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A Nuclear Reactor Power Controller Using a Receding Horizon Control Method

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Abstract

A receding horizon control method is applied to design a fully automatic controller for thermal power in a reactor core. The basic concept of the receding horizon control is to solve an optimization problem for a finite future at current time and to implement as the current control input the first optimal control input among the solutions of the finite time steps. The procedure is then repeated at each subsequent instant. The receding horizon controller is designed so that the difference between the output and the desired output is minimized and the variation of the control rod position is small. The nonlinear PWR plant model (nonlinear point kinetics equation with six delayed neutron groups and the lumped thermal-hydraulic balance equations) was used to verify the proposed controller of reactor power. And a controller design model used for designing the receding horizon controller was obtained by applying a parameter estimation algorithm. From numerical simulation results, the performances of this controller for the 5%/min ramp increase or decrease of a desired load and its 10% step increase or decrease which are design requirements are proved to be excellent.

1. Introduction

Power plants are highly complex, nonlinear, time-varying, and constrained systems. For example, the plant characteristics vary with operating power levels, and ageing effects in plant performance and changes in nuclear core reactivity with fuel burnup generally degrade system performance. Also, if load-following operation is desired, daily load cycles can change plant performance significantly. The fully automatic power tracking control of nuclear reactors has not been accepted mainly due to the safety concerns of imprecise knowledge about the time-varying parameters, nonlinearity, and modeling uncertainty. However, rapid and smooth power maneuvering has its benefits in view of the economical and safe operation of reactors and the importance of load-following strategy.

A digital processor offers flexibility because the control function can be altered by software and this facilitates provisions of sophisticated control. Also, instrumentation and control (I&C) technology has been improved rapidly. In spite of these positive aspects of using a digital controller, for many reasons modern control systems have not been incorporated extensively in nuclear power plants. However, problems created by growing obsolescence of existing technology have stimulated interest in upgrading these systems (EPRI, 1992).

The conventional reactor control system consists of a temperature deviation channel (the difference between the programmed coolant temperature and the average coolant temperature) and a power mismatch channel (difference between the turbine load and the nuclear power). The conventional control method drives the control rods by compensating and filtering these two channels. This method has its own advantages of easy implementation, well-proven technology. However, it is difficult to optimally design compensators and filters for controllers because of variations in nuclear system parameters, nonlinear reactor dynamics, and complex temperature feedback effects. Techniques for the optimal control of nuclear reactors were studied extensively in the past two decades (Cho, et al., 1983; Niar, et al., 1987; Lin, et al., 1989; Park, et al., 1993). But it is difficult or often impossible to design optimal controllers for nuclear systems because of variations in nuclear system parameters and modeling uncertainties.

The receding horizon control methodology has received much attention as a powerful tool for the control of industrial process systems (Kwon, et al., 1977; Richalet, et al., 1978; Clarke, et al., 1991; Garcia, et al., 1989;

Gothare, et al., 1996; Lee, et al., 1997; Lee, et al., 1998). The basic concept of the receding horizon control is to solve an optimization problem for a finite future at current time and to implement the first optimal control input as the current control input. That is, at the present time k the behavior of the process over a horizon N is considered and the process output to changes in the manipulated variable is predicted by using a mathematical design model. The moves of the manipulated variables are selected such that the predicted output has certain desirable characteristics. However, only the first computed change in the manipulated variable is implemented and at each subsequent instant, the procedure is repeated. This method has many advantages over the conventional infinite horizon control because it is possible to handle input and state (or output) constraints in a systematic manner during the design and implementation of the control. In particular, it is a suitable control strategy for nonlinear time varying systems because of the receding horizon concept and recently, the problem of controlling uncertain dynamical systems has been of considerable interest to control engineers. The receding horizon control method has been applied to a nuclear area by Na (2001) for the first time.

In this paper, a receding horizon control method is developed to design a fully automatic controller for thermal power in a reactor core and the proposed control method is applied to the nonlinear PWR plant model (nonlinear point kinetics equation with six delayed neutron groups and the lumped thermal-hydraulic balance equations) developed by Park (1993).

2. Receding Horizon Control Method

The receding horizon control method is to solve an optimization problem for a finite future at current time and to implement the first optimal control input as the current control input. The procedure is then repeated at each subsequent instant. Figure 1 shows this basic concept (Garcia, et al., 1989). As it were, for any assumed set of present and future control moves, the future behavior of the process outputs can be predicted over a horizon N, and the M present and future control moves ($M \le N$) are computed to minimize a quadratic objective function. Although M control moves are calculated, only the first control move is implemented. At the next time step, new values of the measured output are obtained, the control horizon is shifted forward by one step, and the same calculations are repeated. The purpose of taking new measurements at each time step is to compensate for unmeasured disturbances and model inaccuracy, both of which cause the measured system output to be different from the one predicted by the model. At every time instant, receding horizon control requires the on-line solution of an optimization problem to compute optimal control inputs over a fixed number of future time instants, known as the time horizon. The on-line optimization can be typically reduced to either a linear program or a quadratic program. The basic idea of receding horizon control is to calculate a sequence of future control signals in such a way that it minimizes a multistage cost function defined over a prediction horizon.

Also, in order to achieve fast responses and prevent excessive control effort, the associated performance index for deriving an optimal control input is represented by the following quadratic function:

$$J = \frac{1}{2}\sum_{j=1}^{N} Q\left[\hat{y}(t+j \mid t) - w(t+j)\right]^2 + \frac{1}{2}\sum_{j=1}^{M} R\left[\Delta u(t+j-1)\right]^2 , \tag{1}$$

subject to constraints $\begin{cases} y(t+N+i) = w(t+N+i), \ i = 1,\Lambda\ , m \\ \Delta u(t+j-1) = 0, \quad j > M \end{cases}$

where Q and R weight the reactor coolant temperature error $(\hat{y} - w)$ and reactivity (control input) change between time step (control rod step change between time step) at certain future time intervals, respectively, and w is a setpoint (desired coolant average temperature) or reference sequence for the output signal. $\hat{y}(t+j \mid t)$ is an optimum j-step-ahead prediction of the system output (nuclear power level) based on data up to time t; that is, the expected value of the output at time t if the past input and output and the future control sequence are known. N and M are called the prediction horizon and the control horizon, respectively. The prediction horizon represents the limit of the instant in which it is desired for the output to follow the reference sequence. In order to obtain control inputs, the predicted outputs have to be first calculated as a function of past values of inputs and outputs and of future control signals. The constraint, $\Delta u(t+j-1) = 0$ for $j > M$, means that there is no variation in the control signals after a certain interval $M < N$, which is the control horizon concept. The constraint, $y(t+N+i) = w(t+N+i),\ i = 1,\Lambda\ , m$, which makes the output follow the reference input over some range, guarantees the stability of the controller (Kwon, et al., 1977).

The optimal control input that minimizes the foregoing objective function will be derived from now on. The process to be controlled is described by the following Controlled Auto-Regressive and Integrated Moving Average (CARIMA) model, which is widely used as a mathematical model of controller design methods:

$$A(q^{-1})y(t) = B(q^{-1})u(t-1) + C(q^{-1})v(t-1) + \frac{1}{\Delta}D(q^{-1})\xi(t) \ , \tag{2}$$

where y is an output (coolant temperature), u is a control input (reactivity), v is a measurable disturbance (steam flowrate), ξ is a stochastic random noise sequence with zero mean value, q^{-1} is the backward shift operator, e.g., $q^{-1}y(t) = y(t-1)$, and Δ is defined as $\Delta = 1 - q^{-1}$. In Eq. (2), $A(q^{-1})$ and $D(q^{-1})$ are monic polynomials as a function of the backward shift operator q^{-1}, and $B(q^{-1})$ and $C(q^{-1})$ are polynomials. For example, the polynomial $A(q^{-1})$ is expressed as follows:

$$A(q^{-1}) = a_0 + a_1q^{-1} + a_2q^{-2} + \ldots + a_{nA}q^{-nA} \ , \tag{3}$$

where a_0, $a_1, \ldots, a_{nA}$ are coefficients and nA is the order of the polynomial.

The process output at time $t + j$ can be predicted from the measurements of the output and input up to time step t. The optimal prediction is derived by solving a Diophantine equation, whose solution can be found by an efficient recursive algorithm. In this derivation, the most usual case of $D(q^{-1}) = 1$ will be considered. The j-step-ahead output prediction of a process is derived below.

Multiplying Eq. (2) by $\Delta E_j(q^{-1})$ from the left gives

$$y(t+j) - E_j(q^{-1})\xi(t+j) = F_j(q^{-1})y(t) + E_j(q^{-1})B(q^{-1})\Delta u(t+j-1) + E_j(q^{-1})C(q^{-1})\Delta v(t+j-1) \ , \tag{4}$$

where $E_j(q^{-1})$ and $F_j(q^{-1})$ are polynomials satisfying

$$1 = E_j(q^{-1})\tilde{A}(q^{-1}) + q^{-j}F_j(q^{-1}) \ , \tag{5}$$

$$E_j(q^{-1}) = e_{j,0} + e_{j,1}q^{-1} + \ldots + e_{j,j-1}q^{-(j-1)} \ , \tag{6}$$

$$F_j(q^{-1}) = f_{j,0} + f_{j,1}q^{-1} + f_{j,2}q^{-2} + \ldots + f_{j,nA}q^{-nA} \ , \tag{7}$$

$$\tilde{A}(q^{-1}) = A(q^{-1})\Delta \ . \tag{8}$$

Equation (5) is called the Diophantine equation and there exist unique polynomials $E_j(q^{-1})$ and $F_j(q^{-1})$ of order $j-1$ and nA, respectively, such that $e_{j,0} = 1$. By taking the expectation operator and considering that $E\{\xi(t)\} = 0$, the optimal j-step-ahead prediction of $\hat{y}(t+j \mid t)$ satisfies

$$\hat{y}(t+j \mid t) = F_j(q^{-1})y(t) + G_j(q^{-1})\Delta u(t+j-1) + H_j(q^{-1})\Delta v(t+j-1) \ , \tag{9}$$

where

$$G_j(q^{-1}) = E_j(q^{-1})B(q^{-1}) \ ,$$

$$H_j(q^{-1}) = E_j(q^{-1})C(q^{-1}),$$

$$\hat{y}(t+j \mid t) = E\{y(t+j) \mid t\}.$$

$\hat{y}(t+j\,|\,t)$ denotes an estimated value of the output at time step $t+j$ based on all the data up to time step t. The output prediction can easily be extended to the nonzero mean noise case by adding a term $\mathrm{E}_j(q^{-1})E\{\xi(t)\}$ to the output prediction $\hat{y}(t+j\,|\,t)$.

By dividing the matrix polynomials, $G_j(q^{-1})$ and $H_j(q^{-1})$, like the following equations:

$$G_j(q^{-1}) = \overline{G}_j(q^{-1}) + q^{-j}\tilde{G}_j(q^{-1}) \quad \text{with } \delta\left(\overline{G}_j(q^{-1})\right) < j\,,$$

$$H_j(q^{-1}) = \overline{H}_j(q^{-1}) + q^{-j}\tilde{H}_j(q^{-1}) \quad \text{with } \delta\left(H_j(q^{-1})\right) < j\,,$$

the prediction equation, Eq. (9), can now be written as

$$\begin{aligned}\hat{y}(t+j\,|\,t) = {} & \overline{G}_j(q^{-1})\Delta u(t+j-1) + \overline{H}_j(q^{-1})\Delta v(t+j-1) \\ & + \tilde{G}_j(q^{-1})\Delta u(t-1) + \tilde{H}_j(q^{-1})\Delta v(t-1) + F_j(q^{-1})y(t)\,,\end{aligned} \tag{10}$$

where $\delta(\cdot)$ denotes the order of a polynomial. The last three terms of the right hand side of Eq. (10) consist of past values of the process input, measurable disturbance and output variables and correspond to the response of the process if the control and measurable input signals are kept constant. On the other hand, the first two terms of the right hand side consist of future values of the control input signal and the measurable disturbance and correspond to the response obtained when the initial conditions are zero $y(t-j)=0$, $\Delta u(t-j-1)=0$, $\Delta v(t-j-1)=0$ for $j>0$ (Camacho, et al., 1999). Equation (10) can be rewritten as

$$\hat{y}(t+j\,|\,t) = \overline{G}_j(q^{-1})\Delta u(t+j-1) + \overline{H}_j(q^{-1})\Delta v(t+j-1) + f_j\,, \tag{11}$$

where

$$f_j = \tilde{G}_j(q^{-1})\Delta u(t-1) + \tilde{H}_j(q^{-1})\Delta v(t-1) + F_j(q^{-1})y(t)\,. \tag{12}$$

Then a set of N j-step-ahead output predictions can be expressed as

$$\hat{\mathbf{y}} = \overline{\mathbf{G}}\Delta\mathbf{u} + \overline{\mathbf{H}}\Delta\mathbf{v} + \mathbf{f}\,, \tag{13}$$

where

$$\hat{\mathbf{y}} = \left[\hat{y}(t+1\,|\,t) \quad \hat{y}(t+2\,|\,t) \quad \cdots \quad \hat{y}(t+j\,|\,t) \quad \cdots \quad \hat{y}(t+N\,|\,t)\right]^T,$$

$$\Delta\mathbf{u} = \left[\Delta u(t) \quad \Delta u(t+1) \quad \cdots \quad \Delta u(t+j) \quad \cdots \quad \Delta u(t+N-1)\right]^T,$$

$$\Delta\mathbf{v} = \left[\Delta v(t) \quad \Delta v(t+1) \quad \cdots \quad \Delta v(t+j) \quad \cdots \quad \Delta v(t+N-1)\right]^T,$$

$$\mathbf{f} = \left[f_1 \quad f_2 \quad \cdots \quad f_j \quad \cdots \quad f_N\right]^T,$$

$$\overline{\mathbf{G}} = \begin{bmatrix} g_0 & 0 & \cdots & 0 & \cdots & 0 \\ g_1 & g_0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ g_{j-1} & g_{j-2} & \cdots & g_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{N-1} & g_{N-2} & \cdots & \cdots & \cdots & g_0 \end{bmatrix}, \quad \overline{\mathbf{H}} = \begin{bmatrix} h_0 & 0 & \cdots & 0 & \cdots & 0 \\ h_1 & h_0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ h_{j-1} & h_{j-2} & \cdots & h_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ h_{N-1} & h_{N-2} & \cdots & \cdots & \cdots & h_0 \end{bmatrix},$$

$$\overline{G}_j(q^{-1}) = \sum_{i=0}^{j-1} g_i q^{-i}\,,$$

$$\overline{H}_j(q^{-1}) = \sum_{i=0}^{j-1} h_i q^{-i}\,.$$

If all initial conditions are zero, the response $\mathbf{f}$ is zero. If a unit step is applied to the first input at time t; that is, $\Delta\mathbf{u} = [1\,0 \cdots 0]^T$, the expected output sequence $[\hat{y}(t+1)\ \hat{y}(t+2) \cdots \hat{y}(t+N)]^T$ is equal to the first column of the matrix $\overline{\mathbf{G}}$. That is, the first column of the matrix $\overline{\mathbf{G}}$ can be calculated as the step response of the plant when a unit step is applied to the first control signal. The matrix $\overline{\mathbf{H}}$ can be calculated in the same way.

The computation of the control input involves the inversion of an $N \times N$ matrix $\overline{\mathbf{G}}$ that requires a substantial amount of computation. If the control signal is kept constant after the first M control moves (that is, $\Delta u(t+j-1) = 0$ for $j > M$) due to the receding horizon control concept, this leads to the inversion of an $M \times M$ matrix, which reduces the amount of computation. If so, the set of predictions affecting the objective function can be expressed as

$$\hat{\mathbf{y}} = \overline{\mathbf{G}}_s \Delta\mathbf{u}_s + \overline{\mathbf{H}}\Delta\mathbf{v} + \mathbf{f}, \tag{14}$$

where

$$\overline{\mathbf{G}}_s = \begin{bmatrix} g_0 & 0 & \cdots & 0 \\ g_1 & g_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_{N-1} & g_{N-2} & \cdots & g_{N-M} \end{bmatrix},$$

$$\Delta\mathbf{u}_s = [\Delta u(t) \quad \Delta u(t+1) \quad \cdots \quad \Delta u(t+M-1)]^T.$$

The following relationship can be derived from the foregoing equation:

$$\hat{\mathbf{y}}_f = \overline{\mathbf{G}}_{sf} \Delta\mathbf{u}_s + \overline{\mathbf{H}}_f \Delta\mathbf{v} + \mathbf{f}_f, \tag{15}$$

where

$$\hat{\mathbf{y}}_f = [\hat{y}(t+N+1\,|\,t) \quad \hat{y}(t+N+2\,|\,t) \quad \cdots \quad \hat{y}(t+N+m\,|\,t)]^T,$$

$$\mathbf{f}_f = [f_{N+1} \quad f_{N+2} \quad \cdots \quad f_{N+m}]^T,$$

$$\overline{\mathbf{G}}_{sf} = \begin{bmatrix} g_N & g_{N-1} & \cdots & g_{N-M+1} \\ g_{N+1} & g_N & \cdots & g_{N-M+2} \\ \vdots & \vdots & \ddots & \vdots \\ g_{N+m-1} & g_{N+m-2} & \cdots & g_{N-M+m} \end{bmatrix},$$

$$\overline{\mathbf{H}}_f = \begin{bmatrix} h_N & h_{N-1} & \cdots & h_1 \\ h_{N+1} & h_N & \cdots & h_2 \\ \cdots & \cdots & \ddots & \cdots \\ h_{N+m-1} & h_{N+m-2} & \cdots & h_m \end{bmatrix}.$$

The objective function of Eq. (1) can be rewritten as the following matrix-vector form:

$$\begin{aligned} J &= \frac{1}{2}(\hat{\mathbf{y}} - \mathbf{w})^T \tilde{\mathbf{Q}} (\hat{\mathbf{y}} - \mathbf{w}) + \frac{1}{2}\Delta\mathbf{u}_s^T \tilde{\mathbf{R}} \Delta\mathbf{u}_s \\ &= \frac{1}{2}(\overline{\mathbf{G}}_s \Delta\mathbf{u}_s + \overline{\mathbf{H}}\Delta\mathbf{v} + \mathbf{f} - \mathbf{w})^T \tilde{\mathbf{Q}} (\overline{\mathbf{G}}_s \Delta\mathbf{u}_s + \overline{\mathbf{H}}\Delta\mathbf{v} + \mathbf{f} - \mathbf{w}) + \frac{1}{2}\Delta\mathbf{u}_s^T \tilde{\mathbf{R}} \Delta\mathbf{u}_s, \end{aligned} \tag{16}$$

$$\text{subject to } \mathbf{w}_f = \overline{\mathbf{G}}_{sf} \Delta\mathbf{u}_s + \overline{\mathbf{H}}_f \Delta\mathbf{v} + \mathbf{f}_f, \tag{17}$$

where

$$\mathbf{w} = [w(t+1\,|\,t) \quad w(t+2\,|\,t) \quad \cdots \quad w(t+N\,|\,t)]^T,$$

$$\mathbf{w}_f = \left[w(t+N+1\,|\,t) \quad w(t+N+2\,|\,t) \quad \Lambda \quad w(t+N+m\,|\,t) \right]^T .$$

$\tilde{\mathbf{Q}} = diag(Q, \Lambda\ , Q)$ is a diagonal matrix consisting of N diagonal elements, Q, and $\tilde{\mathbf{R}} = diag(R, \Lambda\ , R)$ is a diagonal matrix consisting of M diagonal elements, R. Usually $\tilde{\mathbf{Q}} = \mathbf{I}_{N\times N}$ and $\tilde{\mathbf{R}} = \omega \times \mathbf{I}_{M\times M}$ are used and ω is called an input-weighting factor.

The optimal input can be obtained by the well-known Lagrange multiplier approach. To apply the Lagrange multiplier approach, the objective function is rewritten as

$$\begin{aligned} J' = {} & \frac{1}{2}\left(\overline{\mathbf{G}}_s \Delta\mathbf{u}_s + \overline{\mathbf{H}}\Delta\mathbf{v} + \mathbf{f} - \mathbf{w}\right)^T \tilde{\mathbf{Q}} \left(\overline{\mathbf{G}}_s \Delta\mathbf{u}_s + \overline{\mathbf{H}}\Delta\mathbf{v} + \mathbf{f} - \mathbf{w}\right) + \frac{1}{2}\Delta\mathbf{u}_s^T \tilde{\mathbf{R}} \Delta\mathbf{u}_s \\ & + \boldsymbol{\lambda}^T \left(\overline{\mathbf{G}}_{sf} \Delta\mathbf{u}_s + \overline{\mathbf{H}}_f \Delta\mathbf{v} + \mathbf{f}_f - \mathbf{w}_f\right). \end{aligned} \tag{18}$$

By setting to zero the differential of the foregoing objective function with regard to $\Delta\mathbf{u}_s$, the following equation is obtained:

$$\Delta\mathbf{u}_s = \left(\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \overline{\mathbf{G}}_s + \tilde{\mathbf{R}}\right)^{-1} \left[\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \left(\mathbf{w} - \mathbf{f} - \overline{\mathbf{H}}\Delta\mathbf{v}\right) - \overline{\mathbf{G}}_{sf}^T \boldsymbol{\lambda}\right]. \tag{19}$$

To calculate the control input, $\Delta\mathbf{u}_s$, the Lagrange multiplier $\boldsymbol{\lambda}$ must be known. Therefore, Eq. (19) is substituted into the constraint equation, Eq. (17) as follows:

$$\mathbf{w}_f = \overline{\mathbf{G}}_{sf} \left(\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \overline{\mathbf{G}}_s + \tilde{\mathbf{R}}\right)^{-1} \left[\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \left(\mathbf{w} - \mathbf{f} - \overline{\mathbf{H}}\Delta\mathbf{v}\right) - \overline{\mathbf{G}}_{sf}^T \boldsymbol{\lambda}\right] + \overline{\mathbf{H}}_f \Delta\mathbf{v} + \mathbf{f}_f . \tag{20}$$

From Eq. (20), the Lagrange multiplier $\boldsymbol{\lambda}$ can be expressed as

$$\boldsymbol{\lambda} = \left[\overline{\mathbf{G}}_{sf} \left(\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \overline{\mathbf{G}}_s + \tilde{\mathbf{R}}\right)^{-1} \overline{\mathbf{G}}_{sf}^T\right]^{-1} \left[\overline{\mathbf{G}}_{sf} \left(\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \overline{\mathbf{G}}_s + \tilde{\mathbf{R}}\right)^{-1} \left[\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \left(\mathbf{w} - \mathbf{f} - \overline{\mathbf{H}}\Delta\mathbf{v}\right)\right] + \overline{\mathbf{H}}_f \Delta\mathbf{v} + \mathbf{f}_f - \mathbf{w}_f\right]. \tag{21}$$

By substituting Eq. (21) into Eq. (19), the optimal control input can be expressed as

$$\begin{aligned} \Delta\mathbf{u}_s = {} & \left(\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \overline{\mathbf{G}}_s + \tilde{\mathbf{R}}\right)^{-1} \Bigg[\overline{\mathbf{G}}_s^T \ \tilde{\mathbf{Q}} \left(\mathbf{w} - \mathbf{f} - \overline{\mathbf{H}}\Delta\mathbf{v}\right) \\ & + \overline{\mathbf{G}}_{sf}^T \left[\overline{\mathbf{G}}_{sf} \left(\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \overline{\mathbf{G}}_s + \tilde{\mathbf{R}}\right)^{-1} \overline{\mathbf{G}}_{sf}^T\right]^{-1} \left[\mathbf{w}_f - \mathbf{f}_f - \overline{\mathbf{H}}_f \Delta\mathbf{v} - \overline{\mathbf{G}}_{sf} \left(\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \overline{\mathbf{G}}_s + \tilde{\mathbf{R}}\right)^{-1} \left[\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \left(\mathbf{w} - \mathbf{f} - \overline{\mathbf{H}}\Delta\mathbf{v}\right)\right]\right]\Bigg]. \end{aligned} \tag{22}$$

Calculating the control input requires the inversion of matrices $\left(\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \overline{\mathbf{G}}_s + \tilde{\mathbf{R}}\right)$ and $\overline{\mathbf{G}}_{sf} \left(\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \overline{\mathbf{G}}_s + \tilde{\mathbf{R}}\right)^{-1} \overline{\mathbf{G}}_{sf}^T$. From the definition of matrix $\overline{\mathbf{G}}_{sf}$, it can be derived that the number of output constraint m cannot be bigger than the number of control signal variations M; that is, $m \le M$. Another condition for invertibility must be satisfied; $m \le n+1$ since the coefficient g_i of the step response is a linear combination of the previous $n+1$ values (n is the system order). Therefore, the inversion of matrix $\overline{\mathbf{G}}_{sf} \left(\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \overline{\mathbf{G}}_s + \tilde{\mathbf{R}}\right)^{-1} \overline{\mathbf{G}}_{sf}^T$ requires inverting a matrix of which the dimension m is not usually bigger than three or four. Since only $\Delta u(t)$ is needed at time step t, only the first row of the matrices, $\left(\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \overline{\mathbf{G}}_s + \tilde{\mathbf{R}}\right)^{-1} \overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}}$ and $\left(\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \overline{\mathbf{G}}_s + \tilde{\mathbf{R}}\right)^{-1} \overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \overline{\mathbf{G}}_{sf}^T \left[\overline{\mathbf{G}}_{sf} \left(\overline{\mathbf{G}}_s^T \tilde{\mathbf{Q}} \overline{\mathbf{G}}_s + \tilde{\mathbf{R}}\right)^{-1} \overline{\mathbf{G}}_{sf}^T\right]^{-1}$, is required to be computed. Also, in order to obtain the control input from Eq. (22), it is necessary to calculate the matrices $\overline{\mathbf{G}}_s$, $\overline{\mathbf{G}}_{sf}$, $\overline{\mathbf{H}}$ and $\overline{\mathbf{H}}_f$, and the vectors $\mathbf{f}$ and $\mathbf{f}_f$. These matrix and vector can be calculated recursively. From now on, the derivation will be described.

By taking into account a new Diophantine equation corresponding to the prediction for $\hat{y}(t+j+1\,|\,t)$, Eq. (5) can also be rewritten as follows:

$$1 = E_{j+1}(q^{-1})\tilde{A}(q^{-1}) + q^{-(j+1)} F_{j+1}(q^{-1}) . \tag{23}$$

Subtracting Eq. (5) from Eq. (23) gives

$$0 = \left[E_{j+1}(q^{-1}) - E_j(q^{-1})\right]\tilde{A}(q^{-1}) + q^{-j}\left[q^{-1}F_{j+1}(q^{-1}) - F_j(q^{-1})\right] \; . \tag{24}$$

Since the matrix $E_{j+1}(q^{-1}) - E_j(q^{-1})$ is of order j, the matrix can be written as

$$E_{j+1}(q^{-1}) - E_j(q^{-1}) = \tilde{P}(q^{-1}) + p_j q^{-j} \,, \tag{25}$$

where $\tilde{P}(q^{-1})$ is a polynomial of order smaller than or equal to $j-1$. By substituting Eq. (25) into Eq. (24)

$$0 = \tilde{P}(q^{-1})\,\tilde{A}(q^{-1}) + q^{-j}\left[p_j\tilde{A}(q^{-1}) + q^{-1}F_{j+1}(q^{-1}) - F_j(q^{-1})\right] \; . \tag{26}$$

Since $\tilde{A}(q^{-1})$ is monic, it is easy to see that $\tilde{P}(q^{-1}) = 0$. Therefore, from Eq. (25) the polynomial $E_{j+1}(q^{-1})$ can be calculated recursively by

$$E_{j+1}(q^{-1}) = E_j(q^{-1}) + p_j q^{-j} \,. \tag{27}$$

The following expressions can easily be obtained from Eq. (26):

$$p_j = f_{j,0} \,, \tag{28}$$

$$f_{j+1,i} = f_{j,i+1} - p_j\tilde{a}_{i+1} \text{ for } i = 0, \Lambda\ , \delta(F_{j+1}) \,. \tag{29}$$

Also, it can easily be seen that the initial conditions for the recursion equation are given by

$$E_1 = 1 \,, \tag{30}$$

$$F_1 = q\left(1 - \tilde{A}(q^{-1})\right). \tag{31}$$

The vectors $\mathbf{f}$ and $\mathbf{f}_f$ can be computed by the following recursive relationship:

$$f_{j+1} = q\left(1 - \tilde{A}(q^{-1})\right) f_j + B(q^{-1})\Delta u(t+j) + C(q^{-1})\Delta v(t+j) \,, \tag{32}$$

$$\text{with } f_0 = y(t)\,,\ \Delta u(t+j) = 0 \text{ and } \Delta v(t+j) = 0 \text{ for } j \ge 0 \,.$$

Also, the polynomials, $G_j(q^{-1})$ and $H_j(q^{-1})$, can be obtained recursively as follows:

$$G_{j+1}(q^{-1}) = E_{j+1}(q^{-1})B(q^{-1}) = G_j(q^{-1}) + f_{j,0}\, q^{-j} B(q^{-1}), \tag{33}$$

$$H_{j+1}(q^{-1}) = E_{j+1}(q^{-1})C(q^{-1}) = H_j(q^{-1}) + f_{j,0}\, q^{-j} C(q^{-1}), \tag{34}$$

At every time instant, the receding horizon controller solves *on-line* an optimization problem by using Eqs. (22), (29) and (32) through (34) to compute optimal control inputs.

3. Application to Nuclear Power Control

Numerical simulations were conducted to study the performance of the proposed algorithm. The nonlinear PWR plant model (nonlinear point kinetics equation with six delayed neutron groups and the lumped thermal-hydraulic balance equations) developed by Park (1993) was used to apply the proposed control method. The simplified pressurized water reactor model was developed based on the following assumptions:

1) The primary and secondary loops of a PWR are modeled.
2) A nonlinear lumped parameter model of the primary loop is used.
3) Xenon and fuel depletion effects are not considered.
4) Single-phase heat transfer of the core coolant is considered.
5) Primary loop mass flow rate and pressure are constant.
6) Reactor power and core inlet-outlet temperatures are measured.

The process dynamics based on physical laws result in the following differential equations:

$$\frac{dP}{dt} = \frac{\rho - \beta}{l} P + \sum_{i=1}^{6} \lambda_i \beta_i \,, \tag{35}$$

$$\rho = \rho_0 + \alpha_f T_f + \alpha_c T_{avg} + bu, \tag{36}$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{l} P - \lambda_i C_i \,,\ (i = 1, \cdots, 6) \tag{37}$$

$$\frac{dT_f}{dt} = -\frac{UA}{M_i c_{pf}} (T_f - T_{avg}) + \frac{J}{M_f c_{pf}} P, \tag{38}$$

$$\frac{dT_{avg}}{dt} = \frac{UA}{M_c c_{pc}} (T_f - T_{avg}) - \frac{\dot{m}}{M_c} (T_{out} - T_{in}), \tag{39}$$

$$\frac{dT_{in}}{dt} = \frac{1}{\tau_{cl}} (T_{cl} - T_{in}), \tag{40}$$

$$\frac{dT_{hl}}{dt} = \frac{1}{2\tau_{hl}} (T_{avg} - T_{in}), \tag{41}$$

$$\frac{dT_s}{dt} = -\frac{1}{\tau_s} (T_s - T_{hl}) - D_1 L_T \,, \tag{42}$$

$$T_{cl} = D_2 T_s - D_3 T_{hl} \,. \tag{43}$$

As shown in Fig. 2, a part of the parameters in the preceding equations represent temperatures at specific locations, control input and desired load trajectory and other parameters have their usual meanings. The process is simulated using the fifth-order Runge-Kutta method with adaptive time step sizes to deal with stiffness inherent in nuclear reactor dynamics. A simplified diagram of PWR plants is shown in Fig. 2. The reactor coolant system model is divided into five nodes to simulate the energy balance between fuel and coolant and the transport delays between a reactor core and a steam generator. The steam generator model contains heat transfer between the reactor coolant system and the secondary side. The turbine load variation L_T is performed by changing steam flow to the turbine. The thermal part of this model is an extension of the linear, time-invariant model used by Park, et al. (1986) and the nominal values used in this work are listed in Table 1. All the thermodynamic properties included in the plant model are calculated from the steam table within the range of subcooled state. Nonlinearity in the heat transfer between fuel and coolant is considered from the heat transfer coefficient U of the Dittus-Boelter correlation (Rust, 1979):

$$U = C_v \,\mathrm{Re}^{0.8}\, \mathrm{Pr}^{0.4} \frac{K_c}{D_e} \ \text{where}\ \mathrm{Re} = \frac{D_e v \rho_c}{\mu} \ \text{and}\ \mathrm{Pr} = \frac{c_{pc}\mu}{K_c} \,. \tag{44}$$

The plant dynamics were approximated by a conventional parameter estimation algorithm in order to obtain the controller design model. The design model is as follows:

$$A(q^{-1}) y(t) = B(q^{-1}) u(t-1) + C(q^{-1}) v(t-1) \,, \tag{45}$$

where

$$A(q^{-1}) = 1 - 1.26867 q^{-1} + 0.05129 q^{-2} - 0.07760 q^{-3} + 0.29436 q^{-4} \,,$$

$$B(q^{-1}) = -0.05561 + 0.17524 q^{-1} + 0.04213 q^{-2} - 0.16240 q^{-3} \,,$$

$$C(q^{-1}) = -0.09530 + 0.16062 q^{-1} - 0.27699 q^{-2} - 0.16240 q^{-3} \,.$$

In Eq. (45), $y(k)$ is the average coolant temperature, $u(k)$ the position of the control rods and $v(k)$ the reactor power. The measurable disturbance $v(k)$ must be the steam flow to the turbine but the change of the steam flow brings that of the reactor power which has more close relationship to the average coolant temperature. Therefore, $v(k)$ is considered as the reactor power.

The nuclear reactor is controlled so that the average coolant temperature may track the programmed (desired) coolant temperature versus demand load, while an excessively large effort is not called for. Although most nuclear power plants are usually operated at 100 percent power level (base load), sometimes at startup time, trivial problem occurrences and also if they are operated in load-follow mode, nuclear power plants can be operated at relatively low power levels. The nuclear reactor is usually required to cope with the power variations of 5%/min ramp and 10 % step. Therefore, in this paper, the nuclear power controller was designed to deal with these transients (coolant temperature deviation and load disturbance) and especially, computer simulations were conducted to investigate the output tracking performance. Therefore, it is supposed that the plant to be controlled is initially in a steady state condition and then the reference coolant temperature or the desired power changes. In the computer simulation, the operating condition of the process is in a steady state for initial 200 sec at a demand power of 50% and a rod position of 100 steps. The demand power for which the proposed control algorithm is tested is shown in Fig. 3. The demand power increases continuously at a rate, 5%/min, from 200 to 680 sec and approaches 90 % power level at 680 sec. And the power remains constant for 300 sec and decreases continuously at a rate, 5%/min from 980 to 1160 sec. And then the power remains constant at 75% power level for 300 sec and the 10% step increase of the demand power occurs at 1460 sec. Then the power remains constant at 85% power level for 500 sec and at 1960 sec, the 10% step decrease of the demand power occurs.

In numerical simulations, the sampling time was chosen to be 0.4 sec. The prediction and control horizons and the parameter for the constraint m were chosen as 5, 1 and 1, respectively, and the same values were used regardless of power level. Also, the weighting factors, Q and R, are 1 and 10, respectively, and the same weighting factors were used irrespective of the power level. Since the computer code for the nonlinear model had been written in the Fortran language, in order to perform the numerical simulations, the proposed MATLAB (Mathworks, 1999) control algorithm was interfaced with the code written in the Fortran language.

The average coolant temperature tracks very well its setpoint change according to load as shown in Fig. 4 and from Fig. 5 it is shown that the reactor power tracks the demand load very well. The position of control rods is shown in Fig. 6 and it follows the pattern similar to the power. Figure 7 shows several plant states including fuel temperature, hot-leg temperature and steam generator temperature. It is known that the proposed controller copes with the power variations of 5%/min ramp and 10 % step. Also, it was verified from many simulations that the performance of the controller is not sensitive to the values of the weighting factor and the prediction and control horizons.

4. Conclusions

In this work, the receding horizon controller was developed to control the nuclear power in pressurized water reactor. The developed controller was applied to a nonlinear model for nuclear steam generators. The nonlinear PWR plant model (nonlinear point kinetics equation with six delayed neutron groups and the lumped thermal-hydraulic balance equations) was used to verify the proposed controller for reactor power. And a controller design model used for designing the receding horizon controller was obtained by applying a parameter estimation algorithm and became a fourth-order linear model. It is known that the proposed controller controls the control rod position so that the average coolant temperature tracks very well its setpoint change according to load and also the reactor power tracks the demand load very well. From these numerical simulation results, the performances of this controller for the 5%/min ramp increase or decrease of a desired load and its 10% step increase or decrease which are design requirements are proved to be excellent.

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Table 1. Nominal values of a nonlinear plant model.

Reactor physics Parameters	β	β_1	β_2	β_3	β_4	β_5	β_6
	0.007108	0.000216	0.001416	0.001349	0.00218	0.00095	0.000322
	$\lambda\left[s^{-1}\right]$	$\lambda_1\left[s^{-1}\right]$	$\lambda_2\left[s^{-1}\right]$	$\lambda_3\left[s^{-1}\right]$	$\lambda_4\left[s^{-1}\right]$	$\lambda_5\left[s^{-1}\right]$	$\lambda_6\left[s^{-1}\right]$
	0.078	0.0125	0.0308	0.1152	0.3109	1.24	3.3287
	$l\,[s]$	$\alpha_f\left[{}^{o}C^{-1}\right]$	$\alpha_c\left[{}^{o}C^{-1}\right]$				
	5.0×10^{-4}	-2.0×10^{-5}	-5.0×10^{-5}				
Thermal hydraulic parameters	C_v	$D_e\,[m]$	D_1	D_2	D_3	D_4	
	0.0301	0.01297	3.746	0.7005	–0.2995	102.7	
	$\tau_{cl}\,[s]$	$\tau_{hl}\,[s]$	$\tau_s\,[s]$	$\tau_1\,[s]$	$\tau_2\,[s]$	$\tau_3\,[s]$	$\tau_4\,[s]$
	7.0	5.0	11.3	5.58	2.03	80.5	2.08

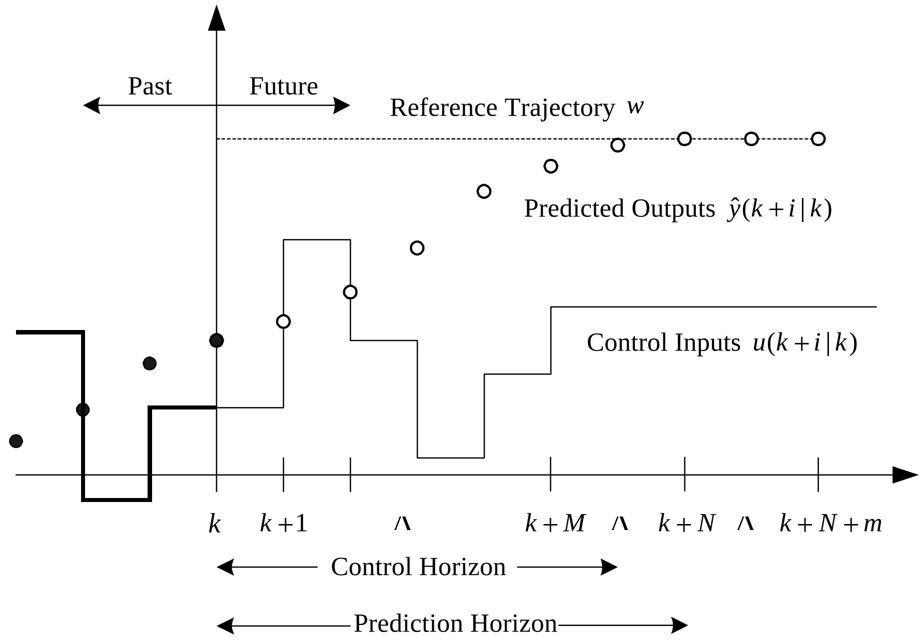

Fig. 1. Basic concept of a receding horizon control method.

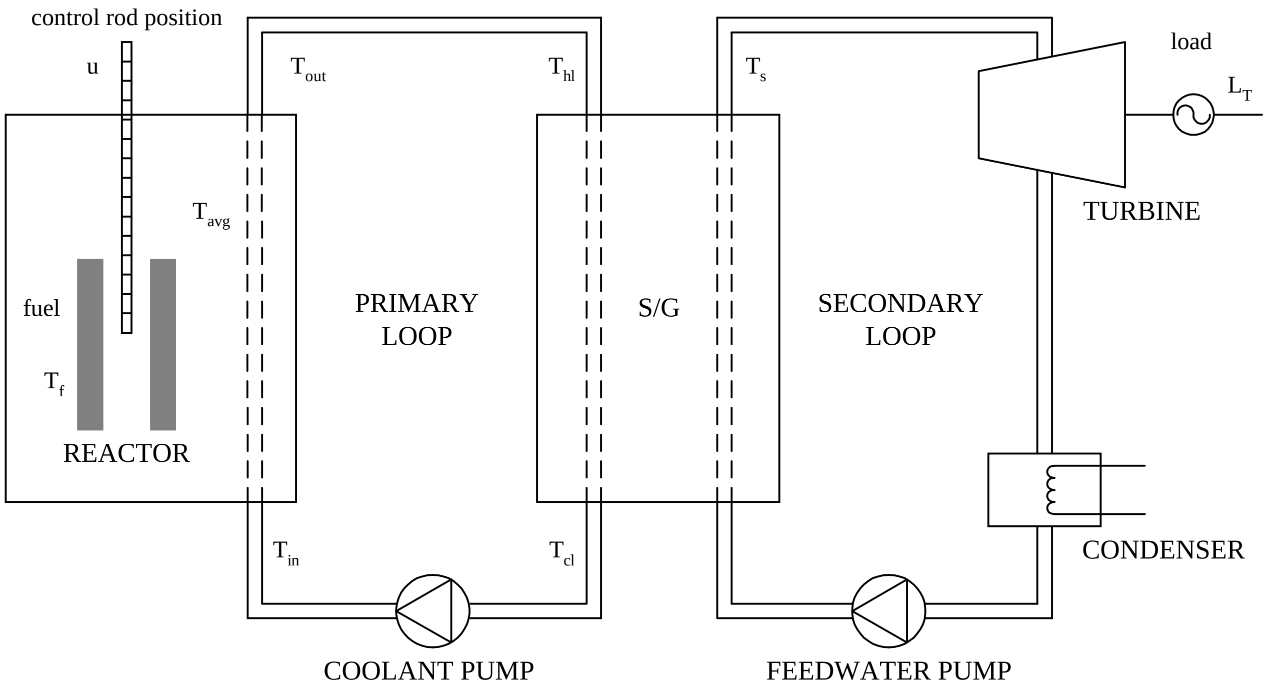

Fig. 2. Simplified diagram of PWR plants.

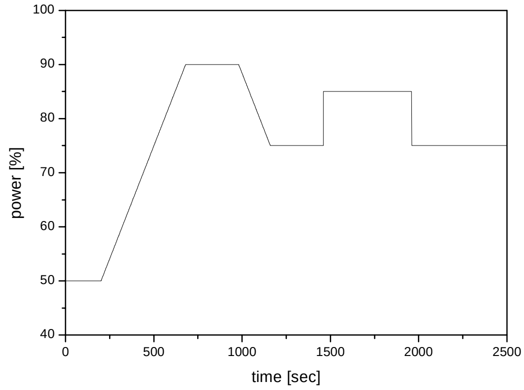

Fig. 3. Desired power trajectory.

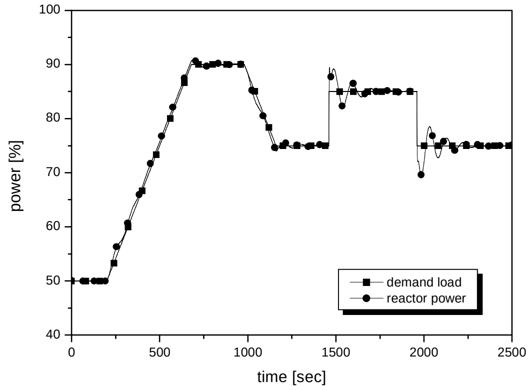

Fig. 4. Nuclear reactor power.

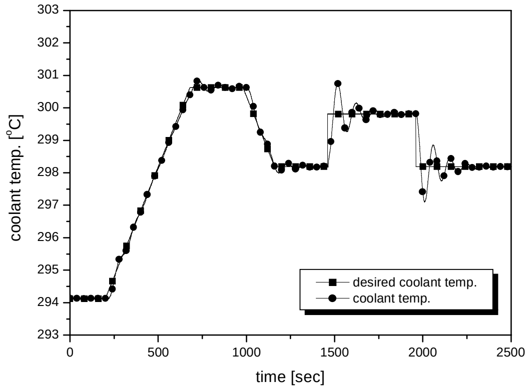

Fig. 5. Reactor coolant temperature.

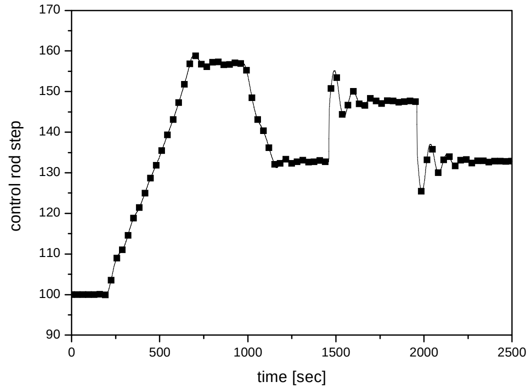

Fig. 6. Control rod step.

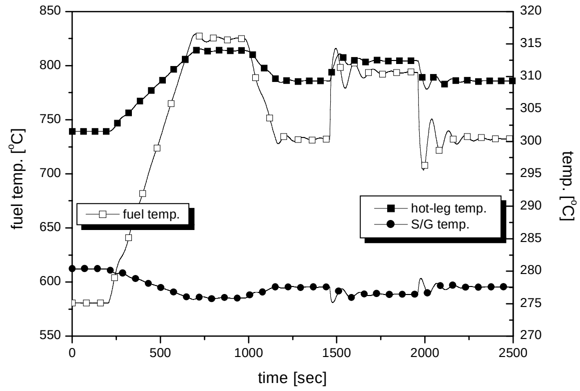

Fig. 7. Various plant states.