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A Fusion Technique of 2-D/1-D Methods for Three-Dimensional Whole-Core Transport Calculations

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ABSTRACT

3-D heterogeneous transport calculation of OECD benchmark problem C5G7 MOX was performed with a fusion technique of 2D/1D methods: the method of characteristics (MOC) for radial 2-D calculation and the diamond difference (DD) scheme for axial 1-D calculation. We further save computer memory and computation time significantly by parallel computation and additive angular dependent rebalance (AADR) acceleration, rendering the 3-D whole-core transport calculation feasible.

1. INTRODUCTION

The CRX code based on the method of characteristics (MOC) has been constructed to treat 2-D whole-core heterogeneous calculation with features of modular ray tracing and parallel computing.[1][2][3]

Recently, a 2D/1D fusion method [4] was implemented in the CRX code for 3-D calculation. The 2D/1D fusion method, devised for 3-D problems, is a synergistic combination of the method of characteristics for radial 2-D calculation and the DD approximation for axial 1-D calculation. Each solver sends and receives surface angular fluxes and thus requires some iterations. However, this new approach requires much less memory and takes advantage of simpler structure of the core in axial direction, resulting in overall efficiency.

In this paper, we presents some results of the C5G7 MOX benchmark problem released by OECD/NEA [5].

2. DESCRIPTION OF THE 3-D CALCULATION

For 3-D calculation, the 2D/1D fusion method [4] as described in the following is used. Let us consider the following directional form of the within-group neutron transport equation:

$$\vec{\Omega}_n \cdot \nabla \psi^g(\vec{r},\vec{\Omega}_n) + \sigma_t^g(\vec{r})\psi^g(\vec{r},\vec{\Omega}_n) = q^g(\vec{r},\vec{\Omega}_n), \tag{1}$$

where standard notations are used.[6] For 3-D cartesian coordinates, we obtain the following equation by integrating Eq. (1) over z-direction interval $(z_{k-1/2}, z_{k+1/2})$:

$$\mu_n \frac{\partial \psi^g_{n,k}(x,y)}{\partial x} + \eta_n \frac{\partial \psi^g_{n,k}(x,y)}{\partial y} + \frac{\chi_n}{\Delta_k}(\psi^g_{n,k+1/2}(x,y) - \psi^g_{n,k-1/2}(x,y)) + \sigma^g_k(x,y)\psi^g_{n,k}(x,y) = q^g_{n,k}(x,y), \tag{2a}$$

where

$$\psi^g_{n,k}(x,y) = \frac{1}{\Delta_k}\int_{z_{k-1/2}}^{z_{k+1/2}} dz\, \psi^g_n(x,y,z), \tag{2b}$$

$$\psi^g_{n,k\pm1/2}(x,y) = \psi^g_n(x,y,z_{k\pm1/2}), \quad \Delta_k = z_{k+1/2} - z_{k-1/2}, \tag{2c}$$

$$q^g_{n,k}(x,y) = \sum_{g'=1}^{G}\sum_{l=0}^{L}\sum_{m=-l}^{l} Y_{lm}(\vec{\Omega}_n)\sigma^{g'\to g}_{sl,k}(x,y)\sum_{n'} w_{n'} Y^*_{lm}(\vec{\Omega}_{n'})\psi^{g'}_{n',k}(x,y) + \frac{\chi^g}{k_{eff}}\sum_{g'=1}^{G}\nu\sigma^{g'}_{f,k}(x,y)\sum_{n'} w_{n'}\psi^{g'}_{n',k}(x,y). \tag{2d}$$

Rewriting Eq. (2a), the equation can be given in two-dimensional form:

$$\mu_n \frac{\partial \psi^g_{n,k}}{\partial x} + \eta_n \frac{\partial \psi^g_{n,k}}{\partial y} + \sigma^g_k(x,y)\psi^g_{n,k}(x,y) = Q^g_{n,k}(x,y), \tag{3a}$$

where

$$Q^g_{n,k}(x,y) = q^g_{n,k}(x,y) - \frac{\chi_n}{\Delta_k}(\psi^g_{n,k+1/2}(x,y) - \psi^g_{n,k-1/2}(x,y)). \tag{3b}$$

Eq. (3a) is solved by the method of characteristics using the existing CRX code system with heterogeneous geometry maintained. But the source term is modified by the z-directional angular fluxes in Eq. (3b) at cell axial interfaces.

Now a 1-D axial equation is derived to update this modifying source term in Eq. (3b). To evaluate the z-directional outgoing angular fluxes, "online cell-wise homogenization" is considered (the approximation due to this cell-wise homogenization will be relaxed later). Integrating Eq. (1) over x-y domain $(x_{i-1/2}, x_{i+1/2})$ and $(y_{j-1/2}, y_{j+1/2})$ of cell (i,j), we obtain

$$\frac{m_n}{\Delta_i}(y^{g}_{n,i+1/2,j}(z)-y^{g}_{n,i-1/2,j}(z))+\frac{h_n}{\Delta_j}(y^{g}_{n,i,j+1/2}(z)-y^{g}_{n,i,j-1/2}(z))$$

$$+x_n\frac{dy^{g}_{n,i,j}(z)}{dz}+\hat{s}^{g}_{i,j,k}\,y^{g}_{n,i,j}(z)=q^{g}_{n,i,j}(z), \tag{4a}$$

where

$$y^{g}_{n,i\pm1/2,j}(z)=\frac{1}{\Delta_j}\int_{y_{j-1/2}}^{y_{j+1/2}}dy\,y^{g}_{n}(x_{i\pm1/2},y,z), \tag{4b}$$

$$y^{g}_{n,i,j\pm1/2}(z)=\frac{1}{\Delta_i}\int_{x_{i-1/2}}^{x_{i+1/2}}dx\,y^{g}_{n}(x,y_{j\pm1/2},z), \tag{4c}$$

$$y^{g}_{n,i,j}(z)=\frac{1}{\Delta_i}\int_{x_{i-1/2}}^{x_{i+1/2}}dx\frac{1}{\Delta_j}\int_{y_{j-1/2}}^{y_{j+1/2}}dy\,y^{g}_{n}(x,y,z), \tag{4d}$$

$$\hat{s}^{g}_{i,j,k}=\frac{\sum_{m\in cell(i,j)}s^{g}_{k}(x,y)\sum_{n}w_n y^{g}_{n,k}(x,y)V_m}{\sum_{m\in cell(i,j)}\sum_{n}w_n y^{g}_{n,k}(x,y)V_m}, \tag{4e}$$

$$q^{g}_{n,i,j}(z)=\frac{\sum_{m\in cell(i,j)}q^{g}_{n,k}(x,y)V_m}{\sum_{m\in cell(i,j)}V_m}. \tag{4f}$$

Eq. (4a) is rewritten as follows:

$$x_n\frac{dy^{g}_{n,i,j}(z)}{dz}+\hat{s}^{g}_{i,j,k}\,y^{g}_{n,i,j}(z)=Q^{g}_{n,i,j}(z), \tag{5a}$$

where

$$Q^{g}_{n,i,j}(z)=q^{g}_{n,i,j}(z)-\frac{m_n}{\Delta_i}(y^{g}_{n,i+1/2,j}(z)-y^{g}_{n,i-1/2,j}(z))-\frac{h_n}{\Delta_j}(y^{g}_{n,i,j+1/2}(z)-y^{g}_{n,i,j-1/2}(z)). \tag{5b}$$

Eq. (5a) can be solved conveniently by using diamond difference approximation.

$$\psi^{g}_{n,i,j,k+1/2}=\left(1+\frac{\hat{\sigma}^{g}_{i,j,k}\,\Delta_k}{2|\xi_n|}\right)^{-1}\left(\psi^{g}_{n,i,j,k-1/2}+\frac{Q^{g}_{n,i,j,k}\,\Delta_k}{2|\xi_n|}\right). \tag{6}$$

These z-directional angular surface fluxes are then used to update the modifying source term in Eq. (3b). Prior to this, the z-directional angular surface-averaged fluxes can be "modulated" to improve the cell-wise homogenization. Fig. 1 shows the calculation procedure of the 2D/1D fusion method for 3-D transport calculation.

3. ACCELERATION

We used AADR acceleration[7] and k_{eff} extrapolation for inner and outer iterations, respectively. The low-order equation of S_2-like AADR is given as follows :

$$k_n^x \frac{d}{dx} f_n^{l+1} + k_n^y \frac{d}{dy} f_n^{l+1} + k_n^z \frac{d}{dz} f_n^{l+1} + \sigma f_n^{l+1} = Q_n^{l+1} - Q_n^l, \tag{7}$$

where

$$f_n^{l+1} = \psi^{l+1} - \psi^{l+1/2} \quad \text{in n'th octant,} \tag{7a}$$

$$k_n^x = \frac{\int_{\Omega_n} W(\mu,\eta,\xi)\mu d\Omega}{\int_{\Omega_n} W(\mu,\eta,\xi) d\Omega}, \; k_n^y = \frac{\int_{\Omega_n} W(\mu,\eta,\xi)\eta d\Omega}{\int_{\Omega_n} W(\mu,\eta,\xi) d\Omega}, \; k_n^z = \frac{\int_{\Omega_n} W(\mu,\eta,\xi)\xi d\Omega}{\int_{\Omega_n} W(\mu,\eta,\xi) d\Omega}, \tag{7b}$$

$$|k_n^x| = |k_n^y| = |k_n^z| = k_n \;, \tag{7c}$$

$$\phi^{l+1} = \phi^{l+1/2} + \frac{1}{8} \sum_{n=1}^{8} f_n^{l+1}, \tag{7d}$$

and Q_n is scattering and fission sources. The low-order equation of AADR was solved by sweeping calculation only once per iteration. Acceleration efficiency of AADR and extrapolation of the eigenvalue is shown in Fig. 2.

To reduce computing time and memory, a parallel computing technique based on angle and domain decomposition is also used.

4. NUMERICAL TESTS AND RESULTS

We implemented the 2D/1D fusion method in the CRX code and applied it to the seven-group three-dimensional OECD benchmark problem (C5G7MOX) which contains a MOX fuel assembly.[5] Table I shows the preliminary results of the calculation for three axial node sizes and fission rate distribution is shown in Fig. 3. Further refinements in the number of axial nodes and angles are of course necessary for desirable accuracy. Nevertheless, the results obtained so far indicate that 3-D whole-core transport calculation is feasible.

5. CONCLUSIONS

The CRX code based on the method of characteristics has been successfully extended to three-dimensional calculation with a fusion technique combining 1-D DD methods for the axial flux calculation. This new approach reduces the huge memory and computing time problem of the direct extension of MOC to

three-dimensional calculation. For better efficiency, solving the low-order equation by Bi-CGSTAB is underway. Acceleration of outer iteration using CMR/CGR is also considered.

ACKNOWLEDGEMENT

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Table I. 3-D Results of C5G7 MOX Benchmark Problem

Number of Axial Nodes in Fuel Region	Max/Min Pin Power	Multiplication Factor
2	2.343/0.220	1.18543
Reference (Monte Carlo Calculation[5])		1.18

Convergence criterion : 1.0E-4,
Mesh division : 32 meshes/cell,
Total computational meshes : 249696,
azimuthal/polar angles : 8/2,
Number of rays for each direction : 50,
Processors used : 24.

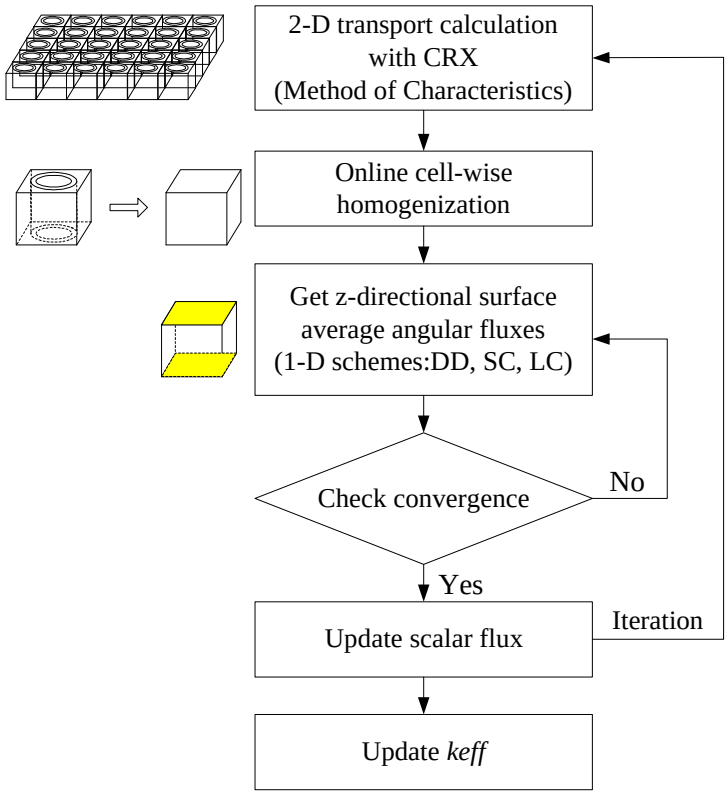

Figure 1. Calculational procedure of the 2D/1D fusion method.

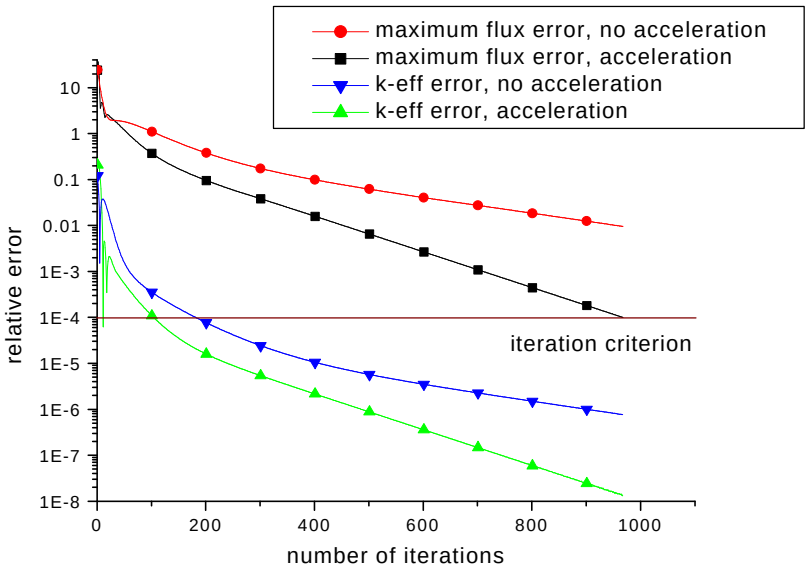

Figure 2. Relative errors of scalar flux and eigenvalue for number of iterations

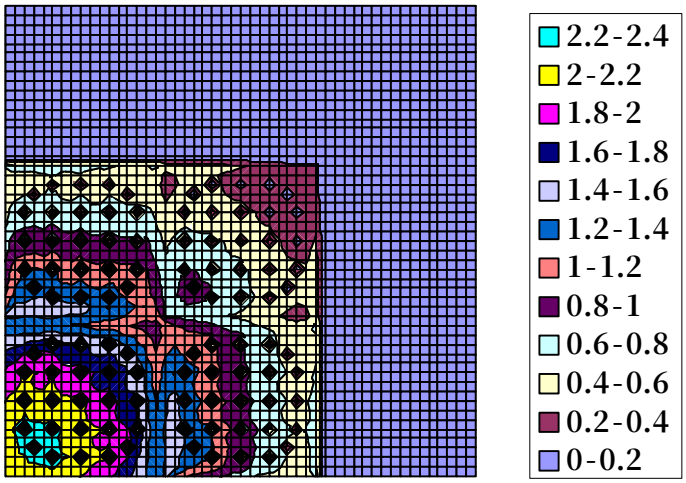

Figure 3. Normalized fission rate distribution