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Constrained Quadratic Programming Method for Determining Shape Annealing Matrix of Ex-core Detectors

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Abstract

A constructive method for determining the Shape Annealing Matrix (SAM) of KSNP is developed with the constrained quadratic programming method with Tikhonov regularization parameter. The current method of SAM determination is using the ordinary least squares method which sometimes gives not physically meaningful and very noise sensitive solutions. Those phenomena come from the poor persistently exciting perturbations in power distribution changes during the start up of the reactor. To circumvent the difficulties, a constrained optimization algorithm is introduced. The method is based on the Tikhonov regularization to reduce the noise sensitivity and the constrained quadratic programming approach to bound the solution within the physical domain. The test results with the real measurement data from KSNPs show remarkable improvement in accuracy and robustness along the cycle burnup.

1. INTRODUCTION

In KSNPs, SAM is used as parameters of reconstructing the 3-level in-core power distribution in the core protection system. The SAM is based on an assumption that the ex-core detector signals and the peripheral core powers have linear relations. Thus the each SAM elements of the top, middle, and bottom ex-core detectors are determined by the linear least squares method with the data measured during the Fast Power Ascension (FPA) test at 20%~80% power or with Xenon oscillation data. The FPA test measurement is a normal process determining the SAM values for reload core. However, the signals measured from FPA test are not persistently exciting since the reactivity perturbations are stepwise increasing inducing small ASI variations. That means the inversion matrix for computing SAM is apt to be singular and ill-posed. This makes the SAM determined very sensitive to the measurement noise and sometimes gives non-physical solutions for SAM.

The non-physical solutions determined at BOC normally invoke very large axial power RMS error exceeding 8% after MOC. This is natural since the measured SAM at BOC is not able to cover the whole cycle it is not optimal for the entire cycle of operation. Also the proper values of SAM should be installed before the increase of the core power up to 80% during FPA test. The power increase sometimes limited due to the difficulty in determining SAM that could give rise to the decrease in plant capacity factor.

In this paper, a constructive method is proposed to resolve the problem by introducing a regularization parameter to reduce the effect of the measurement noise and the physical constraints to bound the solution within the meaningful region. The problem is reconstructed with the damped quadratic programming approach with a regularization parameter. The validity of the proposed method is demonstrated by applying to the real measurement data from KSNPs.

2. REGULARIZATION METHOD FOR LINEAR LEAST SQUARES PROBLEM

A. ILL-POSED LEAST SQUARES PROBLEMS

To demonstrate the ill-posedness of the least square problems induced by measurement noise, consider the following example least squares problem[1)] of finding the over-determined solution x

$$\min_x \| Ax - b \|_2 \tag{1}$$

with coefficient matrix A and right-hand side b given by

$$A = \begin{pmatrix} 0.16 & 0.10 \\ 0.17 & 0.11 \\ 2.02 & 1.29 \end{pmatrix}, \quad b = \begin{pmatrix} 0.27 \\ 0.25 \\ 3.33 \end{pmatrix}.$$

Here, the right-hand side b is small noise corrupted to the exact right-hand side corresponding to the exact solution $x^T = (1 \;\; 1)$.

$$b = \begin{pmatrix} 0.16 & 0.10 \\ 0.17 & 0.11 \\ 2.02 & 1.29 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 0.01 \\ -0.03 \\ 0.02 \end{pmatrix}$$

The condition number of this least squares problem is 1.1×10^3 which implies that the least squares solution is potentially very sensitive to the noise of the data. The condition number of a matrix is always greater or equal to 1. If it is close to one, the matrix is well conditioned which means its inverse can be computed with good accuracy. If the condition number is large, then the matrix is said to be ill-conditioned. Practically, such a matrix is almost singular, and the computation of its inverse, or solution of a linear system of equations is prone to large numerical errors. A matrix that is not invertible has the condition number equal to infinity. The ordinary least-squares solution of this problem is $x^T{}_{LS} = (7.01 \;\; -8.40)$. This solution is obviously worthless, and something must be done in order to compute a better approximation to the exact solution.

Hansen[1] identified the difficulties associates with discrete ill-posed problems.

1. the condition number of the matrix A is large
2. replacing A by a well-conditioned matrix derived from A does not necessarily lead to a useful solution
3. care must taken when imposing additional constraints.

B. DISCRETE ILL-POSED PROBLEMS OF EX-CORE DETECTOR RESPONSE

The Fredholm integral equation of the first kind with a square integrable kernel is the classical example of an ill-posed problem[1,2] ,

$$\int_a^b \mathrm{K}(\mathrm{s},\mathrm{t})\,\mathrm{f}(\mathrm{t})\,\mathrm{dt} = \mathrm{g}(\mathrm{s}) \tag{2}$$

where the right-hand side g and the kernel K are given, and where f is the unknown solution. If the solution f is perturbed by

$$\Delta\mathrm{f}(\mathrm{t}) = \varepsilon\,\sin(2\pi \mathrm{pt}),\qquad \mathrm{p} = 1, 2, ...,\quad \varepsilon = \mathrm{constant}$$

then the corresponding perturbation of the right-hand side g is given by

$$\Delta g(s) = \varepsilon \int_a^b \mathrm{K}(\mathrm{s},\mathrm{t})\sin(2\pi \mathrm{pt})\,\mathrm{dt},\quad \mathrm{p} = 1, 2, ...$$

and due to the Riemann-Lebesgue lemma it follows that $\Delta g \to 0$ as $p \to \infty$.

Hence, the ratio $\|\Delta f\|/\|\Delta g\|$ can become arbitrary large by choosing the integer p large enough, thus showing that (2) is an ill-posed problem. In particular, this example illustrates that Fredholm integral equations of the first kind with square integrable kernels are extremely sensitive to high-frequency perturbations.

The ex-core detector response *D* for a reactor power distribution *P(r)* can be considered as a similar integral equation by

$$\int_V P(r)\omega(r)dr = D, \tag{3}$$

where $\omega(r)$ is the spatial weighting function and V denotes the core volume. For a 3-segment excore detectors, the normalized axial spatial weighting function can be written in the form :

$$\omega_{d,k} = \frac{\int_{V_k} dr_i \int\int \chi(E)\Phi_d^*(r_i,\Omega,E)d\Omega dE}{\sum_{d=1}^{3}\int_V dr_i \int\int \chi(E)\Phi_d^*(r_i,\Omega,E)d\Omega dE}\cdot\frac{V}{V_k}, \tag{4}$$

where $\omega_{d,k}$ is axial spatial weight of d-th detector segment and V_k is volume of the k-th core axial segment. Note that $\Phi_d^*(r_i, \Omega, E)$ is adjoint flux subject to adjoint source at d-th detector segment. Figure 1 shows the axial spatial weights $\omega_{d,k}$ for 3-level detector signals.

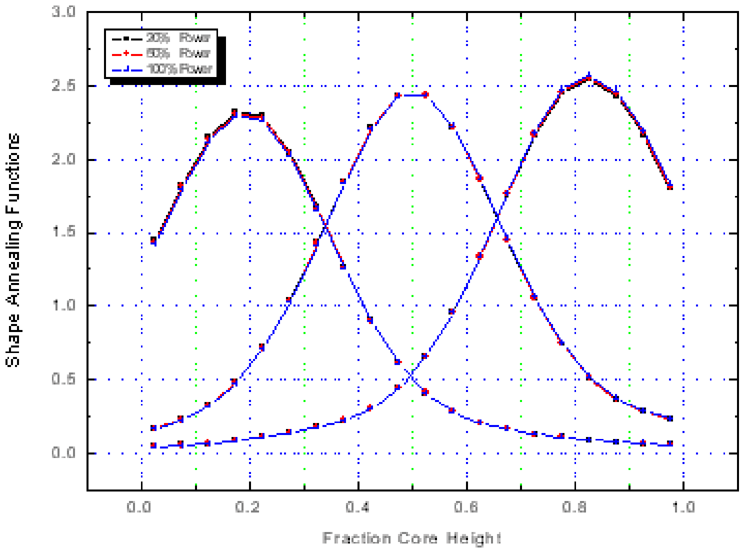

Figure 1.   Axial spatial weights of 3-level detector signals

The linear relations between top, middle, and bottom ex-core detector responses and the 3-level core peripheral powers can be represented by the SAM matrix as follows :

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix}, \tag{5}$$

where S_{ij} is the elements of the SAM, P_i is the peripheral power of core level i and D_i is the excore detector signal of level i. The elements of the inverse SAM mean the integrated values of each axial spatial weights $\omega_{d,k}$.

The determination of the SAM is performed by collecting the measurements (peripheral powers and ex-core detector signals) during the FPA test. Then the corresponding least squares problem becomes

$$\begin{pmatrix} D_{i1}^1 & D_{i2}^1 & D_{i3}^1 \\ D_{i1}^2 & D_{i2}^2 & D_{i3}^2 \\ \cdots & & \\ D_{i1}^N & D_{i2}^N & D_{i3}^N \end{pmatrix} \begin{bmatrix} S_{i1} \\ S_{i2} \\ S_{i3} \end{bmatrix} = \begin{bmatrix} P_i^1 \\ P_i^2 \\ \cdots \\ P_i^N \end{bmatrix} \tag{6}$$

where *i=1,2,3* correspond to the top, middle, bottom detectors, respectively for the snapshots *1,…,N*. If we set Eq.(6) as $Ax = b$, the problem becomes a typical least squares finding the over-determined solution $\mathrm{x}_{LS} = (\mathrm{S}_{i1}\ \ \mathrm{S}_{i2}\ \ \mathrm{S}_{i3})^{\mathrm{T}}$.

The SAM represents the extremely simplified inverse (or adjoint) of the axial spatial weights (kernel) of the neutron transport from core periphery to ex-core detectors. The determination of the SAM matrix can be a typical ill-posed least squares problem corresponding to the classical example of a Fredholm integral equation of the first kind with a extremely simplified transport kernel assumed as a linear summation of the 3-level ex-core detector signals.

This kind of discrete ill-posed problems have the following criteria[1)] .

1. the singular values of A decay gradually to zero
2. the ratio between the largest and the smallest nonzero singular values is large.

Criterion 2 implies that the matrix A is ill-conditioned and that the solution is potentially very sensitive to perturbations. The criterion 1 implies that there is no nearby problem with a well-conditioned coefficient matrix and with well-determined numerical rank. An important aspect of discrete ill-posed problems is that the ill-conditioning of the problem does not mean that a meaningful approximate solution cannot be computed. Rather, the ill-conditioning implies that standard methods in numerical linear algebra cannot be used in a straightforward manner to compute such a solution. Instead, more sophisticated methods must be applied in order to ensure the computation of a meaningful solution. It is the primary goal of this paper to find out a constructive and stable method solving the discrete ill-posed problem of SAM determination.

C. REGULARIZATION METHOD[1,2,3)]

C. 1 PSEUDO INVERSE SOLUTION

The pseudo inverse solution of the least squares problem (1) becomes

$$x_{LS} = (\mathrm{A}^{\mathrm{T}}\mathrm{A})^{-1}\mathrm{A}^{\mathrm{T}}\mathrm{b}\,. \tag{7}$$

By using the SVD (singular value decomposition), this can be written as

$$x_{LS} = V_p \Sigma_p^{-1} U_p^T \mathrm{b}\,. \tag{8}$$

where p is the number of nonzero singular values, Up consists of the first p columns of U, Vp consists of the first p columns of V, and Σ_p^{-1} is a diagonal matrix whose entries are the reciprocals of the nonzero singular values of A.

The elements of the vector $U_p^T \mathrm{b}$ are simply the dot products of the first p columns of U with b. This vector can be written as

$$U_p^T \mathrm{b} = \left[U_1^T \mathrm{b} \quad U_2^T \mathrm{b} \ \cdots \ U_p^T \mathrm{b} \right]^T . \tag{9}$$

Multiplying Σ_p^{-1} times this vector, we obtain

$$\Sigma_p^{-1} U_p^T \mathrm{b} = \left[\frac{U_1^T \mathrm{b}}{\sigma_1} \quad \frac{U_2^T \mathrm{b}}{\sigma_2} \quad \cdots \quad \frac{U_p^T \mathrm{b}}{\sigma_\mathrm{p}} \right]^T . \tag{10}$$

Finally, when we multiply Vp times this vector, we obtain a linear combination of the columns of Vp that can be written as

$$x_{LS} = \sum_{i=1}^{p} \frac{U_i^T \mathrm{b}}{\sigma_\mathrm{i}} V_i . \tag{11}$$

This formula for the least squares solution is helpful because it shows us why small singular values can have a huge effect on the least squares solution in the presence of noise. In the presence of random noise, $U_i^T \mathrm{b}$ is very likely to be nonzero, even if the true data were orthogonal to U_i. When we divide this nonzero value by a very small singular value σ_i, we get a very large number, which is then multiplied by the singular vector V_i. In this way, the least squares solution incorporates large components in the direction V_i. We say that the discrete Picard condition is satisfied when the values of $U_i^T \mathrm{b}$ decay to zero faster than the singular values σ_i. This is an indication that pseudo-inverse solution will not be highly sensitive to noise. If the Picard condition is not satisfied, then our solution is likely to be extremely sensitive to noise in the data.

C. 2 Tikhonov Regularization

Normally, the solution with small norm shows low sensitivity to the measurement noise. This fact implies that we can get more meaningful solution if we bound the norm of the least squares solution. There are many ways to bound the solution through constrained minimization, i.e., $\min \| x \|, \ s.t. \| Ax - b \| \le \delta$ or $\min \| Ax - b \|, \ s.t. \| x \| \le \varepsilon$. The most common and well-known form of regularization method is Tikhonov regularization with the following form of damped least squares problem

$$\min \{ \| \mathrm{Ax - b} \|_2^2 + \lambda^2 \| \mathrm{x} \|_2^2 \} . \tag{12}$$

The damped least squares problem (12) is equivalent to the ordinary least squares problem

$$\min \left\| \begin{bmatrix} \mathrm{A} \\ \lambda \mathrm{I} \end{bmatrix} \mathrm{x} - \begin{bmatrix} \mathrm{b} \\ 0 \end{bmatrix} \right\|^2 . \tag{13}$$

The solution to this least squares problem can be obtained by solving the normal equations

$$\begin{pmatrix} \mathrm{A} & \lambda \mathrm{I} \end{pmatrix} \begin{pmatrix} \mathrm{A} \\ \lambda \mathrm{I} \end{pmatrix} x = \begin{pmatrix} \mathrm{A} & \lambda \mathrm{I} \end{pmatrix} \begin{pmatrix} \mathrm{b} \\ 0 \end{pmatrix} . \tag{14}$$

This system of equations simplifies to

$$(A^T A + \lambda^2 I)x = A^T b\,. \tag{15}$$

In terms of the SVD, this can be written as

$$(V\Sigma^T \Sigma V^T + \lambda^2 I)x = V\Sigma^T U^T b\,. \tag{16}$$

Since this system of equations is nonsingular (a is nonzero), it has a unique solution given by

$$x_\lambda = \sum_{i=1}^{p} \frac{\sigma_i^2}{\sigma_i^2 + \lambda^2} \frac{U_i^T \mathrm{b}}{\sigma_\mathrm{i}} V_i\,. \tag{17}$$

The factors

$$f_i = \frac{\sigma_i^2}{\sigma_i^2 + \lambda^2} \tag{18}$$

are called filter factors. For . $\sigma_i >> \lambda$, the filter factor f_i is approximately one. For $\sigma_i << \lambda$, the filter factor f_i is approximately zero. In between, the filter factors serve to reduce the contribution of the small singular values to the solution. Clearly, a large λ (equivalent to a large amount of regularization) favors a small solution semi-norm at the cost of a large residual norm, while a small λ (i.e., a small amount of regularization) has the opposite effect. The underlying idea is that a regularized solution with small (semi)norm and a suitably small residual norm is not too far from the desired, unknown solution to the unperturbed problem underlying the given problem.

D. Constrained Regularization Method

In real applications the behavior of the solution is sometimes known beforehand. When the behavior of the solution is known beforehand it can be a great advantage to be able to incorporate this information in order to reconstruct the solution. The use of the proper side constraints helps to reconstruct the solution. The linear inequality constraints are imposed as follows

$$\min\{\| \mathrm{Ax} - \mathrm{b} \|_2^2 + \lambda^2 \| \mathrm{x} \|_2^2\}, \quad \mathrm{s.t.}\, \mathrm{Gx} \ge \mathrm{d}\,. \tag{19}$$

For non-negative x, we can choose $G = I, d = 0$.

Since Eq. (19) is equivalent to the damped least squares problem (13), (19) can be transformed into

$$\min\{\| \mathrm{Ax} - \mathrm{b} \|_2^2\}, \quad \mathrm{s.t.}\, \mathrm{Gx} \ge \mathrm{d} \tag{20}$$

where the matrix A and the vector b are augmented as $\begin{bmatrix} \mathrm{A} \\ \lambda \mathrm{I} \end{bmatrix} \rightarrow \mathrm{A}$ and $\begin{bmatrix} \mathrm{b} \\ 0 \end{bmatrix} \rightarrow \mathrm{b}$.

We can further define $H = A^T A$ and $g = -A^T b$, then the following problem becomes equivalent to (20)

$$\min q(x) = \frac{1}{2}\mathrm{x}^{\mathrm{T}}\mathrm{Hx} + \mathrm{g}^{\mathrm{T}}\mathrm{x}, \quad \mathrm{s.t.}\, \mathrm{Gx} \ge \mathrm{d} \tag{21}$$

This is the well-known constrained quadratic programming (QP) solution and can be easily solved by the active set method.

3. Application of Constrained Regularization method To Determine Shape Annealing Matrix

The inverse of the SAM matrix has the well-established physical meaning of inequalities in magnitudes explaining the location dependent detector sensitivities as follows

$$\begin{aligned} &\mathrm{T}_{11} > \mathrm{T}_{12} > \mathrm{T}_{13} \\ &\mathrm{T}_{21} < \mathrm{T}_{22} > \mathrm{T}_{23} \\ &\mathrm{T}_{31} < \mathrm{T}_{32} < \mathrm{T}_{33} \end{aligned} \tag{22}$$

where T_{ij} is the elements of the inverse SAM has to be positive in any conditions since the detector sensitivities have definite positive contributions.

In this context, we can set up the constraints for the SAM matrix. Firstly, the analytic expression of the inverse of the (3x3) matrix can be written by

$$\begin{pmatrix} \mathrm{S}_{11} & \mathrm{S}_{12} & \mathrm{S}_{13} \\ \mathrm{S}_{21} & \mathrm{S}_{22} & \mathrm{S}_{23} \\ \mathrm{S}_{31} & \mathrm{S}_{32} & \mathrm{S}_{33} \end{pmatrix}^{-1} = \begin{pmatrix} \mathrm{T}_{11} & \mathrm{T}_{12} & \mathrm{T}_{13} \\ \mathrm{T}_{21} & \mathrm{T}_{22} & \mathrm{T}_{23} \\ \mathrm{T}_{31} & \mathrm{T}_{32} & \mathrm{T}_{33} \end{pmatrix}, \tag{23}$$

where the matrix determinant is given by

$$\det = S_{11}S_{22}S_{33} + S_{12}S_{23}S_{31} + S_{13}S_{21}S_{32} - (S_{13}S_{22}S_{31} + S_{23}S_{32}S_{11} + S_{33}S_{12}S_{21})\,.$$

For the one of the upper diagonal element of SAM $S_{12} = T_{13}T_{32} - T_{12}T_{33}$. Since $T_{12} \sim T_{32}$ and $T_{13} << T_{33}$, S_{12} should be negative. Similarly, the elements of the SAM matrix $\mathrm{S}_{21}, \mathrm{S}_{23}, \mathrm{S}_{32}$ should be negative too. Since the behavior of the solution of SAM is known beforehand it can be a great advantage to be able to incorporate this information in order to reconstruct the solution.

For the calculation of the SAM matrix, the constrain matrix G for top, middle, and bottom detectors become as follows:

$$G(Top) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad G(Middle) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad G(Bottom) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{24}$$

The simulation result showd the regularization factor id not sensitive to the result in the range of $1.0\times10^{-3} \sim 1.0\times10^{-2}$ so the fixed value of $\lambda = 1.0\times10^{-2}$ was used.

The developed algorithm is tested with three FPA test cases in KSNPs. Table 1 shows the condition numbers of the detector measurement matrix in Eq.(6) which show very large numbers that means the least squares solutions of those problems are extremely sensitive to measurement noise.

Table 1. Condition numbers of the Detector Measurement Matrix in Eq.(6)

Cases \ Channels	A	B	C	D
Case 1	2.6343×10^3	1.4167×10^3	2.4008×10^3	$1.6486e\times10^3$
Case 2	2.8714×10^3	3.0349×10^3	2.4409×10^3	2.3385×10^3
Case 3	2.4409×10^3	3.0349×10^3	2.4409×10^3	2.3385×10^3

Tables 2-1 ~ 4-2 are the calculated values of SAM, Inverse SAM and test value for three sample cases. The test value means the norms of the calculated SAM values with the optimal value of 4.0. The SAM matrix with the test value within the range of 3.0~6.0 can be installed to prevent the excessive noise sensitivity. The most physically meaningful SAM can be considered as the one with test value 4.0 and the inequality (22) is satisfied. The test value of 5.0289 (channel B of case 1) in table 2-1 is improved to 4.2119 in table 2-2 by applying new method. Tables 3 and 4 are the noise contaminated cases and the SAM calculation results are greatly improved. The inverse SAMs have many negative(-) elements with the current method but the new method gives the positiveness of inverse SAM elements satisfying inequality and test values very close to 4.0.

Table 2-1. SAM, Inverse SAM and Test Value for Case 1 (Current Method)

```
============================================================================
  Channel            SAM                      Inverse SAM          Test Value
============================================================================
    A        4.5397 -0.2905 -1.5600    0.2383  0.0269  0.0785      3.8990
            -0.7580  3.6646 -0.2673    0.0524  0.2803  0.0325
            -0.7817 -0.3740  4.8272    0.0426  0.0261  0.2224
---------------------------------------------------------------------------
    B        5.2913 -2.3617  0.6197    0.1960  0.1305 -0.0041      5.0289
            -0.3447  3.6017 -0.8293    0.0410  0.2738  0.0628
            -1.9466  1.7600  3.2096    0.0964 -0.0710  0.2746
---------------------------------------------------------------------------
    C        4.3385 -0.4846 -0.9101    0.2436  0.0370  0.0529      3.9674
            -0.8709  4.1603 -0.9288    0.0581  0.2569  0.0602
            -0.4677 -0.6757  4.8389    0.0317  0.0394  0.2202
---------------------------------------------------------------------------
    D        4.6510 -0.9608 -0.6811    0.2318  0.0583  0.0445      4.0832
            -0.3466  3.3753 -0.4288    0.0326  0.2992  0.0366
            -1.3044  0.5855  4.1098    0.0689 -0.0241  0.2522
---------------------------------------------------------------------------
```

Table 2-2. SAM, Inverse SAM and Test Value for Case 1 (New Method)

Channel	SAM			Inverse SAM			Test Value
A	4.5149	-0.2516	-1.5846	0.2398	0.0240	0.0803	3.8736
	-0.7093	3.5880	-0.2188	0.0501	0.2849	0.0296	
	-0.8148	-0.3220	4.7942	0.0441	0.0232	0.2242	
B	5.2773	-2.3350	0.5975	0.1971	0.1285	-0.0029	4.2119
	-0.3336	3.5805	-0.8116	0.0282	0.2977	0.0480	
	-1.0311	0.0000	4.6793	0.0434	0.0283	0.2131	
C	4.3021	-0.4239	-0.9525	0.2453	0.0337	0.0549	3.9303
	-0.7917	4.0285	-0.8366	0.0552	0.2623	0.0570	
	-0.5238	-0.5822	4.7734	0.0337	0.0357	0.2225	
D	4.6405	-0.9422	-0.6951	0.2287	0.0643	0.0407	3.8057
	-0.3333	3.3517	-0.4110	0.0287	0.3064	0.0321	
	-0.9733	0.0000	4.5515	0.0489	0.0137	0.2284	

Table 3-1. SAM, Inverse SAM and Test Value for Case 2 (Current Method)

Channel	SAM			Inverse SAM			Test Value
A	5.2241	-1.4507	-0.7759	0.2110	0.0810	0.0449	4.6658
	-0.0713	2.9462	-0.1558	0.0110	0.3369	0.0155	
	-2.1529	1.5046	3.9317	0.1113	-0.0846	0.2730	
B	0.9993	5.1356	-4.9302	-0.8392	2.0111	-1.1236	18.9414
	1.3037	0.6094	1.5227	0.7570	-1.0214	0.8252	
	0.6970	-2.7449	6.4075	0.4156	-0.6563	0.6318	
C	2.4951	2.8674	-3.5096	-0.7540	1.8716	-1.0368	9.1595
	1.1706	0.7907	1.4107	1.0502	-1.5788	1.1597	
	-0.6659	-0.6579	5.0987	0.0370	0.0407	0.2104	
D	1.2107	4.9768	-5.0204	-0.2055	0.8390	-0.4045	45.3182
	1.5863	0.1794	1.7793	0.3955	-0.3555	0.4195	
	0.2030	-2.1562	6.2411	0.1433	-0.1501	0.3183	

Table 3-2. SAM, Inverse SAM and Test Value for Case 2 (New Method)

Channel	SAM			Inverse SAM			Test Value
A	5.1873	-1.3873	-0.8221	0.2023	0.0969	0.0354	3.9210
	-0.0421	2.8960	-0.1192	0.0051	0.3477	0.0091	
	-1.2792	0.0000	5.0290	0.0515	0.0246	0.2078	
B	4.3226	0.0000	-1.7054	0.2259	0.0526	0.0609	4.3365
	-0.2618	3.0309	0.0000	0.0195	0.3345	0.0053	
	0.6155	-2.6189	6.3283	-0.0139	0.1333	0.1543	
C	4.3378	0.0000	-1.7013	0.2446	0.0167	0.0820	3.7594
	-0.2619	3.0231	0.0000	0.0212	0.3322	0.0071	
	-0.6922	-0.6169	5.0728	0.0360	0.0427	0.2092	
D	4.4893	0.0000	-1.9642	0.2236	0.0482	0.0708	4.1031
	-0.3133	3.0686	0.0000	0.0228	0.3308	0.0072	
	0.1593	-2.0899	6.2002	0.0019	0.1103	0.1619	

Table 4-1. SAM, Inverse SAM and Test Value for Case 3 (Current Method)

Channel	SAM			Inverse SAM			Test Value
A	5.8865	-2.4271	0.0972	0.1877	0.1120	0.0181	4.7872
	-0.5611	3.8059	-0.7558	0.0472	0.2697	0.0545	
	-2.3254	1.6212	3.6586	0.0984	-0.0483	0.2607	
B	2.4521	2.5744	-3.2089	-0.1247	0.6109	-0.2907	10.4823
	1.6250	0.4934	1.5580	0.4799	-0.4130	0.4695	
	-1.0772	-0.0678	4.6509	-0.0219	0.1355	0.1545	
C	3.3198	1.3147	-2.3667	0.3498	-0.1495	0.1811	4.6928
	0.3123	2.3141	0.4363	-0.0544	0.4452	-0.0655	
	-0.6322	-0.6286	4.9303	0.0379	0.0376	0.2177	
D	2.5073	2.7235	-3.5205	3.4273	-5.1441	3.3011	7.5505
	1.0917	1.2133	1.1201	-2.9606	5.1541	-2.9989	
	-0.5990	-0.9369	5.4005	-0.1335	0.3237	0.0310	

Table 4-2. SAM, Inverse SAM and Test Value for Case 3 (New Method)

Channel	SAM			Inverse SAM			Test Value
A	5.8350	-2.3474	0.0409	0.1852	0.1160	0.0157	4.0922
	-0.5232	3.7473	-0.7144	0.0353	0.2889	0.0427	
	-1.2771	0.0000	4.8033	0.0492	0.0308	0.2124	
B	4.3120	0.0000	-1.6480	0.2552	0.0009	0.0909	3.6747
	-0.2266	3.0590	0.0000	0.0189	0.3270	0.0067	
	-1.1041	-0.0305	4.6283	0.0610	0.0024	0.2378	
C	4.2421	0.0000	-1.5335	0.2487	0.0148	0.0780	3.7019
	-0.1696	3.0017	0.0000	0.0141	0.3340	0.0044	
	-0.6743	-0.5687	4.8923	0.0359	0.0409	0.2157	
D	4.4625	0.0000	-1.8736	0.2372	0.0239	0.0828	3.8326
	-0.2336	3.0619	0.0000	0.0181	0.3284	0.0063	
	-0.6380	-0.8825	5.3676	0.0312	0.0568	0.1972	

Figures 2 and 3 show the RMS errors of cases 1 and 2 reconstructed with the calculated SAM (e=Ax-b) summed for 3-level detectors. Although the calculated SAMs are quite different, the RMS errors show very close trend This means that the SAMs are decided in another optimal sense with constrained optimization technique.

Figures 4 and 5 show the RMS errors of cases 1 and 3 by core follow calculations from BOC to EOC. The decrease in RMS error of channel B in Fig.4 is due to the improvement in SAM value by the new method. The increase in accuracy of the RMS errors for each channel is remarkable for case 3 (Fig. 5) since the SAM calculated with conventional ordinary least squares method has unphysical values for case 3.

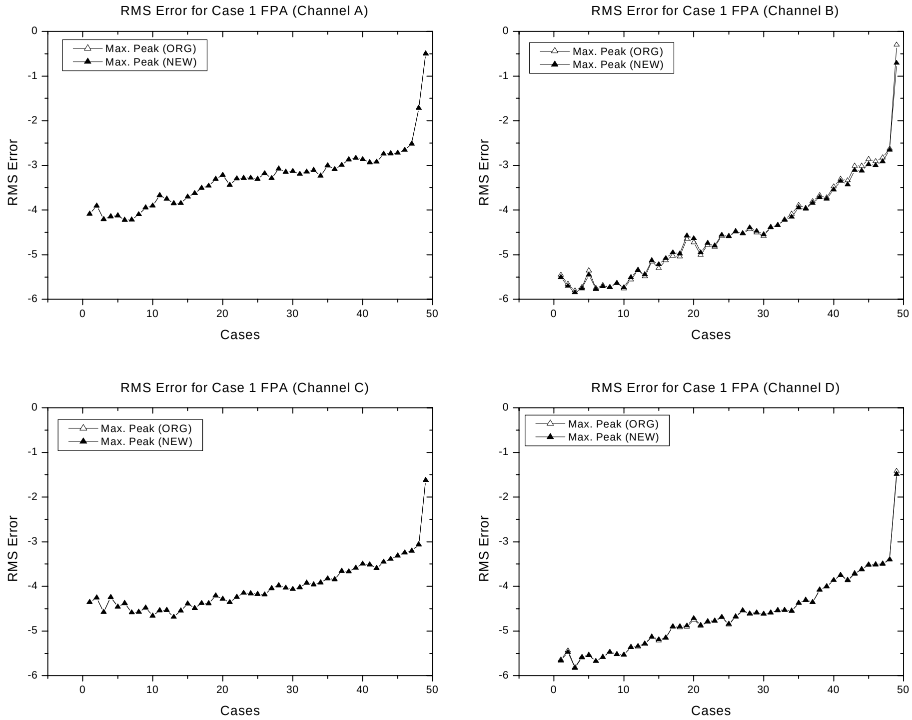

Figure 2. RMS Errors of each Channel for Case 1 FPA (Current & New Methods)

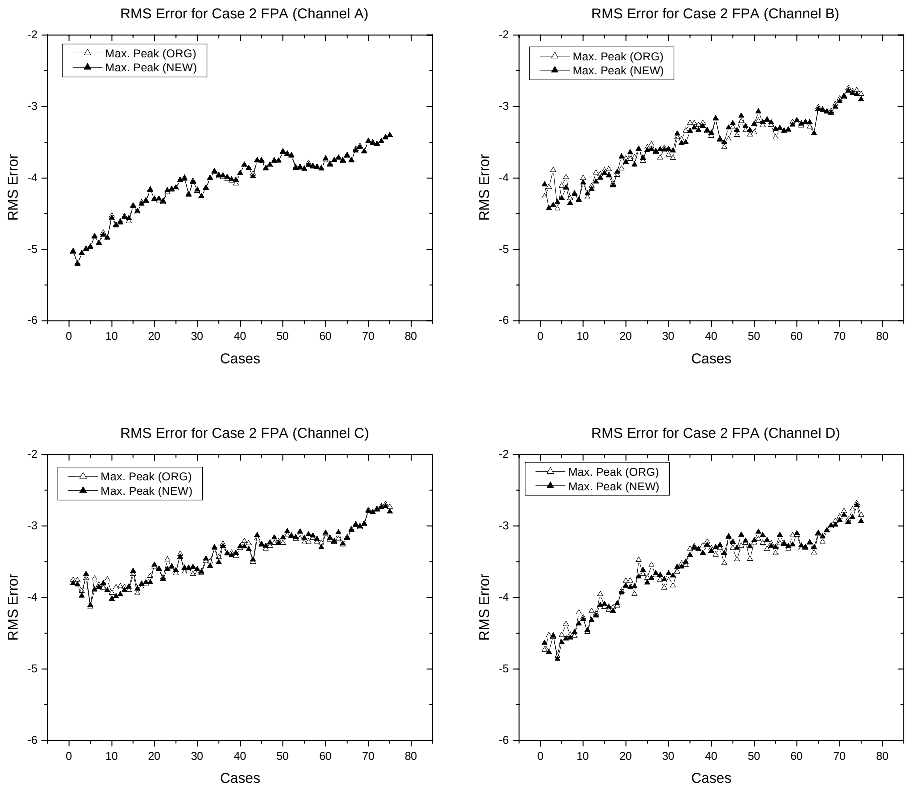

Figure 3. RMS Errors of each Channel for Case 2 FPA (Current & New Methods)

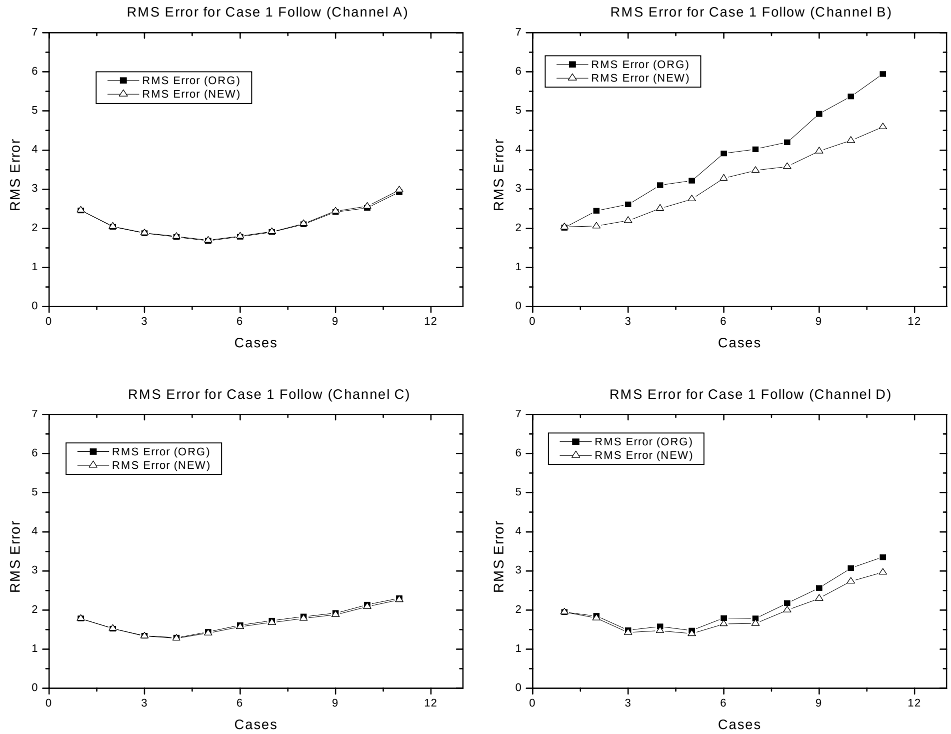

Figure 4. RMS Errors of each Channel for Case 1 Follow (Current & New Methods)

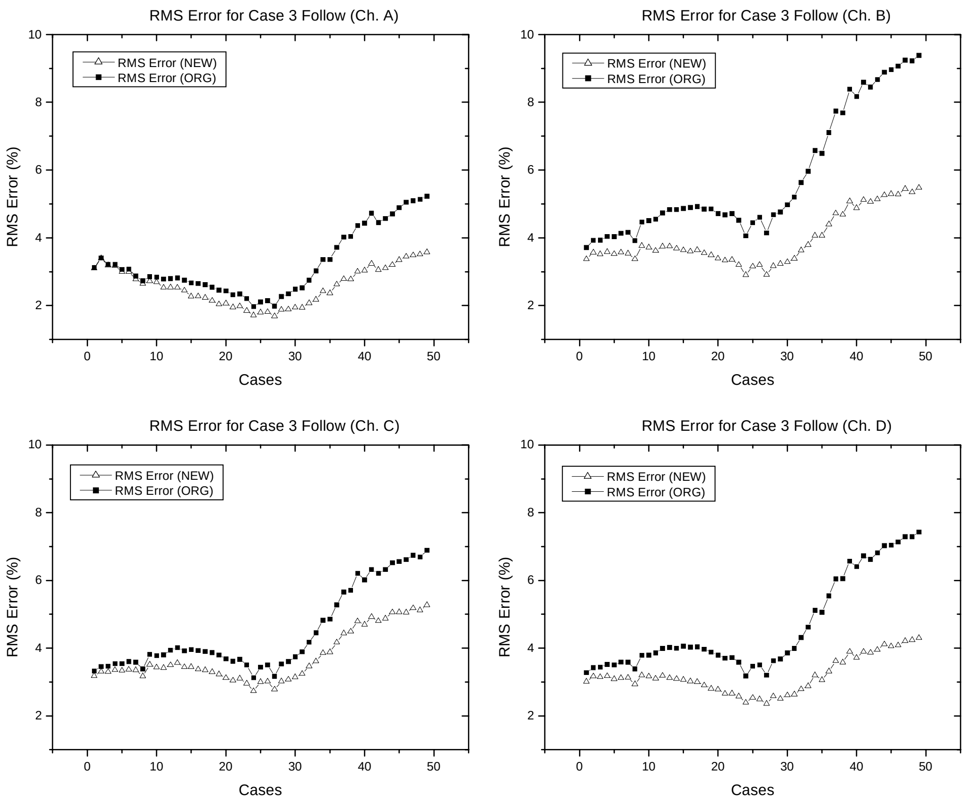

Figure 5. RMS Errors of each Channel for Case 3 Follow (Current & New Methods)

4. Conclusion

The least squares method for parameter identification of linear models has definite applications in nuclear science, for example, modeling the responses of the spatially distributed detectors, nuclear data treatment, response surface modeling of the thermal margin estimation etc. Considering the inevitable measurement noise, the ill-posedness of the least squares method can arise and limit the applicability of the assumed model structures. In this paper, a constructive method is proposed with the constrained quadratic programming approach with Tikhonov regularization parameter. The test results applied to determine the Shape Annealing Matrix of KSNP show the remarkable improvement in accuracy and robustness under the poor persistently exciting perturbations conditions. The developed method can be applied with minimal changes in computer codes for core protection system design. Since the method always gives physically meaningful and robust solution, the reload start-up process can be facilitated. The developed method can be applied to various reconstruction problems, parameter identification in dynamic system modeling, functional neural network training, and computer-assisted tomography etc[3)].

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