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Measurement of Subcriticality by Source Multiplication Method with Consideration of Higher Mode

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Abstract

It is important to develop a method for measuring subcriticality of a system with fissile materials from a viewpoint of nuclear criticality safety, and many techniques have been applied for this purpose. Neutron source multiplication method is one of the methods for subcriticality measurement, which is simple method. However, position of both neutron detector and neutron source cannot be considered in this method and it is possible to applied only to small subcriticality measurement. To overcome these difficulty, new source multiplication method with consideration of higher mode flux was developed. In the present method, neutron flux distribution with an external neutron source in subcritical state was expanded in a series of higher mode flux, and when neutron count of detector was measured at an already-known subcritical state, other unknown subcriticality can be obtained through measuring neutron counts. Numerical simulations were carried out to demonstrate the present method by simulating a KUCA core experiment. It is found that subcriticality can be obtained accurately up to -20%dk/k by the present method, and detector position dependency in measured subcriticality was also greatly decreased compared to the ordinary source multiplication method.

1. Introduction

Measuring subcriticality accurately at nuclear fuel facility such as fuel reprocessing plant is one of the main issues in nuclear criticality safety study. For this purpose, many methods have been proposed and studied so far. For example, neutron source multiplication method, pulse neutron method with use of external pulse generator system, neutron noise analysis method such as Feynman-α method (variance to mean ratio method) [1] and frequency analysis method and so on. Among these method, source multiplication method is a well-known experimental method where an external neutron source and neutron detectors are used and method of data analysis is very simple, and it has been often used in basic reactor experiments. Usually, its formulation of data analysis is based on the one point reactor theory with assumption of a bare homogeneous core, therefore, for accurate subcriticality measurement by this method, it is necessary to locate the neutron detectors carefully to eliminate the effect of higher mode components excited by an external neutron source, for example, the center of core is usually appropriate position for this measurement to obtain subcriticality accurately(2). However, when a detector is located at an arbitrary position of the core, such as in reflector region, the accuracy of measured result is assured only in a small subcritical state, for example less than several dollars. One of the methods for taking into account the spatial dependence of neutron flux distribution in large subcritical state is to consider higher mode components of neutron flux excited by an external neutron source. Recently, new reactor analysis code(3) was developed for calculation of higher mode flux accurately in multi-group and modified higher mode analysis method was successfully applied to estimate the change of flux and reactivity caused by arbitrary perturbation(4). The present study is preliminary application of higher mode analysis to the source multiplication method for accurate subcriticality measurement.

2. Source Multiplication Method with Higher Mode Components

In the ordinary source multiplication method, it is assumed that neutron count of detector (C_0), is proportional to external source intensity (S) and efficiency of detector (ε), and written as follows:

$$C_0 = \frac{\varepsilon S}{1 - k_{eff}}, \tag{1}$$

where k_{eff} is multiplication factor. If neutron count C is measured at already-known subcriticality (ρ_0), it is possible to determine $\varepsilon\, S$ value through Eq. (1), then other unknown subcriticality (ρ) can be obtained using count C by the detector as follows:

$$\rho = \frac{1}{1 - \dfrac{C}{\varepsilon S}} \quad . \tag{2}$$

The derivation of above equation is based on simplified one-point reactor theory where neither neutron detector position nor source position are not taken into account, therefore, it can be applied only to measurement of small subcriticality.

To overcome this difficulty, new source multiplication method with consideration of higher mode flux, namely, higher harmonics of the system was developed. In this new theory, neutron flux distribution with external neutron source is expanded by higher mode fluxes. Therefore, both detector and source position in each measurement can be considered in this theory.

Neutron balance equation of fundamental mode in static state (lambda mode eigenvalue equation) can be written as follows:

$$L\phi = \frac{1}{\lambda_0} M\phi \ , \tag{3}$$

where L is a destruction operator such as absorption and leakage terms, M is a production operator such as fission term, ϕ is neutron flux, and λ_0 is eigenvalue of fundamental mode, which is unity in a critical state. If the system becomes subcritical because of change of destruction and production operators in Eq. (3), neutron balance equation with an external neutron source in subcritical state can be written as follows:

$$(L+\delta L)\phi' - (M+\delta M)\phi' = S\delta(x_0) \ , \tag{4}$$

where δL and δM are change of L and M operators, respectively, ϕ' is new neutron flux under subcritical state, S is source intensity, x_0 is source position, and $\delta(x)$ is the Dirac's delta function.

Higher mode flux φ_i is defined by neutron balance equation of higher mode as follows:

$$L\varphi_i = \frac{1}{\lambda_i} M\varphi_i \ , \tag{5}$$

where λ_i is i-th higher mode eigenvalue. Then, ϕ' in Eq. (4) is expanded in a series of higher mode fluxes of Eq. (5) as follows:

$$\phi' = \sum_{i=0}^{N} c_i \varphi_i \ , \tag{6}$$

where N is maximum number of expansion modes and c_i is an expansion coefficient.

Multiplying adjoint flux of fundamental mode φ_0^+ in both sides of Eq. (4), and integrating over whole space and energy, the following equation can be obtained:

$$<\varphi_0^+ L\sum_{i=0}^{N} c_i\varphi_i> + <\varphi_0^+ \delta L\phi'> - <\varphi_0^+ M\sum_{i=0}^{N} c_i\varphi_i> - <\varphi_0^+ \delta M\phi'> = S\varphi_0^+(x_0) \ . \tag{7}$$

Using the following orthogonality equation for the higher mode flux,

$$<\varphi_i^+ M\varphi_j> = \delta_{ij} \ , \tag{8}$$

Eq. (7) is modified as follows:

$$\left(\frac{1}{\lambda_0} - 1\right)c_0 - <\varphi_0^+(-\delta L + \delta M)\phi'> = S\varphi_0^+(x_0) \ . \tag{9}$$

Here, using perturbation theory without assumption and Eq. (9), reactivity $\Delta\rho$ that is subcriticality can be obtained as follows:

$$\begin{aligned}\Delta\rho = \frac{<\varphi_0^{\ +}(-\delta L+\delta M)\phi^{'}>}{<\varphi_0^{\ +}M\phi^{'}>} &= -\frac{1}{c_0}\left\{S\varphi_0^{\ +}(x_0)-\left(\frac{1}{\lambda_0}-1\right)c_0\right\} \\ &; \ -\frac{1}{c_0}S\varphi_0^{\ +}(x_0) \qquad\qquad .\end{aligned} \tag{10}$$

In derivation of this equation (10), it is assumed that λ_0 is unity. From the above equation, $\Delta\rho$ is proportional to source intensity and adjoint flux of source position and inversely as an expansion coefficient of fundamental mode.

Same as the fundamental mode, in order to derive coefficients of higher order term, multiplying higher mode adjoint flux in both sides of Eq. (7), the following equation can be obtained:

$$\left(\frac{1}{\lambda_i}-1\right)C_i-<\varphi_i^{\ +}(-\delta L+\delta M)\phi^{'}> = S\varphi_i^{\ +}(x_0) \ . \tag{11}$$

Here, assuming that changes of both M and L operators are uniform in the system, second term of left hand side of above equation can be approximated as follows:

$$\begin{aligned}<\varphi_i^{\ +}(-\delta L+\delta M)\phi^{'}> \ &; \ \frac{\delta M-\delta L}{M}<\varphi_i^{\ +}M\phi^{'}> \\ &; \quad \Delta\rho \quad <\varphi_i^{\ +}M\phi^{'}> = \Delta\rho \ \ c_i \ \ .\end{aligned} \tag{12}$$

From these results of Eqs. (10) and (12), neutron flux of Eq. (6) in subcritical state can be written as follows:

$$\phi^{'} = \sum_{i=0}^{N}C_i\varphi_i = \left\{-\frac{\varphi_0^{\ +}(x_0)}{\Delta\rho}\varphi_0+\sum_{i=1}^{N}\frac{\varphi_i^{\ +}(x_0)}{\dfrac{1}{\lambda_i}-1-\Delta\rho}\varphi_i\right\}S \ . \tag{13}$$

Therefore, neutron count at position of x can be written as follows:

$$N_0 = \varepsilon\phi^{'} = \left\{-\frac{\varphi_0^{\ +}(x_0)}{\Delta\rho}\varphi_0(x)+\sum_{i=1}^{N}\frac{\varphi_i^{\ +}(x_0)}{\dfrac{1}{\lambda_i}-1-\Delta\rho}\varphi_i(x)\right\}\varepsilon S \ . \tag{14}$$

When the neutron count is measured at an already-known subcritical state, and higher mode flux and higher eigenvalues are known beforehand through calculations, then εS can be determined by the above equation (14). And when neutron count is measured at other unknown subcritical state, subcriticality is obtained by solving this equation numerically.

3. Simulation of Present Method

To demonstrate the validity of the present new source multiplication method, numerical calculations were carried out to simulate a core of Kyoto University Critical Assembly (KUCA) at Kyoto University Research Reactor Institute[5], because in the near future, systematic subcriticality measurement experiments including subcriticality benchmark experiments will be carried out at this facility.

Horizontal cross section view of the KUCA building is shown in Fig. 1. It is consisted of three different cores and an accelerator. It has been used for joint researches of other Japanese universities, and also used for the joint reactor laboratory course of graduate students of 10 universities of Japan.

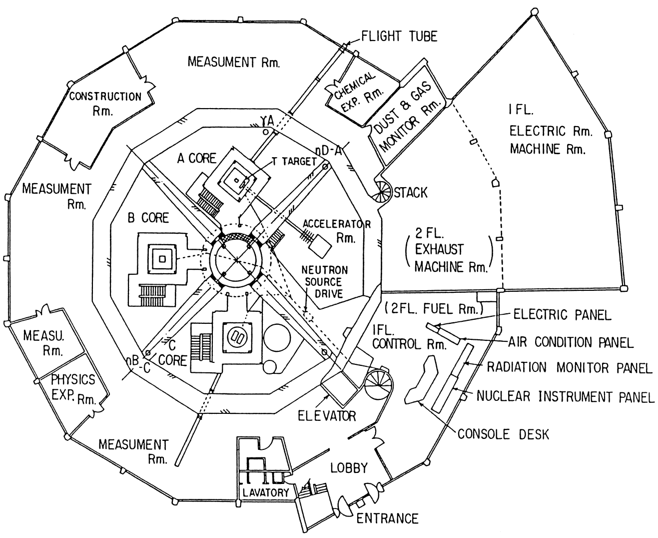

Figure 1 Horizontal cross section of KUCA building

The horizontal core configuration in this simulation is shown in Fig. 2. Fuel element is composed of highly enriched uranium-aluminum alloy plates as fuel and polyethylene as moderator, and polyethylene reflectors surround the fuel elements. Control rods (C1 to S6) are located adjacent the core. An external neutron source in the reflector region is an ^{241}Am-Be and neutron detectors are located at the reflector region as shown in Fig. 2.

Before the source multiplication analysis, higher mode calculations were carried out to obtain higher mode fluxes and eigenvalues. For this purpose, NEUMAC-3[4] (Neutron Modal Analysis Code) that been developed by author was used. NEUMAC-3 code is based on two or three dimensional diffusion theory in multi energy groups, and has an ability to

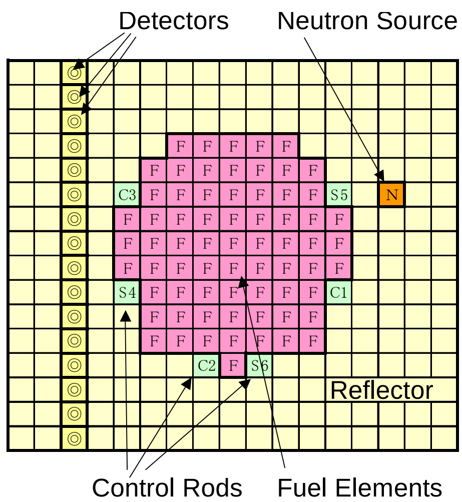

Figure 2 Horizontal cross section of core in the present simulation

calculate higher mode flux and its eigenvalue by the deflation method with power iteration or the double QR method to solve operator matrix of diffusion equation directly. In the present study, 2-dimensional x-y geometry in 2-energy groups was adopted and higher mode fluxes up to 150-th order were solved. Figure 3 is examples of higher mode flux distribution of the present core calculated by NEUMAC-3 and Fig. 4 shows eigenvalues of each higher mode of the present core. It is found that higher mode eigenvalues decrease with an increase of mode number.

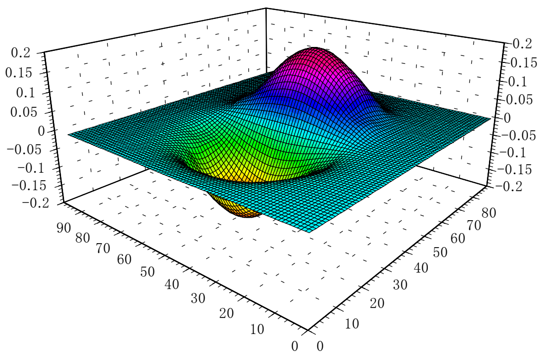

Figure 3 1-st higher mode flux distribution

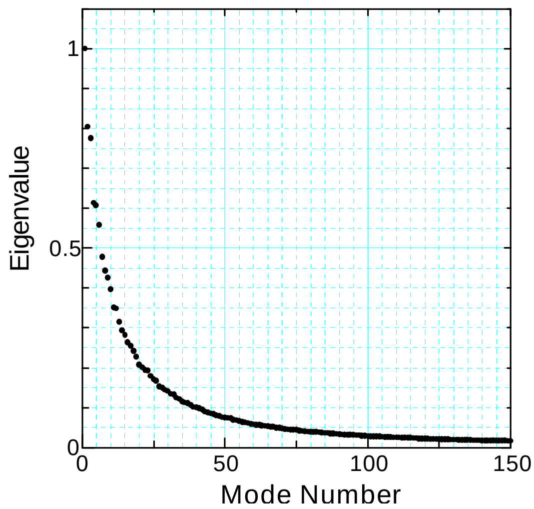

Figure 4 Eigenvalues of each higher mode

In the present analysis, neutron flux distribution with an external neutron source was calculated by CITATION code, which is a multi-dimensional and multi-group diffusion code, and subcriticality of the core which was calculated by the eigenvalue calculation was changed from approximately -1 %dk/k to -20%dk/k by altering the vertical buckling to simulate the change of vertical length of the core. Figure 5 shows an example of neutron flux distribution with an external neutron source calculated by CITATION.

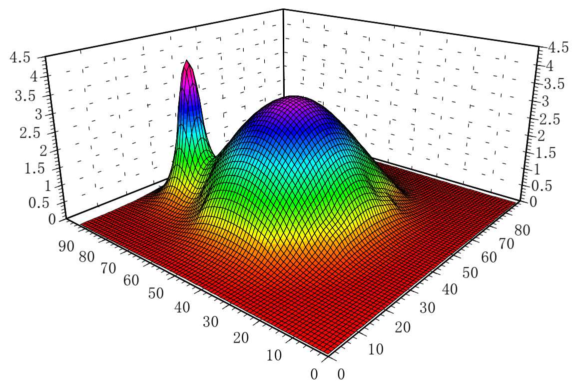

Figure 5 Neutron flux distribution of fast energy group with external neutron source

4. Results and Discussion

The results of the present simulation for new source multiplication method with consideration of higher mode flux to obtain subcriticality of each core are shown in Fig. 6 together with both the results of eigenvalue calculation by CITATION and ordinary source multiplication method. In this analysis, $\varepsilon\, S$ value in Eq. (14) was obtained at the smallest subcritical state by using its calculated subcriticality.

It is found that the results of ordinary source multiplication method are apart from calculated results when subcriticality is more than -7%dk/k and large discrepancy between those results by different detector positions are observed especially when subcriticality is more than -10 %dk/k. On the other hand, the present results by new source multiplication method show fairly good agreement with calculated ones and dependency of detector position is also very small.

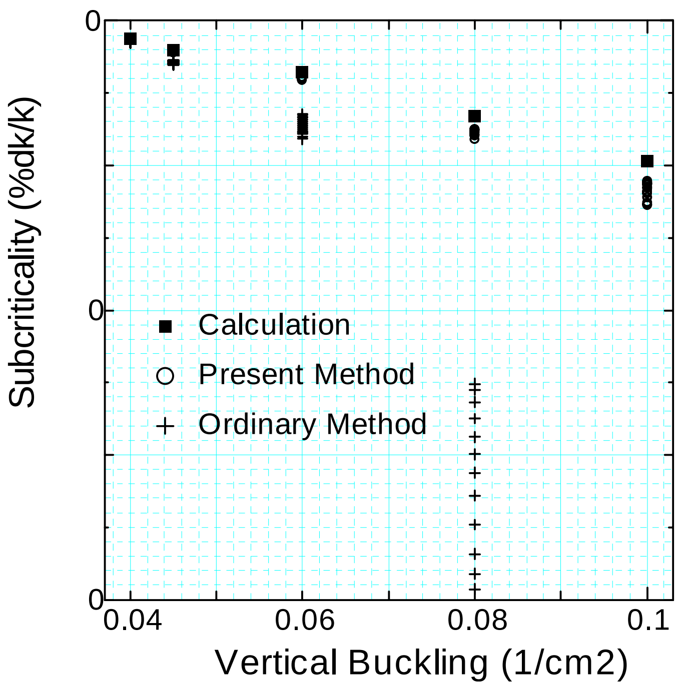

Figure 6 Results of subcriticality by the present source multiplication method together with both the results of eigenvalue calculation by CITATION and ordinary source multiplication method

5. Conclusion

New neutron source multiplication method with consideration of higher mode flux to measure subcriticality was developed. Through the present simulation by a computer code, it is found that compared to the ordinary source multiplication method, the accuracy of the present neutron source multiplication method is much improved even in large subcritical state because the effect of higher mode flux excited by an external neutron source on the neutron flux distribution becomes much remarkable with an increase of subcriticality, and detector position dependency of obtained subcriticality can be greatly decreased.

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