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## Experience with Monte Carlo Variance Reduction Using Adjoint Solutions in HYPER Neutronics Analysis

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### Abstract

The variance reduction techniques using adjoint solutions are applied to the Monte Carlo calculation of the HYPER(HYbrid Power Extraction Reactor) core neutronics. The applied variance reduction techniques are the geometry splitting and the weight windows. The weight bounds and the cell importance needed for these techniques are generated from an adjoint discrete ordinate calculation by the two-dimensional TWODANT code. The flux distribution variances of the Monte Carlo calculations by these variance reduction techniques are compared with the results of the standard Monte Carlo calculations. It is shown that the variance reduction techniques using adjoint solutions to the HYPER core neutronics result in a decrease in the efficiency of the Monte Carlo calculation.

### 1. Introduction

The Monte Carlo (MC) calculation is occasionally adopted as a means to obtain the reference solutions to the neutronics problems because it is capable of providing accurate numerical solutions by handling the complex three-dimensional geometry of the problems exactly and inputting the continuous energy microscopic cross-section data from the up-to-date evaluated nuclear data files. . But it has a major drawback of long computing time by requiring sampling a number of particles that are sufficient enough to obtain the results with sufficient precision. To overcome this drawback, variance reduction techniques[1] are

commonly adopted in the Monte Carlo calculation that allows one to sample less particle histories required than otherwise but can bring out the results having desired precision.

Geometry splitting and weight windows are among the most common variance reduction techniques. Use of these methods requires importance weightings dependent on space, energy and angle. They can be generated from the adjoint solutions either from deterministic adjoint calculations or from the MC adjoint calculations. The variance reduction techniques using the adjoint solutions[2] have been successfully applied to deep penetration problems and have been found very effective in enhancing the computational efficiency of the Monte Carlo calculation.

Motivated by this, we applied the variance reduction techniques using adjoint solution to the neutronics analysis of the HYPER[3] core and examined the efficiency of the adjoint weighting in reducing the variance in the MC neutronics calculations. HYPER is an accelerator driven subcritical reactor system designed for incinerating or transmuting long-lived nuclides contained in the nuclear wastes. The purpose of this paper is to present our experience with the efficiency of the adjoint weighting on the variance reduction in the MC calculation for the multiplication and flux distributions of the HYPER core.

The MC code we used in this study is the MC-CARD[4], the Monte Carlo Code for Advanced Reactor Design. The MC-CARD is a personal computer based MC program which is designed exclusively for neutronics analysis of nuclear fuel and reactor systems. The special feature of it is the capability of depletion analysis and parallel computation. These features of the MC-CARD are verified elsewhere.[5]

## 2. Test of Variance Reduction Techniques

Variance-reducing techniques in Monte Carlo calculations can reduce the computer time required to obtain results of sufficient precision. Yet it is often difficult to properly apply variance reduction techniques. Moreover, they may increase the variance without due precaution in using them.

### 2.1 Variance Reduction Techniques

Geometry splitting and weight windows are the most common variance reduction techniques. They are referred to as methods of controlling particle population.

Geometry splitting requires the assignment of an importance to each geometric cell in the system geometry model. The creation and destruction of particles are based on those values. When a particle enters a cell of higher importance than the cell it has just left, it may be split into two or more particles, each being attached with the share of the original particle's weight. When a particle enters a cell of lower importance, it may be killed or survive by Russian roulette game.

Weight windows are similar to geometry splitting. Each cell has the lower and upper

weight bounds that are inversely proportional to the importance. If a particle's weight is above the upper bound, it is split into a sufficient number of particles such that each individual particle's weight lies within the weight window. If a particle's weight is below the lower bound, it is subjected to Russian roulette. Geometry splitting controls particle's population dependent on its position, whereas the weight window particles' population dependent on its position, energy and direction of motion.

## 2.2 Examination of Adjoint Weighting with Deep Penetration Problem

In order to examine the efficiency of the adjoint weighting on the variance reduction in the MC calculations by the MC-CARD, a shielding problem is taken as the test problem[6] and MC-CARD calculations with the geometry splitting and weight windows with the adjoint weighting are compared with the analog and the standard MC calculations. The test problem is described by a point isotropic neutron source shielded by 180 centimeter thick Saeffer portland concrete. The problem calls for estimation of the neutron flux at the detector location 20 meter away from the source.

Table 1(a) shows that there are no neutrons tallied in the detector by the analog Monte Carlo calculation and the standard Monte Carlo calculation. The standard Monte Carlo calculation here means the MC calculation that is conducted with the use of the implicit capture variance reduction technique. In contrast to these, Table 1(b) and 1(c) show that neutrons are tallied in the detector cell by the Monte Carlo calculation using geometry splitting or weight windows. For this problem, the importance weightings used in geometry splitting and weight windows are properly postulated ones.[6]

## 3. Variance Reduction Using Adjoint Solutions

One has to input the importance weightings or weight bounds in applying geometry splitting or weight windows to Monte Carlo calculation of HYPER core. The direct assignment of importance (or weight bounds) is an art requiring considerable skill, experience, and luck. The adjoint solution can be used as the importance function as suggested as early as 1958 by Geertzel and Kalos [7]. The weight bounds of weight windows are readily converted from cell importance by the following equations.

$$\text{Lower Bound} = \frac{0.25}{\text{Cell Importance}}$$

$$\text{Survival Weight} = 3.0 \times \text{Lower Bound}$$

$$\text{Upper Bound} = 5.0 \times \text{Lower Bound}$$

In practice, the adjoint solution can be obtained by Monte Carlo adjoint calculation or by the deterministic solution method for the adjoint transport equation or the adjoint diffusion equation. Figure 1 shows the computational flowchart of variance reduction using the adjoint solutions.

## 4. Numerical Results

The HYPER core is illustrated in Figure 2. The core is composed of four regions and fuel region is homogenized for simplicity. The HYPER core is divided into as many as 11,000 cells. The adjoint flux for each cell is obtained by TWODANT code[8]. Figure 3 shows the radial flux distributions in the center plane axially and energy spectrum in the fuel cell at the center of the reactor from TWODANT forward and backward calculations.

These adjoint fluxes are normalized such that the maximum adjoint flux outside the source region is unity. Then the normalized adjoint fluxes are used as cell importance in geometry splitting method. Table 2(a) shows the MC-CARD calculations with the geometry splitting weighted by adjoint fluxes. It shows that the relative standard deviations at  $R=26.16\text{cm}$  and  $Z=208\text{cm}$ , are 0.03 and 0.06, respectively, by the standard MC and by the geometry splitting MC calculation. In contrast to the test shielding problem, the use of adjoint weighting in the geometry splitting method does not result in the variance reduction. Instead, It increased the variances of radial fluxes in comparison with the standard MC calculation. For this comparison, two MC calculations are stopped at about the same computer run time.

Inverse values of normalized groupwise adjoint fluxes are inputted as the weight bounds of 25 energy groups. Table 2(b) show the MC-CARD results with the weight windows weighted by adjoint fluxes. It shows that the standard MC and MC calculations with the weight windows weighted by adjoint fluxes have about the same relative standard deviations at  $R=26.16\text{cm}$  and  $Z=208\text{cm}$ . But the two MC calculations show the relative standard deviations of 0.03 and 0.04 at  $R=70.25\text{cm}$  and  $Z=208\text{cm}$ , respectively, which indicates weight windows applied to HYPER core analysis problem decrease the Monte Carlo efficiency as is shown in the geometry splitting

## 5. Conclusion

The variance reduction technique using adjoint solutions has been a powerful method for the deep penetration problem. To our surprise, however, the application of this technique to the HYPER core results in a decrease in the efficiency of the Monte Carlo calculation.

To our best knowledge, the application of the variance reduction techniques utilizing the adjoint weighting to the power reactor like the HYPER core has not yet been reported. Several issues remain yet to be explored. The three-dimensional transport theory code like THREEDANT[8] may be used to get the directional information on adjoint weight function, which may allow the angular biasing in the right direction.

## References

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Table 1. Results of Variance Reduction Test Problem

(a) Analog and Standard Monte Carlo Calculations

Var. Red. Tech. Results	Analog		Implicit Capture	
	Flux	Std. Dev.	Flux	Std. Dev.
Neutron Flux of Detector	0.00E+00	0.00E+00	0.00E+00	0.00E+00
Computer Time	7.85 min.		14.46 min.	
Source Particle Number	1,000,000		1,000,000	

(b) Geometry Splitting Monte Carlo Calculations

Var. Red. Tech. Results	Geo. Split. Only		Geo. Split. / Impl. Cap.	
	Flux	Std. Dev.	Flux	Std. Dev.
Neutron Flux of Detector	2.96E-13	2.07E-14	2.58E-13	1.29E-14
Computer Time	37.64 min.		79.68 min.	
Source Particle Number	200,000		200,000	

(c) Weight Windows Monte Carlo Calculations

Var. Red. Tech. Results	Wgt. Wnd. Only		Wgt. Wnd. / Impl. Cap.	
	Flux	Std. Dev.	Flux	Std. Dev.
Neutron Flux of Detector	2.92E-13	2.34E-14	2.41E-13	1.69E-14
Computer Time	34.55 min.		40.24 min.	
Source Particle Number	200,000		200,000	

Table 2. Results of HYPER Core by Variance Reduction Using Adjoint Solutions

(a) Geometry Splitting Monte Carlo Calculations

Loc(R_dir) Z=208	Analog		Standard		Geo. Split. / Impl. Cap.	
	Flux	Rel. Err.	Flux	Rel. Err.	Flux	Rel. Err.
26.16	6.27	2.87E-02	6.22	3.03E-02	6.06	6.08E-02
28.48	6.15	2.92E-02	6.14	3.08E-02	5.95	6.25E-02
30.80	6.00	2.91E-02	6.04	3.12E-02	5.79	6.40E-02
33.12	5.84	2.97E-02	5.87	3.19E-02	5.70	6.48E-02
35.44	5.75	3.02E-02	5.72	3.22E-02	5.61	6.58E-02
37.76	5.68	3.06E-02	5.70	3.29E-02	5.50	6.68E-02
40.08	5.59	3.12E-02	5.61	3.31E-02	5.43	6.79E-02
42.40	5.52	3.17E-02	5.49	3.36E-02	5.37	6.80E-02
44.72	5.45	3.17E-02	5.36	3.40E-02	5.29	6.89E-02
47.04	5.35	3.21E-02	5.28	3.43E-02	5.19	7.00E-02
49.37	5.24	3.24E-02	5.17	3.49E-02	5.10	7.06E-02
51.69	5.13	3.27E-02	5.05	3.49E-02	5.02	7.17E-02
54.01	5.07	3.30E-02	5.03	3.51E-02	4.93	7.23E-02
56.33	4.98	3.31E-02	4.90	3.56E-02	4.86	7.32E-02
58.65	4.86	3.37E-02	4.85	3.55E-02	4.77	7.36E-02
60.97	4.78	3.40E-02	4.74	3.58E-02	4.69	7.40E-02
63.29	4.66	3.41E-02	4.64	3.59E-02	4.62	7.41E-02
65.61	4.56	3.42E-02	4.57	3.64E-02	4.53	7.55E-02
67.93	4.47	3.45E-02	4.46	3.68E-02	4.45	7.61E-02
70.25	4.39	3.48E-02	4.41	3.68E-02	4.37	7.65E-02
Computer Time	7.37 hr		6.06 hr		5.50 hr	
Src. Part. Num.	30000		10000		40000	

(b) Weight Windows Monte Carlo Calculations

Loc(R_dir) Z=208	Standard		Wgt. Wnd. / Impl. Cap.		Wgt. Wnd. / Impl. Cap.		Wgt. Wnd. / Impl. Cap.	
	Flux	Rel. Err.	Flux	Rel. Err.	Flux	Rel. Err.	Flux	Rel. Err.
26.16	6.22	3.03E-02	5.88	1.02E-01	6.11	4.68E-02	6.25	3.04E-02
28.48	6.14	3.08E-02	5.85	1.05E-01	6.00	4.71E-02	6.13	3.08E-02
30.80	6.04	3.12E-02	5.78	1.02E-01	5.88	4.78E-02	6.00	3.17E-02
33.12	5.87	3.19E-02	5.68	1.06E-01	5.80	4.85E-02	5.88	3.19E-02
35.44	5.72	3.22E-02	5.61	1.08E-01	5.72	4.98E-02	5.80	3.26E-02
37.76	5.70	3.29E-02	5.51	1.08E-01	5.60	5.07E-02	5.67	3.34E-02
40.08	5.61	3.31E-02	5.52	1.08E-01	5.57	5.14E-02	5.62	3.36E-02
42.40	5.49	3.36E-02	5.42	1.11E-01	5.49	5.23E-02	5.53	3.40E-02
44.72	5.36	3.40E-02	5.24	1.13E-01	5.35	5.26E-02	5.41	3.46E-02
47.04	5.28	3.43E-02	5.15	1.13E-01	5.27	5.29E-02	5.32	3.50E-02
49.37	5.17	3.49E-02	5.02	1.14E-01	5.20	5.41E-02	5.21	3.54E-02
51.69	5.05	3.49E-02	4.90	1.20E-01	5.10	5.55E-02	5.14	3.59E-02
54.01	5.03	3.51E-02	4.82	1.19E-01	5.01	5.59E-02	5.05	3.64E-02
56.33	4.90	3.56E-02	4.77	1.22E-01	4.90	5.67E-02	4.94	3.68E-02
58.65	4.85	3.55E-02	4.74	1.23E-01	4.80	5.68E-02	4.86	3.72E-02
60.97	4.74	3.58E-02	4.60	1.24E-01	4.72	5.73E-02	4.80	3.76E-02
63.29	4.64	3.59E-02	4.46	1.27E-01	4.64	5.78E-02	4.71	3.80E-02
65.61	4.57	3.64E-02	4.40	1.27E-01	4.58	5.89E-02	4.63	3.86E-02
67.93	4.46	3.68E-02	4.32	1.29E-01	4.46	5.90E-02	4.54	3.86E-02
70.25	4.41	3.68E-02	4.41	1.28E-01	4.44	6.01E-02	4.47	3.93E-02
Computer Time	6.06 hr		0.60 hr		2.50 hr		5.82 hr	
Src. Part. Num.	10000		10000		40000		100000	

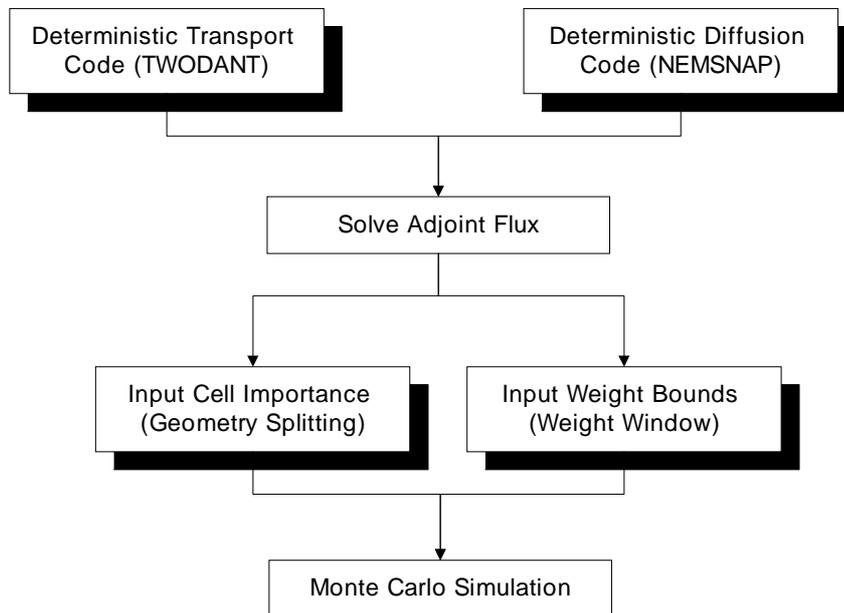


Figure 1. Flowchart of Variance Reduction Using Adjoint Solutions

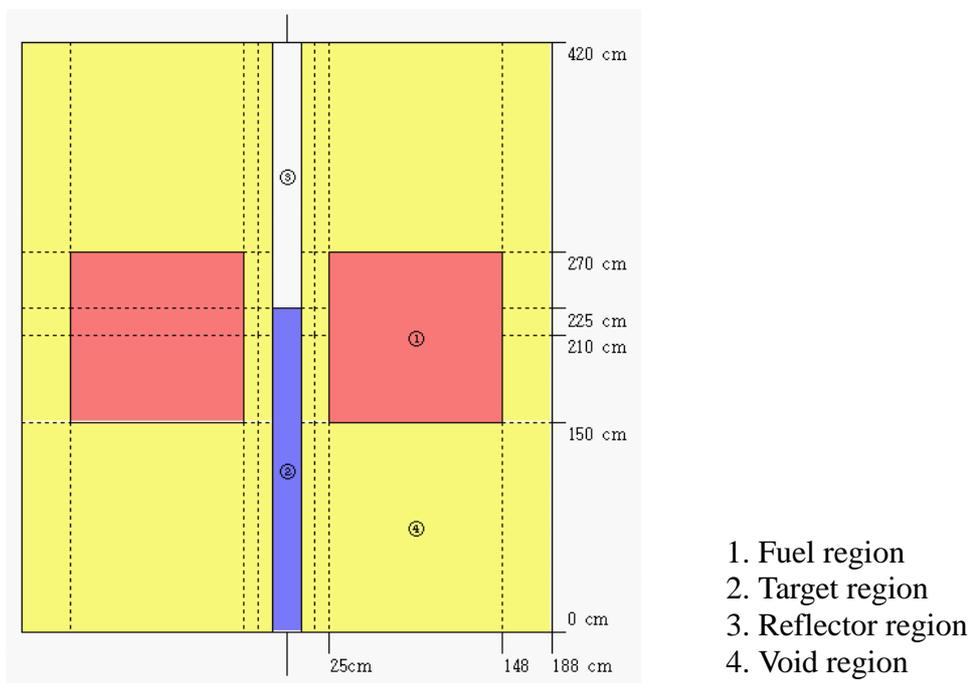


Figure 2. Approximated Geometry of HYPER Core

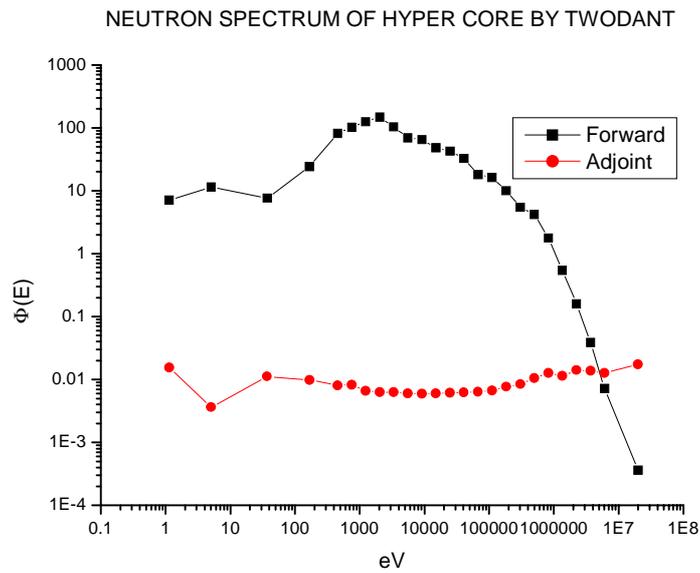
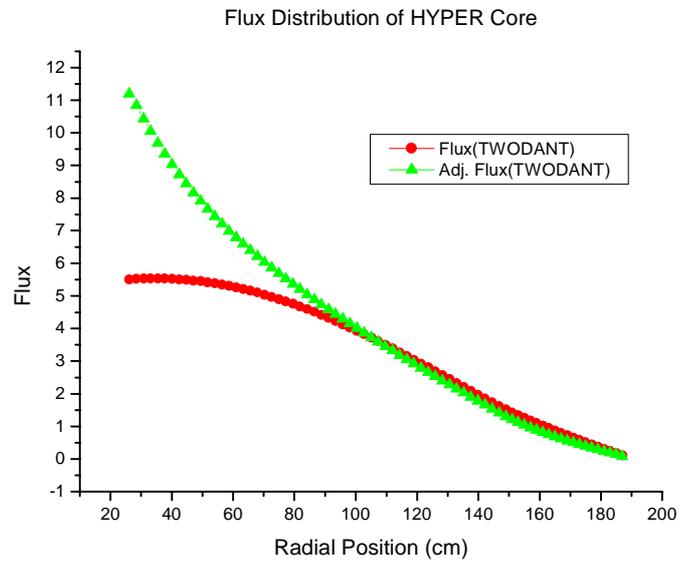


Figure 3. Flux distributions and Energy Spectrum of HYPER Core By TWODANT