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Turbulent Friction Factors for Bare Rods in a Triangular Array

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Abstract

Under an assumption that the universal velocity profile for a circular tube can be applied to a non-circular channel, a theoretical and general friction factor model for an infinite triangular array of a bare rod bundle in turbulent flow region for small P/D, i.e., P/D£1.2 has been obtained from the normalized wall shear stress profile. The model has been extended to be applicable to large P/D conditions, i.e., P/D>1.2, under the assumption of uniform wall shear stress and the equivalent annular zone concept for a large rod distance.

The turbulent friction factor for a triangular array of a bare rod bundle in a hexagonal duct has been developed based on the existing friction factor model for an infinite triangular array of a bare rod bundle by incorporating the effects of the wetted perimeter and the number of rods.

The present and the existing friction factor correlations have been compared with the existing measured friction factor data.

1. Introduction

The development of friction factors is necessary for the prediction of pressure drop through rod bundles. A number of earlier workers have already presented various models for the friction factor of a rod bundle in a triangular array and there have also been efforts to obtain the friction factor from the universal velocity profile.

Maubach [1] developed a friction factor correlation for fully developed turbulent flow in an annulus. He assumed that the universal velocity profile is valid also for an annular zone. He proposed the geometry factor G of a turbulent flow for circular tubes, annular zones, and parallel plates. The geometry factor G has a characteristic value for each shape of the flow channel, independent of the roughness of the channel or the Reynolds number.

Rehme [2] developed a simple method to obtain the friction factor for turbulent flows through hexagonal channels. He used the ‘equivalent’ annular zone concept such that a hexagonal elementary cell around each rod in an infinite rod bundle can be replaced by the annular zone of the same area as shown in Fig.1. And he proposed the geometry parameter G for the equivalent annular zone. The equivalent annular zone is a good approximation for rod bundles with large rod space ratios (P/D>1.2) where the wall shear stress is relatively constant around the perimeter of the rod. The maximum value of the pressure drop coefficient is obtained from a uniform flow distribution throughout the rod bundle, i.e. for nearly identical hydraulic diameters of the subchannels. In rod bundles, the channel wall has the effect of causing the pressure drop coefficients always to be below the values for infinite rod arrays and thus below the values of the equivalent annular zone. The values of the equivalent annular zone represent the upper limit of the pressure drop coefficient whether the rod bundle is surrounded by duct wall or not.

Rehme [3] developed a simple turbulent friction factor for multiple non-circular channels by comparing many theoretical calculations for turbulent friction factor from different investigators.

Cheng and Todreas [4] developed a friction factor for turbulent flow in a triangular array of a bare rod bundle as a function of P/D by fitting the results of Rehme [3].

Lee [5] developed a general and theoretical model for infinite triangular and rectangular array rod bundles with a small P/D from the normalized wall shear stress profile.

2. Infinite Triangular Array of a Rod Bundle

The law of the wall assumes that the velocity distribution in the viscous dominant wall region is independent of the channel shape or pipe diameter. For the turbulent dominant wall region, the law of the wall is

$$u^{+} = 2.5 \ln y^{+} + 5.5 \quad (1)$$

where $u^{+} \equiv u/u^{*}$ is the dimensionless flow velocity, $y^{+} \equiv yu^{*}/\nu$ is the dimensionless distance from the wall, $u^{+} \equiv \sqrt{\tau_w/\rho}$, and $u^{*} = \sqrt{\tau_w/\rho}$ is the friction velocity.

Under an assumption that Eq.(1) is applicable for the entire pipe radius, the friction factor in circular channels can be predicted fairly well. The friction factor for a non-circular channel can be obtained by integrating the velocity profile over the cross-sectional area of the channel.

2.1 A Rod Bundle with Small P/D

Lee [5] developed a general friction factor relationship for P/D<1.2 from the law of the wall. In the present study, some of the errors in his paper are corrected and his model is extended to large P/D.

For an infinite triangular array of a bare rod bundle as shown in Fig.2, the average velocity over the cross-section can be obtained using the universal velocity profile as follows:

$$\bar{u} = \frac{1}{A}\int u dA = \frac{1}{A}\int_0^{\theta_{max}}\int_0^{y_{max}} u(R+y)\,dy\,d\theta \quad (2)$$

where $\theta_{max} = \frac{\pi}{6}$ and $y_{max} = \frac{P}{2}\sec\theta - R$.

The cross sectional area of the triangular array is

$$\begin{aligned} A &= \int_0^{\theta_{max}} \int_0^{y_{max}} (R+y)\,dy\,d\theta \\ &= \int_0^{\theta_{max}} \frac{1}{2} Y_1 Y_2\, d\theta \end{aligned} \quad . \qquad (3)$$

where $Y_1 = \frac{P}{2}\sec\theta - R$ and $Y_2 = \frac{P}{2}\sec\theta + R$.

Using the universal velocity profile Eq.(1), the average velocity over the channel cross-section can be obtained as follows:

$$\bar{u} = \frac{1}{A}\int_0^{\theta_{max}} \int_0^{y_{max}} u^* \left[2.5\ln\left(\frac{u^* y}{\nu} \right) + 5.5 \right] (R+y)\,dy\,d\theta \,. \qquad (4)$$

Fig.3 shows measured local wall shear stress variation for an interior subchannel of a triangular array rod bundle. This figure shows that the normalized wall shear stress profile is a function of azimuthal angle and *P*/*D*. The profile is symmetric and it's amplitude increases as the *P*/*D* decreases. For a given *P*/*D*, the profile varies as a function of angle and the friction velocity is assumed as a multiplication of average friction velocity and a function of azimuthal angle:

$$u^* = \bar{u}^* F(\theta) \qquad (5)$$

where

$$F(\theta) = (1 - a\cos 6\theta - b\cos 12\theta)^{1/2} \quad (P/D \le 1.2)\,. \qquad (6)$$

For small rod space, i.e. *P/D*≤1.2, the coefficients *a* and *b* were correlated as a function of *P/D* by the authors:

$$\begin{aligned} a &= 15.3 - 25.3333\left(\frac{P}{D}\right) + 10.6667\left(\frac{P}{D}\right)^2 \\ b &= -0.75 - 1.4333\left(\frac{P}{D}\right) - 0.6667\left(\frac{P}{D}\right)^2 \end{aligned} \qquad (7)$$

Finally, the friction factor relationship for infinite bare rods array for turbulent flow is

$$\sqrt{\frac{8}{f}} = A_C\left[2.5\ln\left(\mathrm{Re}\sqrt{\frac{f}{8}} \right) + 5.5 \right] - G_C \qquad (8)$$

where

$$A_C = \frac{1}{2A}\int_0^{\theta_{max}} F(\theta) Y_1 Y_2 d\theta \tag{9}$$

$$G_C = \frac{2.5}{2A}\int_0^{\theta_{max}} F(\theta)\left\{\frac{Y_1 + 4R}{2Y_2} - \ln\left(F(\theta)Y_1\right)\right\} Y_1 Y_2 d\theta + A_C\, 2.5 \ln D_e \,. \tag{10}$$

2.2 A Rod Bundle with Large P/D

For a large rod space, i.e., *P/D*>1.2, the normalized friction velocity becomes relatively uniform and considered to be an average friction velocity:

$$F(\theta) \approx 1 \quad (P/D > 1.2)\,. \tag{11}$$

A_C of Eq.(9) is always equals to 1 when $F(\theta)=1$ and Eq.(10) becomes

$$G_C = \frac{2.5}{4}\left[\frac{(P/D)3\ln 3 + \pi}{\sqrt{3}(P/D)^2 - \pi/2} - 1\right] + 2.5\ln(D_e)\,. \tag{12}$$

The friction factor of Eq.(8) is simplified as follows:

$$\sqrt{\frac{8}{f}} = \left[2.5\ln\left(\mathrm{Re}\sqrt{\frac{f}{8}}\right) + 5.5\right] - \frac{2.5}{4}\left[\frac{(P/D)3\ln 3 + \pi}{\sqrt{3}(P/D)^2 - \pi/2} - 1\right] - 2.5\ln(D_e)\,. \tag{13}$$

For even larger rod space between the rods, i.e. *P/D*>1.4, the normalized friction velocity becomes uniform and the equivalent annular zone concept [2] is simple and effective (Fig.1). If we adopt the equivalent annular zone concept on the primary flow cell of an infinite triangular bare rod array, the average velocity Eq.(2) reduces to

$$\bar{u} = \frac{\bar{u}^*}{A}\int_R^{r_0}\left[2.5\ln\left(\frac{\bar{u}^*(r-R)}{\nu}\right) + 5.5\right] r dr\,. \tag{14}$$

By integration of Eq.(14), the following relationship can be obtained.

$$\begin{aligned} \frac{\bar{u}}{\bar{u}^*} &= 2.5\ln\left(\frac{u^*}{\nu}\right) + 5.5 + 2.5\left(\ln\left(\frac{r_0 - R}{D_e}\right) - \frac{r_0 + 3R}{2(r_0 + R)}\right) \\ &= 2.5\ln\left(\frac{u^*}{\nu}\right) + 5.5 - 2.5\left(\ln 2(x+1) + \frac{x+3}{2(x+1)}\right) \end{aligned} \tag{15}$$

where $x = r_0/r_1$.

After arrangement, Eq.(10) becomes

$$G_C = 2.5\ln 2(x+1) + \frac{1.25x + 3.75}{x+1} . \tag{16}$$

Finally, the turbulent friction factor for *P/D*>1.4 is

$$\sqrt{\frac{8}{f}} = 2.5\ln\left(\mathrm{Re}\sqrt{\frac{8}{f}}\right) + 5.5 - 2.5\ln 2(x+1) - \frac{1.25x + 3.75}{x+1} . \tag{17}$$

The above results is almost the same as the friction factor relationship of Rehme [2]:

$$\sqrt{\frac{8}{f}} = 2.5\ln\left(\mathrm{Re}\sqrt{\frac{f}{8}}\right) + 5.5 - 2.5\ln 2(x+1) - \frac{1.25x + 3.966}{x+1} . \tag{18}$$

Fig.4 shows the comparison between the predicted friction factor and the measured friction factor for rod bundles in hexagonal channels over the rod space ratios. The reference, f_R is the friction factor of Maubach [1]:

$$\frac{1}{\sqrt{f_R}} = 2.035\log\left(\mathrm{Re}\sqrt{f_R}\right) - 0.989 . \tag{19}$$

The measured data in Fig.4 are the experimental results of Rehme [2] (1.025≤*P/D*≤2.324, 7≤*N*≤61). Ratio of the friction factor for a triangular array to the friction factor for a circular tube decreases rapidly and the values are lower than those of the circular tube as the ratio of the pitch to the rod diameter decreases, i.e., *P/D*<1.2. In Fig.4, the equivalent zone solution of Rehme is linear over *P/D* and it fails to predict the friction factor in the range of *P/D*<1.2. The present model predicts well the experimental data for overall *P/D* ranges.

Fig.5 shows the comparison of Rehme's equivalent zone solution with the measured data. It always over-predicts (~35%) the measured data for overall ranges and the concept fails especially in the range of low rod distance ratios.

Fig.6 shows the comparison of the predicted value of the present model with the same measured data pool of Fig.5. The results agree with the measured data reasonably well but the scatter is still large. The discrepancies are due mainly to the fact that the measured data are not from infinite rod arrays but from finite rod arrays.

3. A Rod Bundle with Duct Wall

A rod array that is contained in a duct has a different wetted perimeter from that of an infinite rod array. The ratio of the bundle wetted perimeter (wetted perimeter of rod only) to the total wetted perimeter (wetted perimeter of rod and duct wall) is a function of *P/D* and the number of rod rings as shown in Fig.7. The relationship is correlated to be

$$\frac{P_{wt}}{P_{wb}} = 1.44e^{-0.049Z}\left(\frac{P}{D}\right)^{Z^{4.98}/10^4} . \tag{20}$$

where Z is the number of rod rings, i.e., Z=1 for 7-pin and Z=2 for 19-pin rod bundle.

The effects of duct wall on the friction factor can be expressed by the ratio of the total wetted perimeter to the wetted perimeter of a rod bundle:

$$f = f_{\infty}\left(\frac{P_{wb}}{P_{wt}}\right)^{1/4} \tag{21}$$

where f_{∞} is a friction factor of infinite triangular array rod bundle for turbulent flow, i.e. Eqs.(8), (13), and (17) according to the rod distance ratio.

Fig. 8 shows the comparison of the predicted value by Eq.(21) with the same measured data of Figs 5 and 7. The present model agrees well with the experimental data within ±10% for turbulent flow in overall test ranges.

4. Conclusions

A theoretical and general friction factor model for infinite triangular array of a bare rod bundle in turbulent flow region for small P/D, i.e., $P/D \le 1.2$ has been introduced from the normalized wall shear stress profile. The model has been extended to be applicable to large P/D conditions, i.e., $P/D > 1.2$, under the assumption of uniform wall shear stress and the equivalent annular zone concept for a large rod space. The friction factor for a triangular array rod bundle in a duct has been developed based on the friction factor model for an infinite triangular array rod bundle by incorporating the effects of the wetted perimeter and the number of rods.

The results can be summarized as follows:

(1) Infinite triangular array rod bundle:

- Eq.(8) for $P/D \le 1.2$, Eq.(13) for $P/D > 1.2$, and Eq.(17) for $P/D > 1.4$.

(2) Triangular array rod bundle with channel wall:

- Eq.(21).

The present friction factor model shows a good agreement with the measured friction factor data for wide ranges of test conditions.

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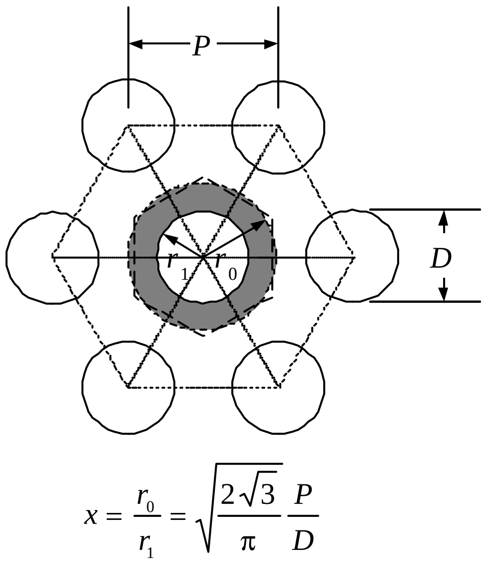

Fig. 1 Equivalent annular zone concept [2].

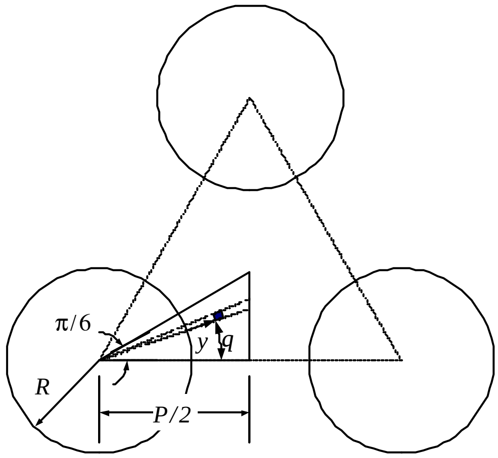

Fig. 2 Schematic diagram of an infinite triangular array for a bare rod bundle [5].

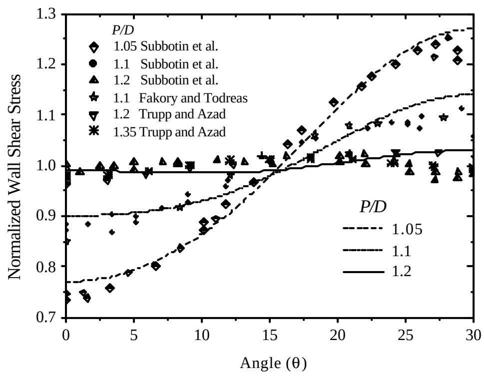

Fig. 3 Normalized wall shear stress variation for a triangular array [5].

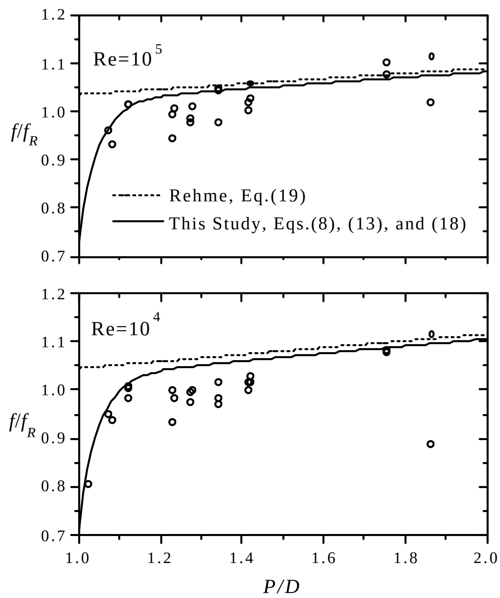

Fig. 4 Comparison between predicted values with measured friction factors for a triangular array of a bare rod bundle at $Re=10^4$ and 10^5

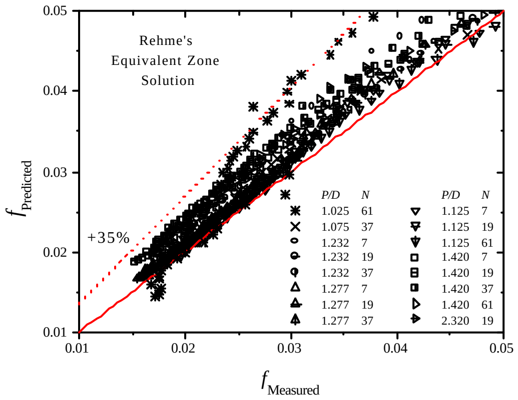

Fig. 5 Comparison of Rehme' s equivalent zone solution, Eq.(18) with the measured friction factor data for a triangular array rod bundle.

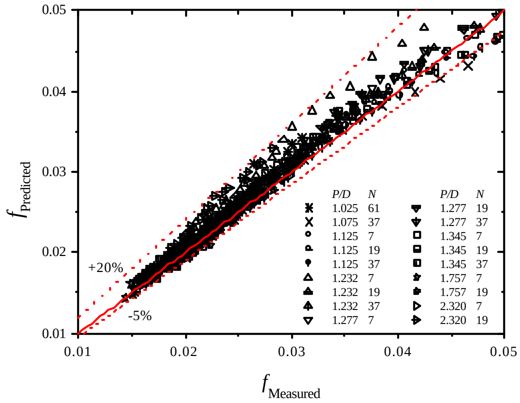

Fig. 6 Comparison of the predicted values by Eqs. (8), (13) and (17) with the measured data for a triangular array rod bundle.

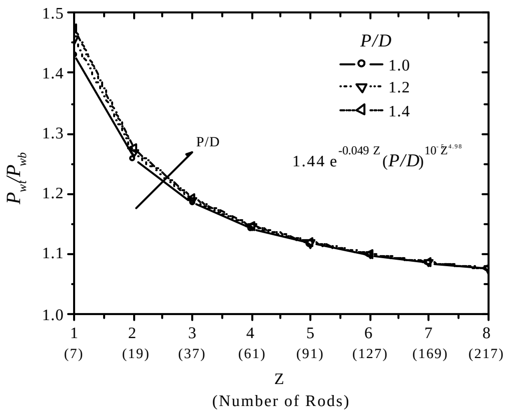

Fig. 7 Effects of duct wall and the number of rods for a hexagonal duct.

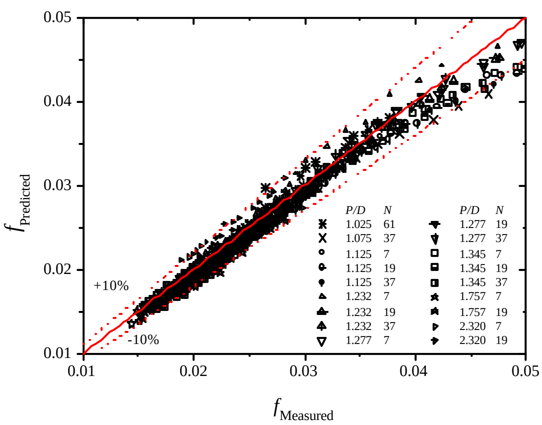

Fig. 8 Comparison of the predicted values of the present model, Eq.(21) with the measured friction factor data for a bare rod bundle with duct.