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A Coherent Remedy for the One Dimensional Two Fluid Model

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ABSTRACT

An improved one-dimensional two-fluid model with appropriate momentum flux parameters is proposed, which is stable in the whole range of flow regime. A linear stability analysis is performed for a two-phase channel flow described by an improved one-dimensional two-fluid model. For a two-phase flow in a dispersed flow regime, an analytical expression for the growth factor is derived as a function of wave number, void fraction, drag coefficient, and relative velocity. It is demonstrated that the two-phase channel flow can be properly described by the proposed one-dimensional two-fluid model in the whole range of flow regime, while the basic form of the one-dimensional two-fluid model renders the system mathematically ill posed. It is also shown that the proposed model is applicable in a practical range of density ratio.

1.Background

The one-dimensional two-fluid model is widely used in describing complex two-phase flow systems[1,2]. However, it is well recognized that the basic form of the governing differential equations for the one-dimensional two-fluid model is mathematically ill posed [3,4,5]. Though there have been efforts to improve the one-dimensional two-fluid model [6, 7, 8, 9, 10], the issue is still unresolved. Recently, Song and Ishii [11, 12] proposed to consider the void fraction and velocity distribution in the flow channel by use of the momentum flux parameters in the one-dimensional two-fluid model.

In the present paper a completely well posed one-dimensional two-fluid model is proposed. A linear stability analysis in a similar manner to that of Ramshaw and Trapp [4] and Pokharna, Mori and Ransom [13], is performed for a channel flow described by an improved one-dimensional two-fluid model, where appropriate momentum flux parameters are proposed. It is shown that the proposed one-dimensional two-fluid model describing a two-phase flow in a flow channel is stable in the whole range of flow regime and in a wide range of density ratio.

2. A general form of the one-dimensional two-fluid model

By defining the area average and void fraction weighted average quantities as below

$$\langle F\rangle = 1/A \int F\, dA \qquad (1)$$

$$\langle\langle F_i\rangle\rangle = \langle \alpha_i F_i\rangle / \langle\alpha_i\rangle \qquad (2)$$

Let us denote $\alpha=\langle\alpha_i\rangle$, $u_i=\langle\langle u_i\rangle\rangle$ and assume that the density of each phase is uniform such that $\rho_i=\langle\langle\rho_i\rangle\rangle$. The incompressible two-phase flow in a vertical channel is described by the generalized one-dimensional two-fluid model [14, 15, 16] as below

$$\alpha\rho_g\partial u_g/\partial z + \rho_g\partial\alpha/\partial t + \rho_g u_g\partial\alpha/\partial z = 0 \qquad (1)$$

$$\alpha_f\rho_f\partial u_f/\partial z - \rho_f\partial\alpha/\partial t - \rho_f u_f\partial\alpha/\partial z = 0 \qquad (2)$$

$$\alpha\rho_g\partial u_g/\partial t + \alpha\rho_g(2C_{vg}-1)u_g\partial u_g/\partial z + \rho_g(C_{vg}-1)u_g^2\partial\alpha/\partial z = -\alpha\partial p/\partial z + \alpha\rho_g g - F_I + M_{ig} \qquad (3)$$

$$\alpha_f\rho_f\partial u_f/\partial t + \alpha_f\rho_f(2C_{vf}-1)u_f\partial u_f/\partial z - \rho_f(C_{vf}-1)u_f^2\partial\alpha/\partial z = -\alpha_f\partial p/\partial z + \alpha_f\rho_f g + F_I + M_{if} \qquad (4)$$

where $\alpha_f=1-\alpha$, F_I is the inter-phase drag, and M_{ig} and M_{if} are the generalized drag force including the transient forces and wall drag. The C_{vf} and C_{vg} are the momentum flux parameters [15, 16] defined as

$$C_{vi}=\langle\alpha_i u_i^2\rangle/\langle\alpha_i\rangle\langle\langle u_i\rangle\rangle^2 \qquad (8)$$

There is a fundamental diffeence between the single phase flow and two-phase flow in the role of momentum flux parameters. For the single-phase flow, the momentum flux parameter indicates only the effect of velocity profile. On the other hand, the momentum flux parameters for the gas and liquid phase in the one-dimensional two-fluid model for the two-phase flow do not only indicate the effect of velocity profile but also the coupling between the velocity profile and void fraction profile.

As the values of these parameters are close to 1, the parameters are assumed to be unity in the conventional one-dimensional two-fluid models. So, the role of these momentum flux parameters are neglected. In the present analysis, we will explicitly model the momentum flux parameters to consider the coupling of void fraction profile and velocity profile by using existing correaltion for the dsitribution parameter[16] and velocity profile[17]. It is physically correct. And we will see the benefit of using them in terms of the stability of the one-dimensional two-fluid model.

3. A Linear Stability Analysis

By defining a vector $\boldsymbol{\varphi} = (\alpha, u_g, u_f, p)$. The system of continuity and momentum equations in equations (1) to (4) can be written as

$$\mathbf{A}\,\partial\boldsymbol{\varphi}/\partial t + \mathbf{B}\,\partial\boldsymbol{\varphi}/\partial z + \mathbf{C} = 0 \qquad (11)$$

where **A**, **B**, and **C** are matrices. To investigate the stability of two-phase flow described by these equation, a linear stability analysis is performed. The linear differential equation for a perturbation, $\delta\boldsymbol{\varphi}=\boldsymbol{\varphi}-\boldsymbol{\varphi}^{\mathbf{o}}$ is written as

$$\mathbf{A}_o\partial\delta\boldsymbol{\varphi}/\partial t + \mathbf{B}_o\partial\delta\boldsymbol{\varphi}/\partial t + [(\partial\mathbf{A}/\partial\boldsymbol{\varphi})_o(\partial\boldsymbol{\varphi}/\partial t)_o + (\partial\mathbf{B}/\partial\boldsymbol{\varphi})_o(\partial\boldsymbol{\varphi}/\partial x)_o+(\partial\mathbf{C}/\partial\boldsymbol{\varphi})_o]\delta\boldsymbol{\varphi} = 0 \qquad (12)$$

where subscript o denotes the quantities at initial state.

Let's consider a perturbation in the form of a traveling wave

$$\delta\boldsymbol{\varphi}=\delta\boldsymbol{\varphi}^{o}\exp[i(kx-\omega t)] \qquad (13)$$

where $\delta\boldsymbol{\varphi}^{o}$ denotes the initial amplitude of the perturbation, k is the wave-number, and ω is the frequency in a complex number. The imaginary part of ω will govern the growth or decay of the perturbation and the real part determines the speed of propagation. On substitution of equation (13) into equation (12), a compatibility condition for $\delta\boldsymbol{\varphi}^{o}$ is obtained

$$-i\omega\mathbf{A}_o\delta\boldsymbol{\varphi}^{\mathbf{o}}+ ik\mathbf{B}_o\delta\boldsymbol{\varphi}^{\mathbf{o}} + [(\partial\mathbf{A}/\partial\boldsymbol{\varphi})_o(\partial\boldsymbol{\varphi}/\partial t)_o + (\partial\mathbf{B}/\partial\boldsymbol{\varphi})_o(\partial\boldsymbol{\varphi}/\partial x)_o+(\partial\mathbf{C}/\partial\boldsymbol{\varphi})_o]\delta\boldsymbol{\varphi}^{\mathbf{o}} = 0 \qquad (14)$$

The condition for the existence of a non-trivial solution for $\delta\boldsymbol{\varphi}^{\mathbf{o}}$ is that the determinant of the coefficient matrix must vanish; i.e.,

$$\text{Determinant of } (-i\omega\mathbf{A} + ik\mathbf{B} + \mathbf{D}) = 0 \quad (15)$$

$$\mathbf{D} = [(\partial\mathbf{A}/\partial\boldsymbol{\varphi})_o(\partial\boldsymbol{\varphi}/\partial t)_o + (\partial\mathbf{B}/\partial\boldsymbol{\varphi})_o(\partial\boldsymbol{\varphi}/\partial x)_o + (\partial\mathbf{C}/\partial\boldsymbol{\varphi})_o]^T \quad (16)$$

By defining $\lambda = \lambda_R + i\lambda_I = \omega_R/k + i\omega_I/k$, equation (15) can be written as below for nonzero wave number k

$$\text{Determinant of } (\mathbf{A}\lambda - \mathbf{B} + i/k\,\mathbf{D}) = 0 \quad (17)$$

For the case of a perturbation wave-length much smaller than the length-scale of the initial steady state or for an initial uniform steady state, $(\partial\boldsymbol{\varphi}/\partial t)_o$ and $(\partial\boldsymbol{\varphi}/\partial x)_o$ will be negligible so that **D** becomes

$$\mathbf{D} = (\partial\mathbf{C}/\partial\boldsymbol{\varphi})_o \quad (18)$$

Let's use a simple Darcy model suggested by Ishii and Zuber[18] for the interfacial drag to compare the analysis results with those of Pokharna et al. [13],

$$F_I = 1/2 C_D \rho_f (u_g - u_f)|u_g - u_f| A_p/V_b = K\alpha\rho_f u_r^2 \quad (19)$$

Then, the matrix **C** and **D** becomes

$$\mathbf{C} = [0, 0, -\alpha\rho_g g + K\alpha\rho_f u_r^2, -\alpha_f\rho_f g - K\alpha\rho_f u_r^2] \quad (20)$$

$$D_{11} = D_{12} = D_{13} = D_{14} = 0. \quad (21)$$

$$D_{21} = D_{22} = D_{23} = D_{24} = 0. \quad (22)$$

$$D_{31} = K\rho_f u_r^2,\ D_{32} = 2K\alpha\rho_f u_r,\ D_{33} = -D_{32},\ D_{34} = 0 \quad (23)$$

$$D_{41} = -D_{31},\ D_{42} = -D_{32},\ D_{43} = D_{32},\ D_{44} = 0. \quad (24)$$

where it is assumed that C_D is constant, $A_p/V = 3\alpha/4r_b$. K equals $3/8C_D/r_b$. Let's denote $u_r = u_g - u_f$ and $\alpha_f = 1-\alpha$. Let $\Phi^* = iD_{31}/k$, $\Omega_1^* = iD_{32}/k$. The matrix $(\mathbf{A}\lambda - \mathbf{B} + i/k\,\mathbf{D})$ becomes

$$\begin{matrix} \rho_g(\lambda - u_g) & -\alpha\rho_g & 0 & 0 \\ -\rho_f(\lambda - u_f) & 0 & -\alpha_f\rho_f & 0 \\ -\rho_g(C_{vg}-1)u_g^2 + \Phi^* & \alpha\rho_g[\lambda - (2C_{vg}-1)u_g] + \Omega^* & -\Omega^* & -\alpha \\ \rho_f(C_{vf}-1)u_f^2 - \Phi^* & -\Omega^* & \alpha_f\rho_f[\lambda - (2C_{vf}-1)u_f] + \Omega^* & -\alpha_f \end{matrix} \quad (25)$$

The determinant of the matrix (**A**λ – **B** + *i*/k **D**) is calculated as

$$f(\lambda,k)=-\alpha\rho_g\alpha_f\rho_f\,[\rho_g\alpha_f\,(\lambda^2-2C_{vg}u_g\lambda+C_{vg}u_g^{\,2})+\alpha\rho_f(\lambda^2-2C_{vf}u_f\lambda+C_{vf}u_f^{\,2})]$$
$$-\,\rho_g\rho_f\,[\Omega^*\alpha_f(\lambda-u_g)+\Omega^*\alpha(\lambda-u_f)]-\alpha\rho_g\alpha_f\rho_f\Phi^* \qquad (26)$$

Note that for finite ω/k in the limit $k \to \infty$, equation (17) reduces to the characteristic equation. And the dependence of the solution on the initial data can be reduced to an investigation of the roots of the equation

$$\text{Determinant of } (\mathbf{A}\,\lambda - \mathbf{B}) = f(\lambda) = 0 \qquad (27)$$

The determinant of this matrix is calculated as

$$f(\lambda)= -\alpha(1-\alpha)\rho_g\rho_f[(1-\alpha)\rho_g(\lambda^2-2\lambda C_{vg}u_g+C_{vg}u_g^{\,2}) + \alpha\rho_f\,(\lambda^2-2C_{vf}u_f\lambda+C_{vf}u_f^{\,2}\,)] \qquad (28)$$

In the case of the basic form of the one-dimensional two-fluid model, where $C_{vg}=C_{vf}=1$, the equation $f(\lambda)= 0$ can have real roots only if $\lambda=u_g=u_f$. So, the two-fluid model is mathematically ill posed. On the other hand, we can have real roots for λ for the equation $f(\lambda)= 0$, if the momentum flux parameters satisfy the following equation.

$$P = (\alpha_f\rho_g\,C_{vg}u_g+\alpha\rho_f\,C_{vf}u_f\,)^2 - (\alpha_f\rho_g+\alpha\rho_f)(\alpha_f\rho_gC_{vg}u_g^{\,2} +\alpha\rho_fC_{vf}u_f^{\,2}\,) \geq 0 \qquad (29)$$

It suggests that the one-dimensional two-fluid model allows two real characteristic roots for λ by incorporating appropriate momentum flux parameters, which are related to the speed of the void fraction wave observed in the two-phase flow system [11, 19]. So, the necessary condition for the stability of the one-dimensional two-fluid model is defined by above inequality.

4. Dispersion Relation and Stability Criteria

Let $\Omega=D_{32}/(\alpha\alpha_f)=2K\rho_fu_r/\alpha_f$ and $\Phi=D_{31}$. The solution for $f(\lambda,k) =0$ is determined from the real and imaginary parts of equation (26) as below

$$\rho_g\alpha_f\,(\lambda_R^{\,2}-2C_{vg}u_g\lambda_R+C_{vg}u_g^{\,2}-\lambda_I^{\,2})+\alpha\rho_f(\lambda_R^{\,2}-2C_{vf}u_f\lambda_R+C_{vf}u_f^{\,2}-\lambda_I^{\,2})-\Omega/k\,\lambda_I=0 \qquad (30)$$
$$2\rho_g\alpha_f\lambda_I(\lambda_R-C_{vg}u_g)+2\alpha\rho_f\lambda_I(\lambda_R-C_{vf}u_f)+[\lambda_R\Omega-\Omega\alpha_fu_g-\Omega\alpha u_f]/k+\Phi/k=0 \qquad (31)$$

For convenience, let $\underline{\rho\alpha}= \rho_g\alpha_f+\alpha\rho_f$, $\underline{\alpha u}= u_g\alpha_f+\alpha u_f$. We can obtain the dispersion relation between wave number k and growth factor ω_I from equation by combining equation (30) and (31)

$$g(\omega_I,k)=4\underline{\rho\alpha}^3\omega_I^4+8\underline{\rho\alpha}^2\Omega\omega_I^3+5\underline{\rho\alpha}\Omega^2\omega_I^2+\Omega^3\omega_I+k^2(4\underline{\rho\alpha}\omega_I^2P+4\Omega\omega_IP-Q)=0 \quad (32)$$

where P is the same one as that in equation (29) and Q is defined as

$$Q = \alpha_f\rho_g[(\Omega\underline{\alpha u}-\Phi)^2-2C_{vg}u_g(\Omega\underline{\alpha u}-\Phi)\Omega+\Omega^2C_{vg}u_g^2] +\alpha\rho_f[(\Omega\underline{\alpha u}-\Phi)^2-2C_{vf}u_f(\Omega\underline{\alpha u}-\Phi)\Omega+\Omega^2C_{vf}u_f^2] \quad (33)$$

From equation (32), it is seen that the solution of ω_I for $g(\omega_I,k)=0$ at given k is determined by the intersection of following two curves

$$g1(\omega_I)= \omega_I\,[4\underline{\rho\alpha}^3\omega_I^3+8\underline{\rho\alpha}^2\Omega\omega_I^2+5\underline{\rho\alpha}\Omega^2\omega_I+\Omega^3] \quad (34)$$

$$g2(\omega_I,k) = - k^2(4\underline{\rho\alpha}\omega_I^2P+4\Omega\omega_IP- Q) \quad (35)$$

Above equations can be rearranged as below

$$g1(\omega_I)= 4\Omega^4/\underline{\rho\alpha}\; x(x^3+2x^2+5/4x+1/4) = 4\Omega^4/\underline{\rho\alpha}(x+1/2)^2(x+1)x = 0 \quad (36)$$

$$g2(\omega_I,k) = -k^2(4\underline{\rho\alpha}\omega_I^2P+4\Omega\omega_IP-Q)= - k^2\{4P\Omega^2/\underline{\rho\alpha}(x+1/2)^2-P\Omega^2/\underline{\rho\alpha}-Q\} \quad (37)$$

where $x=\omega_I\underline{\rho\alpha}/\Omega$.

4.1 Stability of the basic form of the conventional one-dimensional two-fluid model

In this case $C_{vf}=C_{vg}=1$ and $P= - \alpha_f\rho_g\alpha\rho_f\,(u_g^2 - u_f)^2$. If there is no interfacial drag, $\Omega=0$ and equation (32) becomes

$$g(\omega_I,k) = 4\underline{\rho\alpha}^3\omega_I^4 - 4k^2\underline{\rho\alpha}\omega_I^2\alpha_f\rho_g\alpha\rho_f\,(u_g-u_f)^2 = 0 \quad (38)$$

The growth factor is determined as

$$\omega_I= k/(\rho_g\alpha_f+ \alpha\rho_f)u_r\,(\rho_g\alpha_f\alpha\rho_f)^{1/2} \quad (39)$$

It can be seen that the growth factor is proportional to the wave number and relative velocity.

When the interfacial drag is present, Q is determined as $Q=\alpha_f\rho_g[(\Omega\underline{\alpha u}-\Phi)-\Omega u_g]^2+ \alpha\rho_f[(\Omega\underline{\rho u}-\Phi)-\Omega u_f]^2$. So, P is negative and Q is positive always. By using this information, the typical shapes of curves in equation (34) and (35) can be represented by Fig. 1. We assumed k=0.5 or 1.0, P=-1.0, Ω=1.0, $\langle\rho\alpha\rangle$=1.0, Q=1.0 for convenience. $x=\omega_I\underline{\rho\alpha}/\Omega$ represents the growth factor ω_I. The function g1 has zeros at x=0, -1/2, -1.0 and is always positive for the positive value of *x*, as it has the minimum value of 1/16 at $-0.5+\sqrt{2}/4$. The function g2 has a positive value of k^2Q at x=0. Therefore, the function g1 and g2 will always intersect at the positive value of x. It means that the growth factor is always positive. So, the perturbation will grow with time and the system becomes unstable. From Fig. 1 we can see that the growth factor increases as the wave number increases. As the function g1(x) is proportional to Ω^4, while g2 is proportional to Ω^2, the growth factor decreases as $\Omega=2K\rho_f u_r/\alpha_f$ increases.

These results are basically the same as that of Pokharna and et. al.[13], where nuemrical analysis was used to determine the growth factor. The present analysis has an advantage, as the same results are obtained in a much straightforward manner.

4.2 An improved one-dimensional two-fluid model

Let's consider an improved one-dimensional two-fluid model with appropriate momentum flux parameters. Assume that the momentum flux parameters satisfty the necessary condition for the stability in equation (29). P is positive.

In the case of no interfacial drag force, the dispersion relation in equation (32) becomes

$$g(\omega_I,k)= 4\underline{\rho\alpha}\omega_I^2\,[\underline{\rho\alpha}^2\omega_I^2+ k^2 P\,] =0 \qquad (40)$$

As P is positive, it has a solution of ω_I=0. The perturbation does not grow. When we consider the interfacial drag, the growth factor is determined from the intersection of the two curves in equation (34) and (35). Figure 2 illustrates the typical shape of the curves represented by equation (34) and (35). Here we assume that k=0.5, P=1.0, Ω=1.0, $\langle\rho\alpha\rangle$=1.0, Q=1.0 or -0.5 for convenience.

It can be seen that the curve g1 and g2 will intersect at positive value of x if Q is positive. If Q is negative, the two curves would not intersect at the positive value of x, because the curve g2 is always negative at the positive value of x. In this case the growth factor cannot be positive and the disturbance does not grow with time. So, the two-fluid model is stable to the disturbances at all wave numbers.

So, it can be concluded that the sufficent condition for the stability is that Q is negative.

$$Q = \alpha_f\rho_g[(\Omega\underline{\alpha u}-\Phi)^2-2C_{vg}u_g(\Omega\underline{\alpha u}-\Phi)\Omega+\Omega^2C_{vg}u_g^2] + \alpha\rho_f[(\Omega\underline{\alpha u}-\Phi)^2-2C_{vf}u_f(\Omega\underline{\alpha u}-\Phi)\Omega+\Omega^2C_{vf}u_f^2] \le 0 \quad (41)$$

The first criterion in equation (29) makes the system of governing differential equations hyperbolic, which enables the propagation of void fraction wave. The second criterion in equation (41) gives a criterion whether the flow is stable to the small perturbations or not. If the second criterion is not met, it will cause the disturbance to grow and it might lead to a change in flow regime.

5 Application of proposed arguments for a two phase flow in a channel

To demonstrate the feasibility of using an improved one-dimensional two-fluid model, we apply the stability criteria to a typical steam-water two-phase flow in a chanel described by the proposed model.

The first stability criterion in equation (29) and the second stability criterion in equation (41) can be written in a non-dimensional form by introducing the parameters $S = u_g/u_f$ and $R= \alpha\rho_f/((1-\alpha)\rho_g)$. The criteria can be presented in a non-dimensional form as functions of void fraction and density ratio as below

$$P^*(\alpha,\rho_g/\rho_f) = P/(\alpha\rho_f u_f)^2 = (C_{vg}S/R + C_{vf})^2-(1/R+1)(C_{vg}S^2/R+C_{vf}) \ge 0 \quad (42)$$

$$Q^*(\alpha,\rho_g/\rho_f) = Q/(\alpha\rho_f\Omega^2u_f^2) = 0.25(1/R+1)(1+\alpha+\alpha_f S)^2 - S/R[1-\alpha(S-1)]C_{vg} - [S+\alpha(1-S)]C_{vf} \le 0 \quad (43)$$

By using above two criteria, we can investigate the adequacy of the improved one-dimensional two-fluid model in describing the two-phase flow in the vertical channel. We need to determine R, S, and momentum flux parameters a function of void fraction to evaluate above criteria. The effect of void profile over a cross-section is typically presented by the volumetric distribution parameter defined as

$$C_o=\langle\alpha j\rangle/\langle\alpha\rangle\langle j\rangle \quad (44)$$

The global slip ratio S can be determined by using the Ishii correlation[15] for C_o.

$$C_o= (1.2-0.2\sqrt{(\rho_g/\rho_f)})(1-e^{-18\alpha}), \quad 0< \alpha < 0.7 \quad (45)$$

$$S=(1-\alpha)C_o/(1- C_o\alpha), \quad (46)$$

The momentum flux parameters are not easily determined mathematically or experimentally in complex flow condition. Fortunatley, there are corrlations for distribution parameter C_o[15] and power of velocity profile [19] so that the momentum flux parameters for gas and liquid phase can be approximated by neglceting the local slip as discussed by Song and Ishii[11, 12]. It is a reasonable assumption for a two-phase flow system, when the flow rate is not very slow.

5.1 Bubbly and Slug Flow

The velocity and void fraction distribution in a flow channel can be represented by a power law profile. By using experimental results of Welle [19], Song and Ishii [11] suggested that the power of velocity profile and void fraction profile for bubbly and slug flow in the void fraction range of 0.2 – 0.7 can be determined as

$$m=10(1-\alpha) \qquad (47)$$

$$n= 0.5*(C_o-1) - m - 2 \qquad (48)$$

Then momentum flux parameters can be caculated from the equations below

$$C_{vg} = (m+2)/(m+1)[1+ (m+n+2)(C_o-1)I(m,n)]/C_o^2 \qquad (49)$$

$$C_{vf} = (1-\alpha)(m+2)/(m+1)[1-\alpha - \alpha(m+n+2)(C_o-1)I(m,n)]/(1-C_o\alpha)^2 \qquad (50)$$

where $I(m,n)=[1+(2m+2)/(m+n+2)]/(2m+2+n)$. So, we know all the parameters to evaluate the stability criteria.

It is interesting that these parameters are determined as a function of void fraction and density ratio. By picking up a saturated two-phase flow at 1.17 Mpa, the properties are determined as ρ_g=5.9795 kg/m^3, ρ_f=879.55 kg/m^3. The calculated liquid and gas momentum flux parameters for bubbly and slug flow are shown in Fig. 3. By using these momentum flux parameters, R, and S, we can calculate P* and Q* as a function of void fraction and density ratio.

Figure 4(a) shows the non-dimensional quantity $P^*(\alpha,\rho_g/\rho_f)$. To look at the general applicability of the argument in other thermodynamic condition, the density ratio is chosen between 0.01 and 0.001, the range of which is between the system pressure of the nuclear reactor and atmospheric pressure.

It is shown that $P^*(\alpha,\rho_g/\rho_f)$ is always positive, which meets the first stability criterion in equation (42). Figure 5(b) shows the non-dimensional quantity $Q^*(\alpha,\rho_g/\rho_f)$. It is shown that $Q^*(\alpha,\rho_g/\rho_f)$ is always negative, which meets the second stability criterion in equation (43). It means that the system of governirng differential equations of one dimensional two fluid model for describing the two phase flow is stable to the small disturbances. It is consistent with the existence of bubbly and slug flow regime in a physical sense, while the conventional basic form of the one-dimensional two fluid model renders the system unstable.

5.2 Wall-peaked bubbly flow

When the void fraction is very low, the local void fraction tends to be in wall-peaked profile [20, 21]. By following the procedure described in Song and Ishii [11] we can determine the void fraction and velocity profiles for the simplified wall-peaked bubbly flow at typical steam-water two-phase system. The parameters of n, m, and wall void fraction α_w are proposed as

$$m=8 \qquad (51)$$

$$n= 0.4836/\alpha\ ,\ \alpha \geq 0.05 \quad \text{and} \quad n= 0.4836/0.05\ ,\ \alpha < 0.05 \qquad (52)$$

$$\alpha_w = \alpha[1 + 0.5(m+n+2)(1- C_o)] \qquad (53)$$

As the power of void fraction profile at very low void fraction is maintianed the same as that of void fraction at 0.05. Also, the distribution parameter is modified as below at fraction below 0.05.

$$C_o= (1.2-0.2\sqrt{(\rho_g/\rho_f)})(1-e^{-0.9}), \qquad 0< \alpha < 0.05 \qquad (54)$$

The momentum flux parameters are determined from equation (49) and (50). The calculated liquid and gas momentum flux parameters for the wall-peaked bubbly flow are shown in Fig. 5 for a saturated two-phase flow at 1.1.7MPa. By using these momentum flux parameters, R, and S, we can calculate P* and Q* as a function of void fraction and density ratio.

Figure 6(a) shows the non-dimensional quantity $P^*(\alpha,\rho_g/\rho_f)$ in the void fraction between 0.0 and 0.2. The density ratio is chosen between 0.01 and 0.001. It is shown that $P^*(\alpha,\rho_g/\rho_f)$ is always positive. So, the first stability criterion in equation (42) is met. Figure 6(b) shows the non-dimensional quantity $Q^*(\alpha,\rho_g/\rho_f)$. It is shown that $Q^*(\alpha,\rho_g/\rho_f)$ is always negative, which satisfies the second stability criterion in equation (43).

It is concluded that the proposed one-dimensional two-fluid model is stable in desribing the wall-peaked bubbly flow in a wide range of density ratio. This is consistent with the existence of wall-peaked bubbly flow at low void fraction.

5.3 Discussions

The argument in the previous sections indicate that we can siginificantly improve one-dimensional two-fluid model by considering the coupling of velocity and void profile. The proposed model is mathematically well-posed and can describe the propagation of the void fraction wave in a wide range of flow regime including wall-peaked bubbly flow, bubbly flow, and slug flow within a wide range of density ratio. Also as the proposed model is stable to the small disturbances at all wave lengths, it would not cause unphysical instability accompanied in the conventional one-dimensional two-fluid model. So, the proposed model would be very powerful in describing the complex two-phase flow system, where multi-dimensional approach might not be pratical. The proposed argument could be extended to the separated flow, such as, annular flow as challenged in Song and Ishii [11]. However, it might be conveneint to use seprated flow model for those flow regime.

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Nomenclature

A, flow area of a channel

A, B, C, matrices

C_D, drag coefficient

C_{vi}, momentum flux parameter of the i-th phase

r_b , radius of a bubble

F_I, interfacial drag

M_{ik}, generalized drag force for the i-th phase

t , time

u_i , local velocity of the i-th phase, or
void fraction weighted average velocity of the i-th phase

z , axial direction along a flow channel

Greek symbols

α , local void fraction of gas phase or area averaged void fraction of gas phase

ρ_i , density of the ith phase

$\boldsymbol{\varphi}$, a vector

Subscripts

f,g,i; liquid, gas, i-th phase

o; quantities in the initial state

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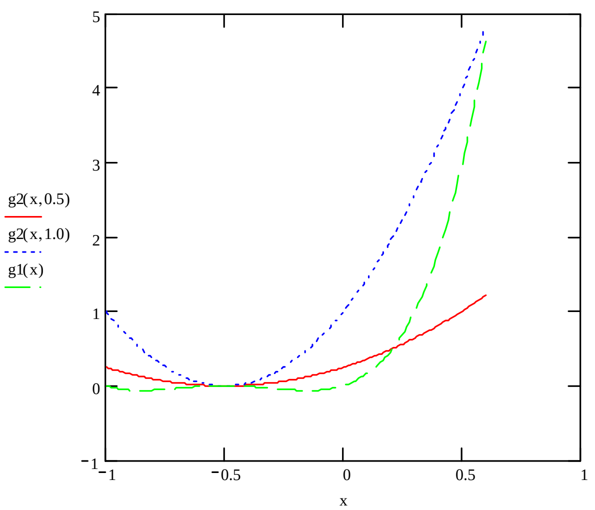


Fig.1 Plots of functions g1(x) and g2(x,k) at k=0.5, Q=1.0 or Q=-0.5

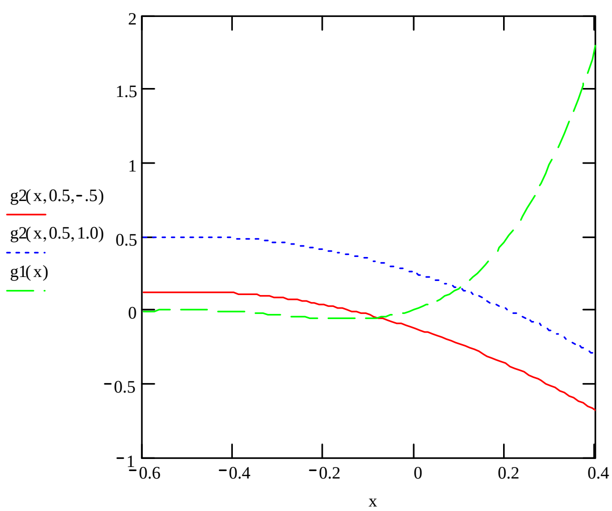
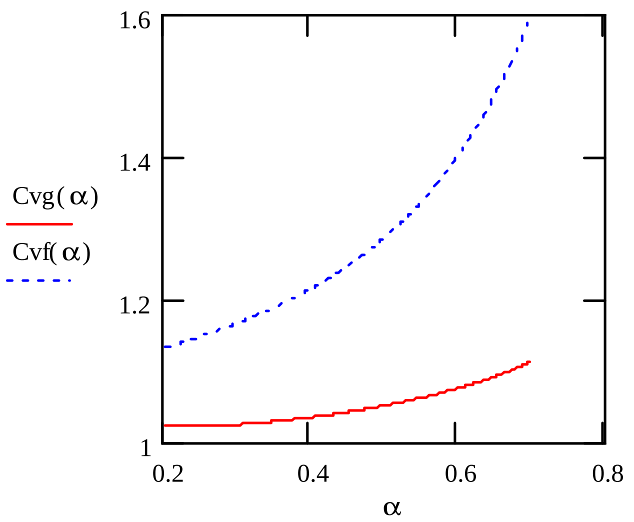


Fig.2 Plots of functions g1(x), g2(x,k,Q) with Ω=1 and k=0.5 or 1.0



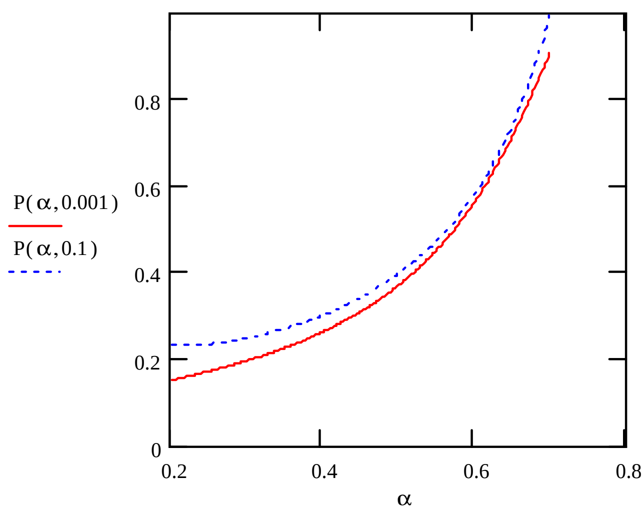


Figure 3 Gas and liquid momentum flux parameters slug flow for bubbly and slug flow

Figure 4(a) Plots of P* for bubbly and at density ratio of 0.1 and 0.001

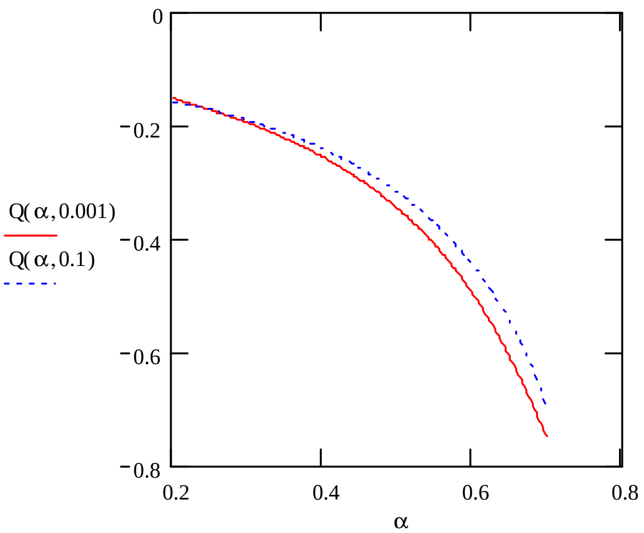


Figure 4(b) Plots of Q* for bubbly and slug flow at density ratio of 0.1 and 0.001

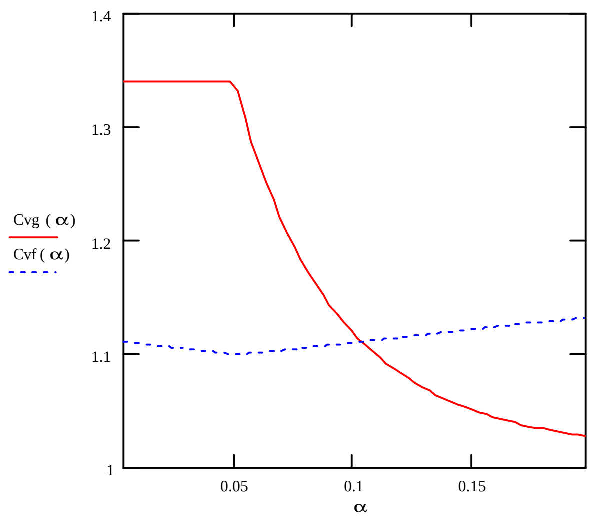


Figure 5 Gas and liquid momentum flux at density parameters for wall-peaked bubbly flow

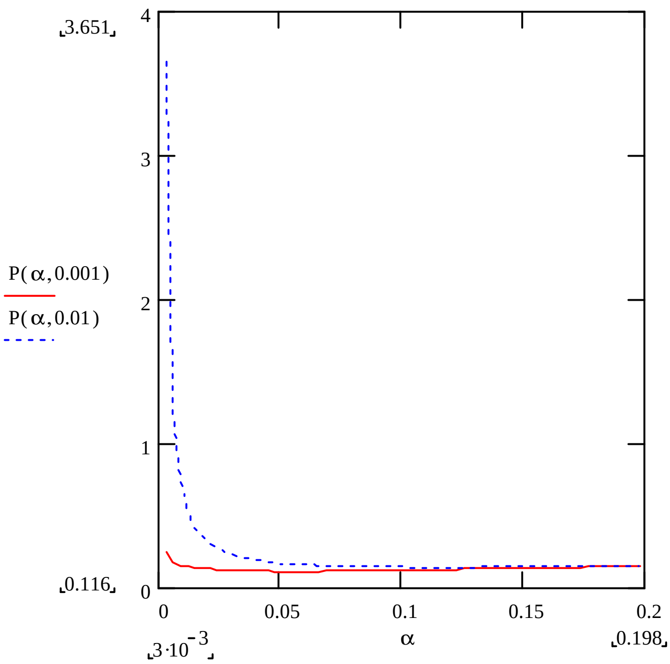


Figure 6(a) Plots of P* for wall-peaked bubbly flow at density ratio of 0.01 and 0.001

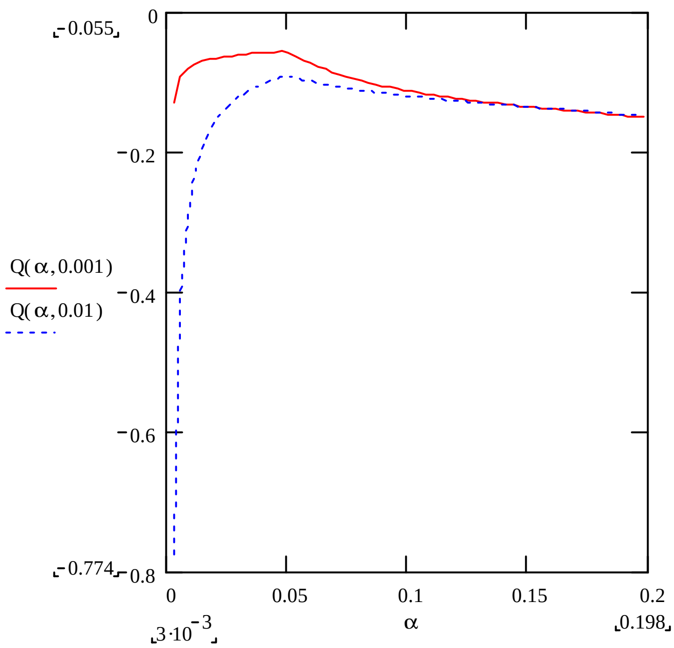


Figure 6(b) Plots of Q* for wall-peaked bubbly flow at density ratio of 0.1 and 0.001