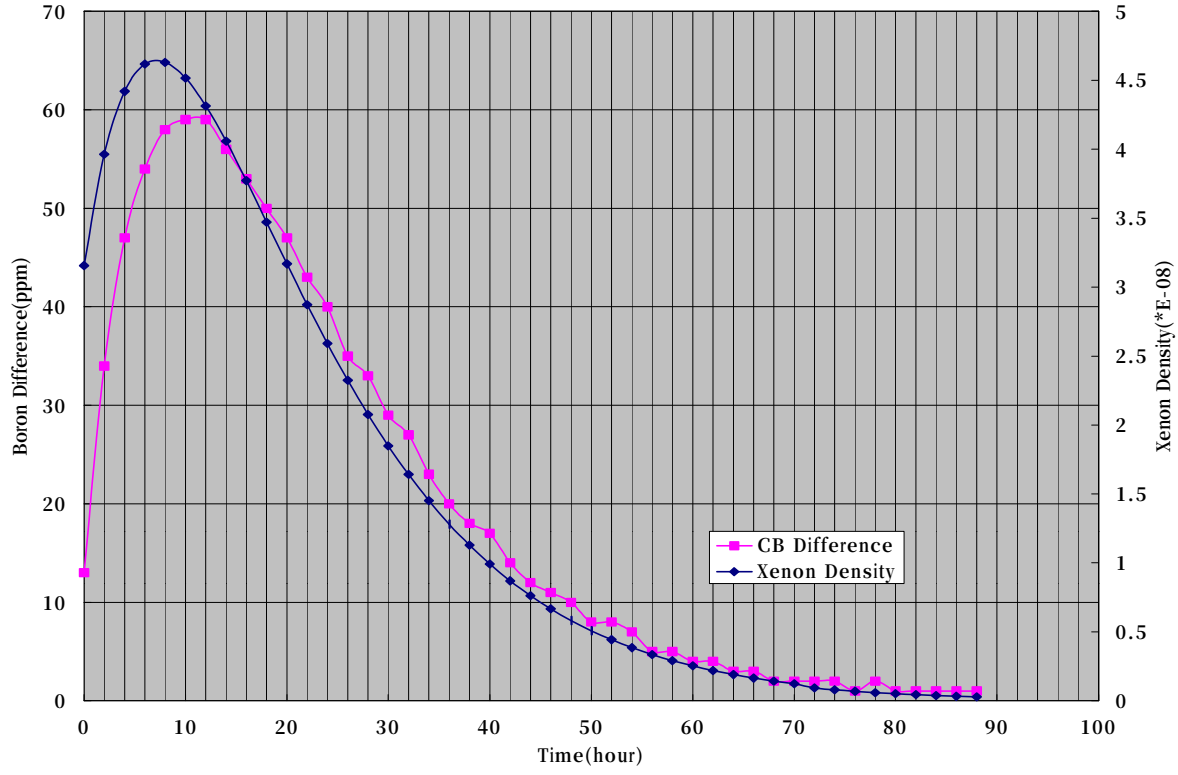


Table 1. Buckling Correction Parameters for YGN Unit 1 Cycle 12

	C_0(ppm)	C_m(ppm)	ΔB^2	Δ PPM
BOL	13	59	0.08	13
MOL	17	59	0.08	21
EOL	14	64	0.08	27

Table 2. Input for ECP Calculation for Kori Unit 2 Cycle 14 at 4238 MWD/MTU

	Pre Shutdown	Post Shutdown	Critical Position
Time	07/14/98 11:18	-	07/14/98 16:45
Criticality	Critical (1.0)	Subcritical (<1.0)	Critical (1.0)
Boron Concentration	1103	1103	1103
Reactor Power	100% RTP	0% RTP	0% RTP
Tavg	306.1	-	291.7
Xenon	Equilibrium	Transient	Transient
D Bank Step	226	0	181

Table 3. Buckling Correction Parameters for Kori Unit 2 Cycle 14

	C_0(ppm)	C_m(ppm)	ΔB^2	Δ PPM
BOL	4	45	0.08	35
MOL	6	41	0.08	40
EOL	3	43	0.08	42

Table 4. ECP Calculation Results for Kori Unit 2 Cycle 14 at 4238 MWD/MTU

	Pre Shutdown			Critical Position			Difference
	Calc.	Meas.	Bias	Calc.	Meas.	With Bias	Meas. – Calc.(Bias)
ANC	1062	1103	41	1057	1103	1098	5
APOLLO	1061		42	1094		1136	-33
OASIS	1060		43	1055		1098	5

Figure 3. Critical Boron Change during BOL, HZP ARO, HFP EQ XE Xenon Depletion for Y1C12

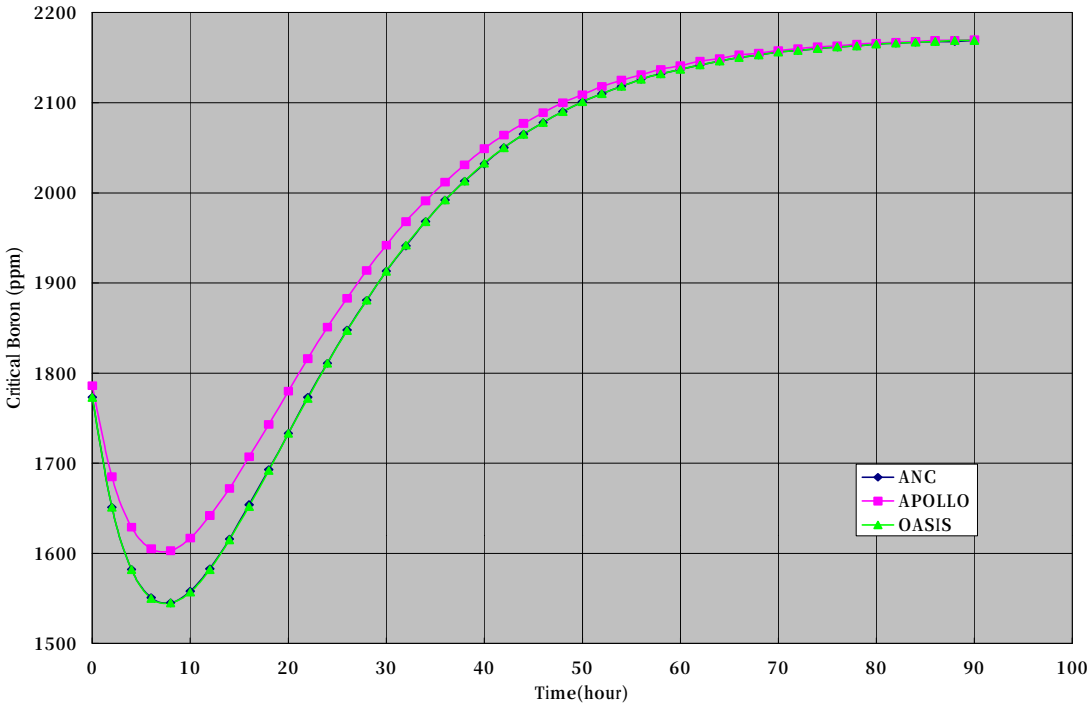


Figure 4. Critical Boron Change during MOL HZP ARO, HFP EQ XE Xenon Transient for Y1C12

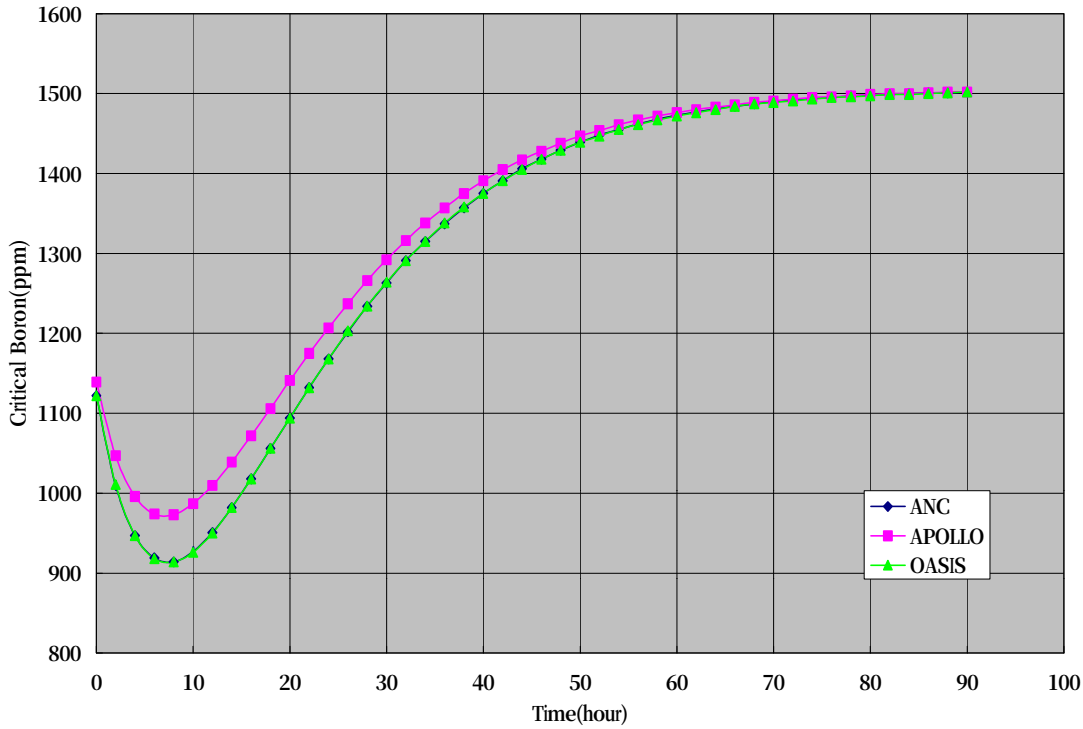
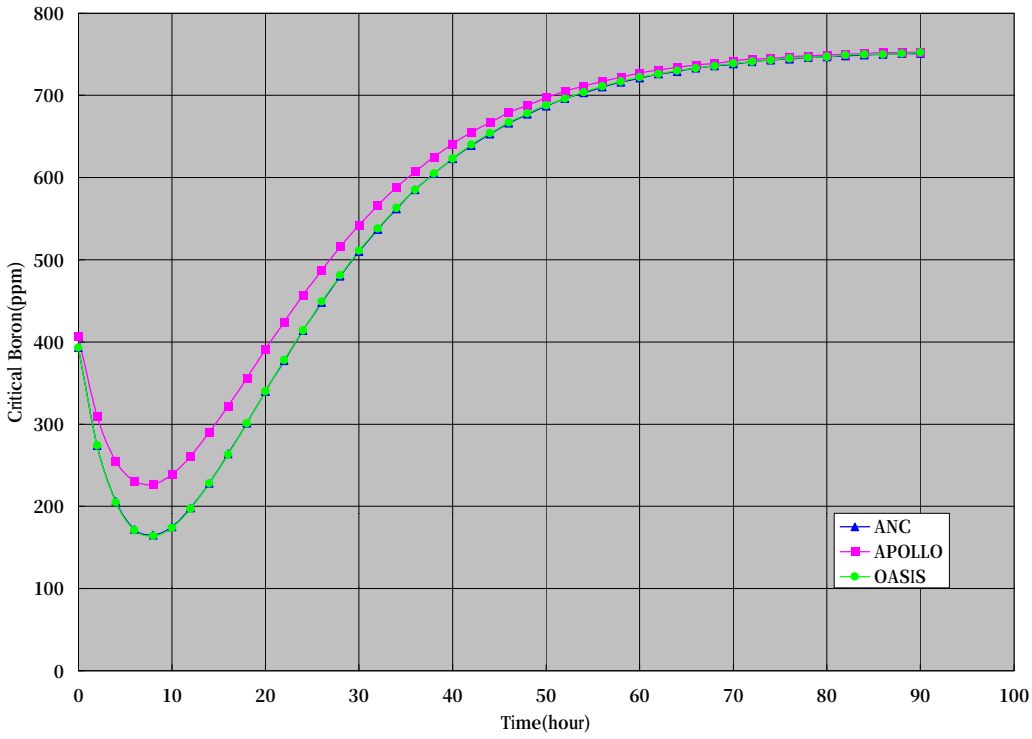


Figure 5. Critical Boron Change during EOL HZP ARO, HFP EQ XE Xenon Transients for Y1C12



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