

SMART

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Equivalent Bending Stiffness of Discontinuous Beam with Periodic Cross Sections for Dynamic Analysis of SMART CEDM

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Abstract

Equivalent bending stiffness of discontinuous beam with periodic cross sections is derived and its usability is verified by finite element analysis for natural frequency. It is general to use FE analysis with beam model for dynamic analysis of slender structures. The analysis efficiency is very low, however, if the beam structure has many discontinuous cross sections. The degree of freedom of FE model of the structure with many discontinuous points is very large. The mover of linear pulse motor type CEDM of SMART is a beam structure with many periodically repeated cross sections and its FE model is very complex due to the discontinuity between sections. The equivalent bending stiffness of this paper makes it possible to build simple FE model of the beam structure with many periodically repeated cross sections. The equation driven for the equivalent bending stiffness is very simple and useful as the results of example study show.

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[1,2].

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[3].

2. 가

2.1 가

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EI

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EI_{eq}

I_{eq}

A

B

I_A

I_B

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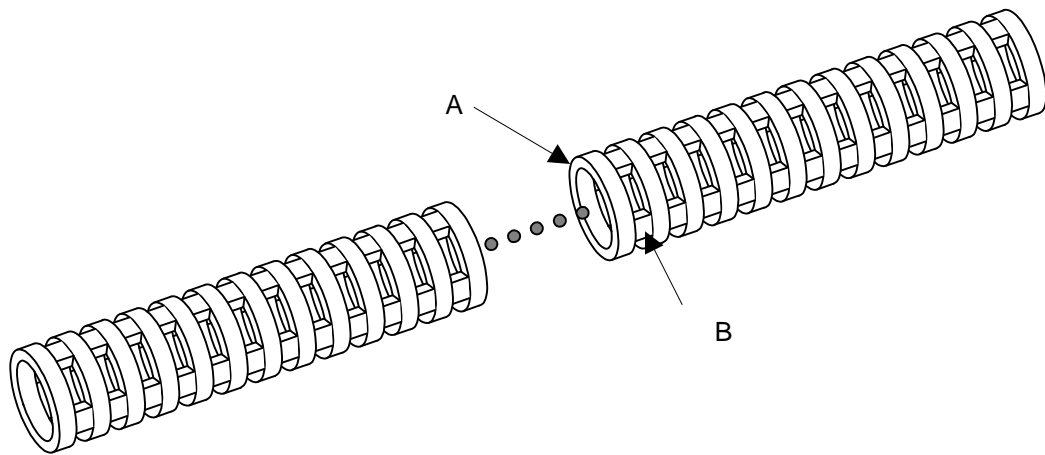
2

I_A

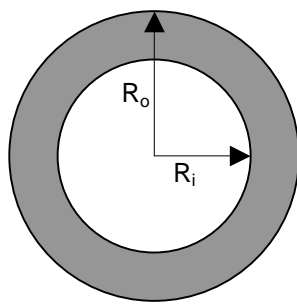
I_B

$$I_A = \frac{P}{4}(R_o^4 - R_i^4)$$

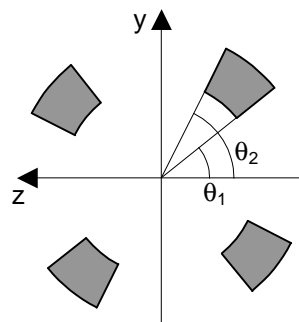
(1)



1. 가



A



B

2. 가

$$I_B = \frac{1}{2}(R_o^4 - R_i^4)(q_2 - q_1) \quad (2)$$

$$A_A = p(R_o^2 - R_i^2) \quad (3)$$

$$A_B = 2(R_o^2 - R_i^2)(q_2 - q_1) \quad (4)$$

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2.2

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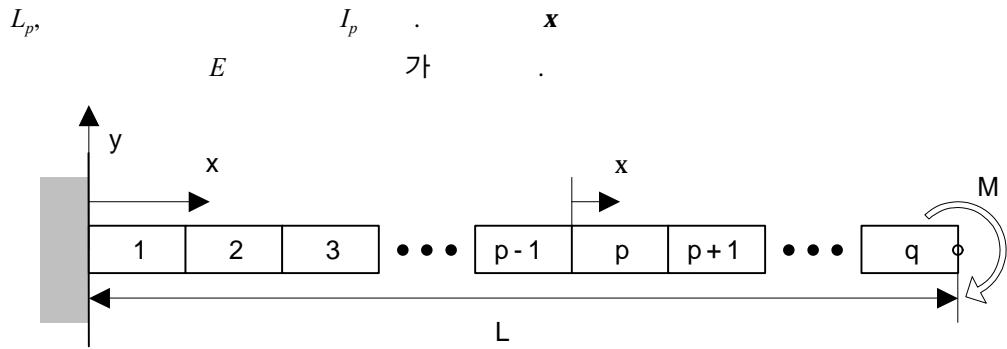
3

M 가

x

3 q

L



3. M 가 x $q(x)$ $y(x)$
 [4].

$$q(x) = \int_0^x \frac{M}{EI(x)} dx \quad (5)$$

$$y(x) = \int_0^x q(x) dx \quad (6)$$

(5) $q(x)$

$$q(x) = \frac{M}{E} \left(\int_0^{L_1} \frac{1}{I_1} dx + \int_{L_1}^{L_1+L_2} \frac{1}{I_2} dx + K + \int_{L_1+K+L_{p-1}}^x \frac{1}{I_p} dx \right) \quad (7)$$

$$= \frac{M}{E} \left[\sum_{i=1}^{p-1} \frac{L_i}{I_i} + \frac{1}{I_p} \left(x - \sum_{i=1}^{p-1} L_i \right) \right] \quad (8)$$

$$q(p, x) = \frac{M}{E} \left(\sum_{i=1}^{p-1} \frac{L_i}{I_i} + \frac{x}{I_p} \right) \quad (8)$$

p 가

$$x = x - \sum_{i=1}^{p-1} L_i \quad (9)$$

(7) (6) x $y(x)$

$$y(x) = \frac{M}{E} \int_0^x \left[\sum_{i=1}^{p-1} \frac{L_i}{I_i} + \frac{1}{I_p} \left(x - \sum_{i=1}^{p-1} L_i \right) \right] dx \quad (10)$$

p \mathbf{x}

$$\begin{aligned}
 y(p, \mathbf{x}) &= \frac{M}{E} \left[\sum_{k=1}^{p-1} \int_0^{L_k} \left(\sum_{i=1}^{k-1} \frac{L_i}{I_i} + \frac{\mathbf{x}}{I_k} \right) d\mathbf{x} + \int_0^{\mathbf{x}} \left(\sum_{i=1}^{p-1} \frac{L_i}{I_i} + \frac{\mathbf{x}}{I_p} \right) d\mathbf{x} \right] \\
 &= \frac{M}{E} \left[\sum_{k=1}^{p-1} \left(\sum_{i=1}^{k-1} \frac{L_i L_k}{I_i} + \frac{L_k^2}{2I_k} \right) + \sum_{i=1}^{p-1} \frac{L_i}{I_i} \mathbf{x} + \frac{1}{2I_p} \mathbf{x}^2 \right]
 \end{aligned} \tag{11}$$

$$\begin{aligned}
 \mathbf{q}_q &= \mathbf{q}(q, L_q) = \frac{M}{E} \left(\sum_{i=1}^{q-1} \frac{L_i}{I_i} + \frac{L_q}{I_q} \right) \\
 &= \frac{M}{E} \sum_{i=1}^q \frac{L_i}{I_i}
 \end{aligned} \tag{12}$$

$$\begin{aligned}
 y_q &= y(q, L_q) = \frac{M}{E} \left[\sum_{k=1}^{q-1} \left(\sum_{i=1}^{k-1} \frac{L_i L_k}{I_i} + \frac{L_k^2}{2I_k} \right) + \sum_{i=1}^{q-1} \frac{L_i L_q}{I_i} + \frac{L_q^2}{2I_q} \right] \\
 &= \frac{M}{E} \sum_{k=1}^q \left(\sum_{i=1}^{k-1} \frac{L_i L_k}{I_i} + \frac{L_k^2}{2I_k} \right)
 \end{aligned} \tag{13}$$

2.3

가 가

가

$$L_{2n-1} = L_A, L_{2n} = L_B \quad n = 1, 2, 3, \Lambda \tag{14}$$

$$I_{2n-1} = I_A, I_{2n} = I_B \quad n = 1, 2, 3, \Lambda \tag{15}$$

가

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m

1 : $q = 2m$

$$\mathbf{q}_q = \frac{M}{E} \sum_{i=1}^m \left(\frac{L_A}{I_A} + \frac{L_B}{I_B} \right) = \frac{M}{E} m \left(\frac{L_A}{I_A} + \frac{L_B}{I_B} \right) \tag{16}$$

$$\begin{aligned}
y_q &= \frac{M}{E} \left[\sum_{k=1}^{2m} \left(\sum_{i=1}^{k-1} \frac{L_i}{I_i} \right) L_k + \frac{m}{2} \left(\frac{L_A^2}{I_A} + \frac{L_B^2}{I_B} \right) \right] \\
&= \frac{M}{E} \left[\frac{m^2(L_A + L_B)}{2} \left(\frac{L_A}{I_A} + \frac{L_B}{I_B} \right) + \frac{m}{2} L_A L_B \left(\frac{1}{I_A} - \frac{1}{I_B} \right) \right]
\end{aligned} \tag{17}$$

가 (18) (19), (20)

$$L_A = L_B = \frac{L}{q} = \frac{L}{2m} \tag{18}$$

$$\mathbf{q}_q = \frac{ML}{E} \frac{I_A + I_B}{2I_A I_B} \tag{19}$$

$$y_q = \frac{ML^2}{2E} \left(\frac{I_A + I_B}{2I_A I_B} - \frac{I_A - I_B}{4mI_A I_B} \right) \tag{20}$$

가 m

2 : $q = 2m - 1$

$$\mathbf{q}_q = \frac{M}{E} \left[\sum_{i=1}^m \left(\frac{L_A}{I_A} + \frac{L_B}{I_B} \right) - \frac{L_B}{I_B} \right] = \frac{M}{E} \left[m \left(\frac{L_A}{I_A} + \frac{L_B}{I_B} \right) - \frac{L_B}{I_B} \right] \tag{21}$$

$$\begin{aligned}
y_q &= \frac{M}{E} \left[\sum_{k=1}^{2m-1} \left(\sum_{i=1}^{k-1} \frac{L_i}{I_i} \right) L_k + \frac{m}{2} \left(\frac{L_A^2}{I_A} + \frac{L_B^2}{I_B} \right) - \frac{L_A^2}{2I_A} \right] \\
&= \frac{M}{E} \left[\frac{m^2(L_A + L_B)}{2} \left(\frac{L_A}{I_A} + \frac{L_B}{I_B} \right) + \frac{m}{2} L_A L_B \left(\frac{1}{I_A} - \frac{1}{I_B} \right) \right] \\
&\quad - \frac{M}{E} \left[m \left(\frac{L_A}{I_A} + \frac{L_B}{I_B} \right) L_B - \frac{L_B^2}{2I_B} \right]
\end{aligned} \tag{22}$$

가 (28) (24), (25)

$$L_A = L_B = \frac{L}{q} = \frac{L}{2m-1} \tag{23}$$

$$\mathbf{q}_q = \frac{ML}{E} \left[\frac{m}{2m-1} \frac{I_A + I_B}{I_A I_B} - \frac{1}{(2m-1)I_B} \right] \tag{24}$$

$$y_q = \frac{ML^2}{2E} \left[\frac{2m^2}{(2m-1)^2} \frac{I_A + I_B}{I_A I_B} - \frac{m}{(2m-1)^2} \frac{3I_A + I_B}{I_A I_B} + \frac{1}{(2m-1)^2 I_B} \right] \tag{25}$$

2.4

가

(19), (24)

(20), (25)

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$$\lim_{m \rightarrow \infty} q = \frac{ML}{E} \frac{I_A + I_B}{2I_A I_B} = \frac{ML}{EI_{eq}} \quad (26)$$

$$\lim_{m \rightarrow \infty} y_p = \frac{ML^2}{2E} \frac{I_A + I_B}{2I_A I_B} = \frac{ML^2}{2EI_{eq}} \quad (27)$$

가

가

EI_{eq}

가

I_{eq}

$$I_{eq} = \frac{2I_A I_B}{I_A + I_B} \quad (28)$$

(28)

가

가

가

가

3. 가

3.1

2

가

ABAQUS

가 2m

17

가

가

가

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가

(28)

1.

	E (GPa)	r (kg/m ³)	R_i (m)	R_o (m)	A (m ²)	I (m ⁴)
A	210	7800	0.08	0.10	2.830×10^{-3}	2.900×10^{-6}
B	210	7800	0.08	0.10	0.566×10^{-3}	0.580×10^{-6}
가	210	7800	•	•	1.698×10^{-3}	0.967×10^{-6}

가

A가

100

가

3.

()	Bending Mode (Hz)							Longitudinal Mode (Hz)		
	1	2	3	4	5	6	7	1	2	3
1(3)	18.561	141.85	349.58	879.63	1320.9	1856.0	2898.8	473.06	1945.8	3418.5
2(5)	17.738	121.95	374.68	733.35	1115.3	2217.5	2850.6	479.83	1336.3	3243.0
3(7)	17.529	114.92	342.73	716.88	1214.0	1777.0	2316.5	481.61	1397.0	2128.1
4(9)	17.445	112.27	326.27	672.59	1169.1	1798.3	2516.7	482.34	1419.2	2258.3
5(11)	17.403	110.99	318.31	645.36	1112.3	1731.3	2489.3	482.70	1429.8	2316.0
6(13)	17.380	110.27	313.96	629.94	1073.7	1661.8	2403.5	482.91	1435.8	2346.8
7(15)	17.365	109.83	311.33	620.67	1049.3	1611.4	2321.3	483.04	1439.5	2365.3
8(17)	17.355	109.54	309.62	614.71	1033.6	1577.1	2258.7	483.13	1441.9	2377.3
9(19)	17.348	109.34	308.45	610.66	1022.9	1553.6	2213.8	483.19	1443.6	2385.5
10(21)	17.343	109.19	307.60	607.78	1015.4	1537.1	2181.6	483.23	1444.8	2391.4
11(23)	17.339	109.08	306.97	605.65	1009.9	1525.0	2158.0	483.27	1445.8	2395.8
12(25)	17.336	109.00	306.5	604.04	1005.8	1516.0	2140.4	483.29	1446.5	2399.1
13(27)	17.334	108.93	306.12	602.79	1002.6	1509.0	2126.9	483.31	1447.0	2401.8
14(33)	17.329	108.80	305.38	600.35	996.38	1495.7	2101.1	483.35	1448.1	2406.9
15(35)	17.328	108.77	305.22	599.8	995.02	1492.8	2095.6	483.36	1448.3	2408.1
16(51)	17.324	108.65	304.53	597.54	989.37	1480.8	2072.9	483.40	1449.4	2412.9
17(99)	17.321	108.57	304.08	596.08	985.78	1473.3	2059.0	483.42	1450.1	2416.0
가	17.323	108.56	303.97	595.66	984.67	1470.9	2054.4	648.59	1945.8	3243.0

3.3

4(a) 3 가 가
 가 가 50
 7 가 가 1% 가
 4(a) 가
 B
 가
 가 가
 5 가
 7 7
 4(a)
 4(b) 가 가
 가
 가 (29) 가 [3].

$$A_{eq} = \frac{2A_A A_B}{A_A + A_B} \quad (29)$$

(29) 가 가
 가 . 가
 가 가

(30)

$$A_{eq} = \frac{A_A + A_B}{2} \quad (30)$$

(30) 가 가
 가 가
 가 가 4(b)
 가 가
 가 가

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가 가 가 가
 가 가 가 가
 (28) 가 가 가
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[1] , SMART-CD-DW620-00, Rev.00, CEDM

[2] , SMART-CD-DW620-02, Rev.00,

[3] , SMART-CD-CA620-05, Rev.00, 가 가

[4] Gere & Timoshenko, "Mechanics of Materials," Second Edition.