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Sensitivity Theory approach to Implementation of the Subgroup Method for Resonance Treatment in Heterogeneous Systems

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Abstract

As an effort to resolve the whole-core flat flux assumption introduced in the formulation of the slowing down fixed source problem appearing in the application of the subgroup method for resonance treatment in heterogeneous systems, a new definition of the equivalence cross section is introduced which establishes equivalence between a heterogeneous and a homogeneous system in terms of equal sensitivity of reaction rate on the perturbation in the resonance cross section. The derivation to obtain the heterogeneous sensitivity coefficient is carried out through the use of the sensitivity theory to yield a fixed source problem which is different only in the right hand side source term from that of the conventional formulation. This derivation guarantees positive equivalence cross sections unlike the conventional formulation. The new approach is evaluated by employing an analytic P_n solver for a simple one dimensional two-region problem consisting of fuel and moderator. The results indicate that the new approach produces conditionally better results than the conventional formulation especially when the fluxes at the high energy range are greater in the moderator regions than in the fuel regions. .

1. Introduction

In multigroup calculations, the group averaged cross sections for the resonance energy regions should be properly generated a priori considering the resonance self shielding of the numerous resonances appearing in the broad resonance group. The resonance self-shielding is primarily influenced by the composition of the material and the temperature of the medium. It is, however, also affected by the surroundings characterized by the geometrical configuration and the contents of the slowing down materials. In order to properly estimate the resonance self shielding in heterogeneous systems, the subgroup method[1] has been used in numerous lattice physics codes.

In the application of the subgroup method in a heterogeneous system, the local flux corresponding to each specified subgroup level has to be determined to generate the local condensed cross section for the resonance group.[2] Traditionally, the local flux has been determined employing the equivalence theorem which states that the local heterogeneous flux can be determined by the homogeneous flux vs. background cross section relation as long as the background cross section includes the equivalence cross section. The equivalence cross section that dictates the equivalence between a heterogeneous system and the corresponding homogeneous system is to be determined from the heterogeneous flux solution to a fixed source problem (FSP) in which the slowing down source from the upper energy groups is specified as the fixed source. The FSP describes neutron transport in a medium for neutrons belonging to certain energy ranges for which the cross section is the same as the prespecified resonance cross section level.

In the HELIOS[2] implementation of the subgroup method, it is assumed that the neutron flux in the upper energy groups is flat everywhere so that the slowing down source is represented just by the potential cross section of each region. This is an approximation introduced because the flux distribution for the upper energy groups can not be known before solving the entire problem. In reality, however, the upper energy group fluxes are not uniform so that the slowing down source obtained with flat flux would have some error. Consequently, the resulting flux distribution for the prespecified resonance level would have some error and so would the equivalence cross section. In addition to the inaccuracy of resulting equivalence cross section, there is a potential danger of negative equivalence cross section in case that the prescribed resonance level is very low whereas the potential cross section in the coolant region is very high.

This paper addresses the two problems of the conventional formulation of the FSP by introducing a new definition of equivalence cross section. Equivalence between a heterogeneous system and a homogeneous system is forced such that a unit change in local cross section in the heterogeneous system would lead the same relative change in the reaction rate as in the homogeneous system. This definition involves a sensitivity coefficient which can be determined by solving another form of FSP as derived employing the sensitivity theory (ST) in Sections 2 and 3. In order to evaluate the ST based subgroup FSP formulation, a simplified one-dimensional two region problem which is a representative of the fuel-moderator cell is examined in Section 4 employing analytic P_n transport solver. With this tool, the error of the conventional and ST approaches in the equivalence cross section are evaluated as a function of flux difference between the fuel and moderator region. As the result, the conditional superiority of the ST based approach is drawn considering the real environment.

2. Equivalence Cross Section based on Sensitivity

The benefit of the concept of equivalence cross section (XS) consists in possibility to replace a heterogeneous system by an equivalent homogeneous system in order to provide a simple and acceptably accurate way for calculation of resonance integral (RI). The crucial point of this approach resides in correct estimation of equivalence cross section Σ_e. Owing to Σ_e we can get true estimation of flux dip in the vicinity of a resonance. Hence we can calculate the within-group spectrum for XS condensation.

Equivalence in conventional sense is defined as equivalence with respect to the value of RI. The concept has no practical benefit in that there is no answer to the question about how to get the accurate value of the equivalence cross-section Σ_e and how to get it numerically in an efficient way compared to direct heterogeneous calculations. Numerous efforts were devoted to the methods for Σ_e calculation. Unfortunately, the conventional way urges us to make some approximations and assumptions during Σ_e derivation. Therefore we use alternative definition, very similar to the original one, but promising some benefits in accuracy of Σ_e due to canceling inherent approximations related to the standard approach.

Our definition is based on clear, accurate and reasonable assumptions. It gives an answer to the question how to get Σ_e accurately and efficiently. We derive influence of moderator on the spectrum in the resonant area via sensitivity theory calculations. Therefore we can cancel conventional approximations during Σ_e calculation, namely, the flat source approximation, the assumption of a regular geometry, and the assumption of separability of space-energy variables within one-material region.

Now we can give a formal definition of equivalence. Heterogeneous and homogeneous systems are equivalent if an increase in XS in the resonant material causes an equal flux dip under the same other conditions. In other terms we regard heterogeneous and homogeneous systems are equivalent if they have the equal sensitivity of reaction rate in the resonant material with respect to the perturbation XS in the resonant region. That means equal efficiency of one resonance.

Consider two one-group problems with a given source of potential scattering. The first one corresponds to the homogeneous media with the reflective boundary condition (0-dimensional problem) and the second one corresponds to the heterogeneous media with real boundary conditions. The basic equation for getting equivalence cross-section Σ_e immediately follows from the definition:

$$b^{\hom} = b^{het}, \qquad (1)$$

where β is the dimensionless sensitivity coefficient which associates the relative change in reaction rate to the relative change in XS. The superscripts “hom” and “het” label homogeneous and homogeneous cases respectively. Calculation of b^{het} is a standard problem of sensitivity theory calculations for real 2 or 3D geometries. Hence b^{het} takes into account real geometry and real material properties. The LHS coefficient $b^{\hom}$ is a simple algebraic function of Σ_e and equation (1) can be easily solved for Σ_e.

Consider 0-dimensional problem and influence on the reaction rate due to the following factors: 1) direct influence of XS perturbation and 2) flux perturbation caused by XS

perturbation. In case of no dilution,

$$\Phi(u) = \frac{C}{\Sigma(u)}, \tag{2}$$

where C is a constant. Perturbation in the reaction rate, $A = \Sigma \cdot \Phi$ caused by $d\Sigma$ is given by

$$d\,A = d\,(\Sigma\Phi) = d\Sigma \cdot \Phi + \Sigma \cdot d\Phi = d\Sigma \cdot \frac{C}{\Sigma} + \Sigma \cdot \left(-\frac{C \cdot d\Sigma}{\Sigma^2} \right) = 0\,. \tag{3}$$

This means that the direct influence of XS perturbation will be completely compensated by the flux perturbation. Hence, $\frac{dA}{d\Sigma} = 0$.

In case of diluted homogeneous media, assume $\Phi_m = \Phi(\Sigma_m)$ in the spirit of subgroup approach, where Σ_m is the prespecified removal XS in the resonant region corresponding to Subgroup Number *m*. Let Σ_b be the unknown background cross-section. By putting $\Phi\big|_{\Sigma_m = 0} = 1$, namely making the upper group unity, the reaction rate is given by

$$A_m = \Sigma_m \cdot \Phi_m = \Sigma_m \cdot \frac{\Sigma_b}{\Sigma_m + \Sigma_b} \tag{4}$$

Now define the dimensionless sensitivity coefficient β, as the coefficient which relates the relative change in the reaction rate $\frac{d\,A_m}{A_m}$ with the relative change $\frac{d\Sigma_m}{\Sigma_m}$,

$$b \equiv \frac{\Sigma_m}{A_m} \frac{dA_m}{d\Sigma_m} \tag{5}$$

Noting that $\Sigma_m = \Sigma_{am} + l\Sigma_p$ and $d\Sigma_m = d\Sigma_{am}$, where $l\Sigma_p$ is the constant potential scattering XS, Σ_{am} is the absorption cross-section, λ is the Goldstein-Cohen intermediate resonance parameter, and Σ_e is the equivalence XS, we can derive:

$$b^{\,\mathrm{hom}} = \frac{\Sigma_m}{A_m} \frac{dA_m}{d\Sigma_m} = \frac{\Sigma_m}{A_m} \frac{dA_m}{d\Sigma_{am}} = \frac{\Sigma_e}{\Sigma_m + \Sigma_e}. \tag{6}$$

Combining Eq. (1) and (6), we obtain the explicit expression for the background XS $\Sigma_e = \Sigma_m \frac{b^{\,het}}{1 - b^{\,het}}$ with the assumption that $b^{\,het}$ is known from the sensitivity theory calculation for the heterogeneous system. Note that in case of infinite dilution we have $b^{\,\mathrm{hom}} = 1$.

3. Sensitivity Theory Formulation in Heterogeneous System

Consider a one-group equation for a heterogeneous media with a given source of the

potential scattering which reads:

$$H\Phi = S \tag{7}$$

where

$$H\Phi \equiv \Omega\nabla\Phi(\vec{r},\Omega) + \Sigma\cdot\Phi(\vec{r},\Omega) \tag{8}$$

and S is the potential scattering source. Note that we make no assumption on the spatial dependence of the source. Let $c_k d\Sigma$ be the perturbation in one pin designated with index k. This consists of a delta function:

$$c_k = c_k(\vec{r}) = \begin{cases} 1, & \text{if } \vec{r} \in \text{Pin number k} \\ 0, & \text{if } \vec{r} \notin \text{Pin number k} \end{cases}. \tag{9}$$

The equation for the perturbed flux then reads:

$$(H + c_k d\Sigma)(\Phi + d\Phi) = S \tag{10}$$

Hold on the first-order terms only and subtract Eq. (7) from Eq. (4). This yields:

$$d\Phi = -H^{-1}(c_k d\Sigma\cdot\Phi) \tag{11}$$

The perturbation in the total reaction rate A over the whole media (or core) due to the perturbation $c_k d\Sigma$ is then given by

$$dA = <\Phi, c_k d\Sigma> - <\Sigma c, H^{-1}\Phi c_k d\Sigma> \tag{12}$$

where

$$c = c(\vec{r}) = \begin{cases} 1, & \text{if } \vec{r} \in \text{resonant region} \\ 0, & \text{if } \vec{r} \notin \text{resonant region} \end{cases} \tag{13}$$

and

$$<f,g> \equiv \int_{V_{core}} \int_{4p} f(\vec{r},\Omega)\cdot f(\vec{r},\Omega) d\vec{r} d\Omega . \tag{14}$$

The first term in equation (12) represents the perturbation of reaction rate due to the XS change, while the second term expresses the influence of the flux dip on the reaction rate. Note that we used the term $<\Sigma c,.>$ in the second brackets because we need to know the total response of all pins. The reaction rate of pin number k is given by

$$A_k = <c_k\Sigma,\Phi> . \tag{15}$$

From this, the heterogeneous sensitivity coefficient is derived as:

$$b^{het} \equiv \frac{\Sigma_k}{A_k}\frac{dA}{d\Sigma} = \frac{\Sigma}{d\Sigma}\frac{<\Phi, c_k d\Sigma> - <\Sigma c, H^{-1}\Phi c_k d\Sigma>}{<c_k\Sigma, \Phi>} \quad (16)$$

We can carry $d\Sigma$ and Σ outside the brackets, because they are constant within the integration area. Hence, it is simplified to

$$b^{het} = \frac{<\Phi, c_k> - <\Sigma c, H^{-1}\Phi c_k>}{<c_k, \Phi>} = 1 - \frac{<\Sigma c, H^{-1}\Phi c_k>}{<c_k, \Phi>} = 1 - \frac{<H^{-1^+}\Sigma c, \Phi c_k>}{<c_k, \Phi>} \quad (17)$$

We can assume that one of two functions $H^{-1^+}\Sigma c$ or Φ is smooth within the pin (or within subregion of pin, if c_k corresponds to subregion). Then we can carry the average value of Φ out the brackets and cancel Φ in the last ratio[1]. As far as the brackets give angular-integrated values, we will receive the identical value of the scalar product if we cancel sign of conjugate operator in the upper brackets.[3]
Finally we have

$$b^{het} = 1 - \frac{<H^{-1}\Sigma c, c_k>}{<c_k, 1>} \quad (18)$$

and

$$\Sigma_e = \Sigma_m \frac{b^{het}}{1 - b^{het}} \quad (19)$$

Note that for the evaluation of the term in the numerator of Eq. (18), we have to solve

$$H\Phi = c\Sigma \quad (20)$$

instead of Eq. (7). This is the primary difference that distinguishes our approach from the conventional one. In addition to this difference we can make the following remarks on the new approach.

Remark 1. Our approach gives an alternative way to realizing the "equivalence theorem" without invoking collision probabilities and related with collision probabilities assumption of flat flux approximation.

Remark 2. It is true $y^+ \equiv H^{-1}\Sigma c < 1$ (due to "leakage" and absorption in the moderator). Hence, $0 < b_k^{het} < 1$.

Remark3. This approach has the same numerical requirements as the conventional approach.

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[1] Or we can carry average value $\frac{<H^{-1}\Sigma c, c_k>}{<c_k, 1>}$ out the brackets. This yields the same resulting expression like in case of canceling flux.

The only difference the source term is replaced by the fuel XS for the solution of the conjugate equation.

$$b^{het} = 1 - \frac{< H^{-1}\Sigma c, c_k >}{< c_k, 1 >} = 1 - \frac{< H^{-1^{+}}\Sigma c, c_k >}{< c_k, 1 >} = 1 - \frac{< \Sigma c, H^{-1} c_k >}{< c_k, 1 >} \qquad (21)$$

Expression $H^{-1}c_k$ gives distribution from the unit source located at pin (or flat source region) k. Expression $< \Sigma c, H^{-1}c_k >$ gives the total collision rate in the fuel caused by unit source located in pin k, Expression $\frac{< \Sigma c, H^{-1}c_k >}{< c_k, 1 >}$ gives the fuel-to-fuel collision probability, and expression $1 - \frac{< \Sigma c, H^{-1}c_k >}{< c_k, 1 >}$ gives the fuel-to-coolant collision probability. Hence formula (18) is completely consistent with the collision-probability formula[4]:

$$\Sigma_e^{eff} = \frac{P_0 \Sigma_{tot}}{1 - P_0} \qquad (22)$$

Thus, our results are completely compatible with collision probability results, but our formula (17) was derived under less restrictive assumptions. This formula is more general in that no flat-flux approximation, no assumption about isotropic source and isotropic scattering are assumed.

4. Evaluation of the Sensitivity Theory Approach

Ideally, the equivalence cross section can be obtained exactly by solving the forward FSP Eq. (7) as long as the source is obtained with the space dependent fluxes of the upper energy groups. Suppose a certain definition of the degree of nonuniformity of the upper group fluxes. The conventional formulation of the FSP has no error in the equivalence XS only if the degree of nonuniformity is zero, namely if the flux distribution is really flat. The error of the conventional formulation would increase as the degree of nonuniformity increases. In case of the ST based formulation employing Eq. (20), however, the error in the equivalent XS will not be zero for the flat flux case since a different FSP equation is solved and also the different definition of the equivalent XS, Eq. (19), is used. In both formulations, once the equivalence XS is calculated for a heterogeneous configuration and given XS, this value will remain constant irrespective of the degree of nonuniformity since there is no place in any of the two formulations the actual upper group flux distribution is reflected.

Noting that the error in the equivalence XS is not zero for the zero degree of nonuniformity in the ST formulation and the equivalence XS remains constant irrespective of the degree of nonuniformity, one can imagine that there would be a certain degree of nonuniformity for which the error becomes zero for the ST formulation. Therefore, near this

degree of nonuniformity, the ST formulation would yield more accurate equivalence XS whereas the conventional formulation would be better near zero degree of nonuniformity. The superiority of one formulation to the other can not be determined unless the range of the degree of nonuniformity and the errors of both formulations are known. In this regard, a simple one-dimensional two-region problem consisting of fuel and moderator regions is examined below for which an analytic solution can be obtained using the P_n transport solution method for any degree of nonuniformity. In the following the P_n solution method realized by Mathematica programming is presented briefly and the test problem and results are given in the subsequent subsections.

4.1 One-Dimensional One-Group P_n Solver

Consider a two-region problem of which the left-hand side is for fuel and the other is for moderator. Write the 1D P_N equation using the standard notations:

$$\begin{cases} \mathbf{\acute{a}}\vec{\mathbf{J}}'_x(x) + \mathbf{A}\ddot{\mathbf{O}}(x) = \vec{\mathbf{f}}(x) \\ \mathbf{\hat{a}}\ddot{\mathbf{O}}'_x(x) + \mathbf{b}\vec{\mathbf{J}}(x) = \underbrace{\vec{\mathbf{F}}(x)}_{\vec{0}} \end{cases} \tag{23}$$

where

$$\ddot{\mathbf{O}}(x) = \begin{pmatrix} f_0(x) \\ f_1(x) \\ \cdots\cdots \\ f_{2M}(x) \end{pmatrix}; \quad \vec{\mathbf{J}}(x) = \begin{pmatrix} f_1(x) \\ f_3(x) \\ \cdots\cdots \\ f_{2M+1}(x) \end{pmatrix}; \quad \vec{\mathbf{f}}(x) = \begin{pmatrix} f_0(x) \\ f_2(x) \\ \cdots\cdots \\ f_{2M}(x) \end{pmatrix}; \quad \vec{\mathbf{F}}(x) = \begin{pmatrix} f_1(x) \\ f_3(x) \\ \cdots\cdots \\ f_{2M+1}(x) \end{pmatrix};$$

$$\mathbf{A} = \mathbf{diag}(\Sigma - S_0, \quad 5(\Sigma - S_2), \quad \ldots \quad (4M+1)(\Sigma - S_{2M}))$$

$$\mathbf{B} = \mathbf{diag}(3(\Sigma - S_1), \quad 7(\Sigma - S_3), \quad \ldots \quad (4M+3)(\Sigma - S_{2M+1}))$$

$$\mathbf{\acute{a}} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 & \ldots \\ 2 & 1+2 & 0 & 0 & \ldots \\ 0 & 2\cdot 2 & 1+2\cdot 2 & 0 & \ldots \\ 0 & 0 & 2\cdot 3 & 1+2\cdot 3 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix}}_{M+1 \text{ columns}} \Bigg\} \begin{matrix} M+1 \\ \text{rows} \end{matrix} \qquad \mathbf{\hat{a}} = \underbrace{\begin{pmatrix} 1 & 2 & 0 & 0 & \ldots \\ 0 & 1+2 & 2\cdot 2 & 0 & \ldots \\ 0 & 0 & 1+2\cdot 2 & 2\cdot 3 & \ldots \\ 0 & 0 & 0 & 1+2\cdot 3 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix}}_{M+1 \text{ columns}} \Bigg\} \begin{matrix} M+1 \\ \text{rows} \end{matrix}$$

Excluding **J** from (23) yields:

$$\begin{cases} -\vec{\Phi}''_{xx}(x) + \mathbf{B}\vec{\Phi}(x) = \vec{g}(x) \\ \mathbf{B} = \mathbf{b}^{-1}\mathbf{A}^{-1} \\ \vec{g}(x) = \mathbf{b}^{-1}\vec{f}^{-1} \end{cases}$$

Solve for $\vec{J}(h)$ the vector-matrix boundary-value problem.

$$\begin{cases} -\vec{\Phi}''_{xx}(x) + \mathbf{B}\vec{\Phi}(x) = \vec{g}(x) \\ \vec{\Phi}(0) = \vec{\Phi}_0; \;\; \vec{\Phi}(h) = \vec{\Phi}_1; \\ \text{Find } \vec{J}: \;\; \vec{J}(h) \equiv -\mathbf{b}^{-1}\vec{\Phi}'_x(h) \end{cases} \tag{24}$$

It can be solved analytically in terms of matrix functions:

$$\vec{J}(h) = \mathbf{P}_0\vec{\Phi}_0 + \mathbf{P}_1\vec{\Phi}_1 - \vec{f}_3 \tag{25}$$

where h is the size of the region and

$$\begin{cases} \mathbf{P}_0 = -\mathbf{b}^{-1}\sqrt{\mathbf{B}}\,Coth[h\sqrt{\mathbf{B}}] \\ \mathbf{P}_1 = +\mathbf{b}^{-1}\sqrt{\mathbf{B}}\,Csch[h\sqrt{\mathbf{B}}] \\ \mathbf{P}_2 = +\mathbf{b}^{-1}(\sqrt{\mathbf{B}})^{-1}\mathbf{B}\,Tanh[\tfrac{1}{2}h\sqrt{\mathbf{B}}]\,\mathbf{B}^{-1}\mathbf{b} \end{cases}$$

Write the flux symmetry conditions at the left (labeled by L) and right (labeled by R) region in the two-region problem:

$$\mathbf{P}_{L1}\vec{\Phi}_0 + \mathbf{P}_{L0}\vec{\Phi}_1 + \mathbf{P}_{L3}\vec{f}_L = \vec{0} \tag{26}$$

$$\mathbf{P}_{R0}\vec{\Phi}_1 + \mathbf{P}_{R1}\vec{\Phi}_2 + \mathbf{P}_{R3}\vec{f}_R = \vec{0} \tag{27}$$

Flux continuity condition for odd moments at interface is:

$$\mathbf{P}_{L0}\vec{\Phi}_0 + \mathbf{P}_{L1}\vec{\Phi}_1 + \mathbf{P}_{L3}\vec{f}_L = -(\mathbf{P}_{R2}\vec{\Phi}_2 + \mathbf{P}_{R1}\vec{\Phi}_1 + \mathbf{P}_{R3}\vec{f}_R) \tag{28}$$

where the fluxes $\vec{\Phi}_0$, $\vec{\Phi}_1$, $\vec{\Phi}_2$ correspond to the LHS boundary, fuel-moderator interface, RHS boundary respectively.

Solve equations (23)-(28) for $\vec{\Phi}_0$, $\vec{\Phi}_1$, $\vec{\Phi}_2$ and substitute $\vec{\Phi}_0$, $\vec{\Phi}_1$ into the vector-matrix boundary-value problem, formulated for the left (fuel) region. This yields the explicit expression for flux in the fuel region. The entire solution process can be easily implemented using the Mathematica package.

4.2 Two-Region Problem

The two-region problem is constructed with the parameters given in Table 1. In this problem, reflective boundary condition is imposed at both ends and a step change in the upper

group flux level is considered. In the fuel region, the flux level is kept to 1.0 whereas the flux level of the moderator region varies from 0.8 to 1.2 to simulate various degree of nonuniformity.

Table 1. Two-Region Problem Parameters

Parameter	Fuel Region	Moderator Region
Width, cm	1.0	1.0
$\lambda\Sigma_p$, 1/cm	0.42429	5x0.42429
Σ_a, 1/cm	1.0	0.05
Upper group flux	1.0	0.8~1.2

The exact value of the equivalence XS based on the conventional definition is obtained by solving Eq. (7) with the source defined by the product of $\lambda\Sigma_p$ and the upper group flux. The upper group flux in the moderator region was varied by an increment of 0.05. For the value of 1.0 in the moderator, the conventional equivalence XS is obtained. For the ST formulation, the source is defined only in the fuel region by Eq.. (20), and the resulting equivalent XS is calculated by Eq. (19).

Since the true value of the equivalence XS varies with the flux value at the moderator region which is a measure of degree of nonuniformity, the error of the ST and conventional equivalence XS, which is represented by only one value, changes. The errors of the two approaches are plotted in Figure 1 for the equivalence XS averaged over the peripheral 1/3 of the fuel region.

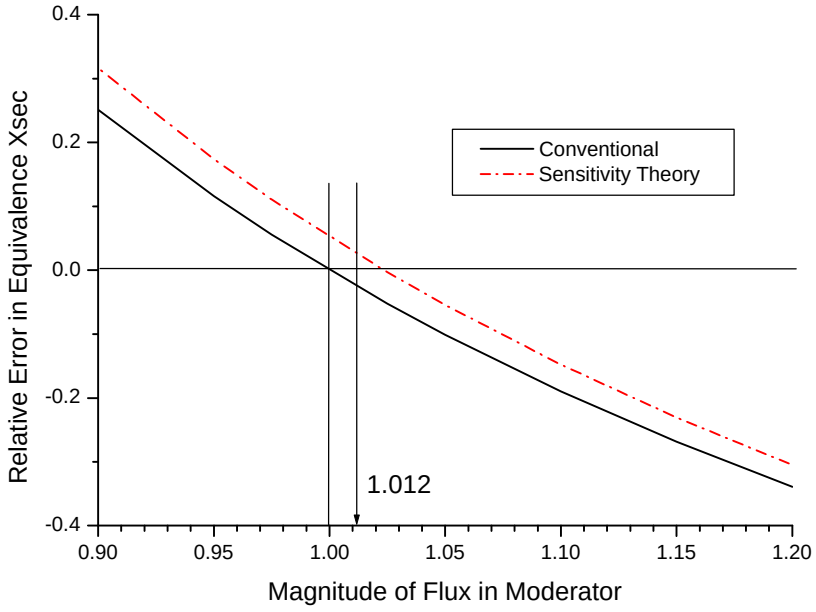

Figure 1. Comparison of Equivalence XS Error Behavior

As shown in the figure, the error of the conventional approach is zero for the uniform flux case as expected whereas the error of ST approach is about +6%. As the flux level in the moderator region increases, the absolute magnitude of the error increases for both cases and it is not negligible. For example, the error is over 10% for the conventional case if the flux in the moderator is higher than 1.05 which is considered a small degree of nonuniformity. For the ST case, the error becomes zero when the flux in the moderator is 1.024. From the error behaviors, it can be concluded that the error of the ST case is smaller than the conventional case if the flux in the moderator is higher than 1.012. But it is worse vice versa.

Now the concern is if the flux in the moderator region is really higher than in the fuel region in reality in the slowing down energy range. In a consideration of an extreme case of no slowing down in the fuel region due to heavy mass, it would be obvious that the flux in the moderator region is higher in the slowing down energy region. In general, this will be true as can be confirmed from a real multigroup calculation for a pin cell shown in Figure 2.

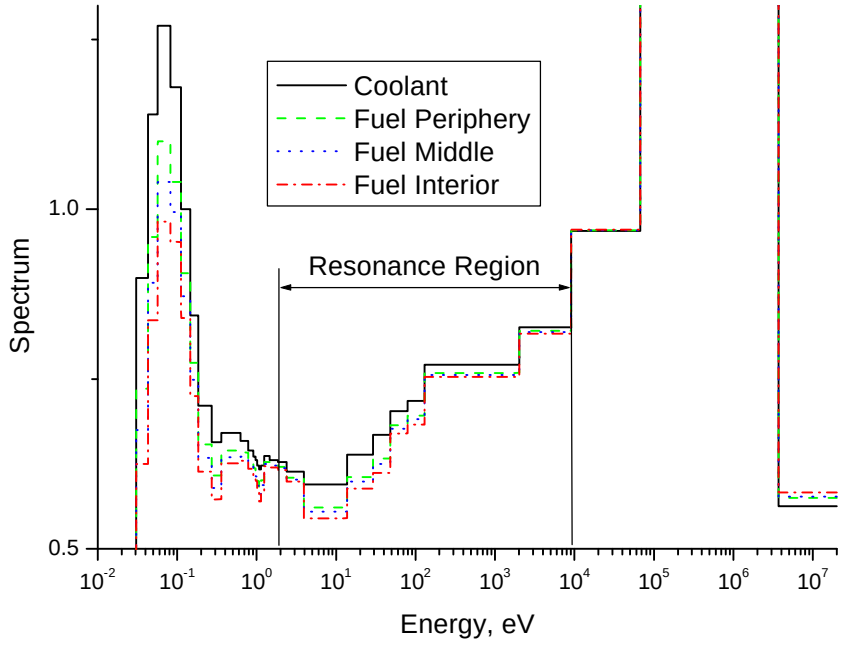

Figure 2. Typical spectrum in the moderator and fuel regions

This figure shows that the flux in the moderator region is higher in all the resonance energy regions. Therefore, it can be stated that the ST based approach would have less error than the conventional approach in real cases.

5. Conclusions

A new definition of the equivalence cross section was introduced which establishes equivalence between a heterogeneous and a homogeneous system in terms of equal sensitivity of reaction rate on the perturbation in the resonance cross section. The derivation to obtain the heterogeneous sensitivity coefficient carried out through the use of the sensitivity theory yields a fixed source problem which is different only on the right hand side source term from that of the conventional formulation. The new definition of the equivalence cross section and the associated fixed source problem provides better estimates of the equivalence cross sections than the conventional method in most real heterogeneous systems in which the fluxes at the high energy range are greater in the moderator regions than in the fuel regions. In addition, the formulation guarantees positive escape cross sections which was not always true with the conventional formulation.

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References

1. Alain Hebert. Advances in the Development of a Subgroup Method for the Self-Shieldingof Resonant Isotopes in Arbitrary Geometries. Nuclear Science and Engineering, 126, 245-263 (1997).

2 .R. J. Stamml'er et al., "HELIOS Methods," Studsvik Scanpower(1998).

3. James J. Duderstadt and William R. Martin, Transport Theory, Wiley and Sons, 1979. *P 371,372.*

4. P.F.Zweifel, Reactor Physics, McGraw-Hill, 1973. *P 245.*