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On the Evaluation of Free Vibration Characteristics of Containment Building with a Lower-order Solid Finite Element

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Abstract

The main purpose of this paper is to provide the vibration characteristic of the containment building by using a lower-order solid finite element (FE). The 8-node solid FE is formulated with B bar method and the strain-displacement matrix based on B bar method is provided in this paper. The lumped mass matrix is derived by the special lumping technique and the subspace iteration method is adopted to calculate the natural frequencies of containment building. The sensitivity of FE mesh refinement to the natural frequencies is at first investigated to determine an appropriate FE mesh for the vibration analysis of containment building. The allowable size of artificial opening at the apex of containment building is then examined to facilitate the solid FE mesh generation. Finally, the natural frequencies and the mode shapes of containment building are calculated and proposed as a benchmark.

1. Introduction

In engineering practice, it is often of crucial importance to conduct a free vibration analysis of structures and use the resulting information in the design process so that resonant behaviour of the structures can be prevented for the given loading conditions [1]. So far, many attempts have been made to predict the ultimate pressure capacity of containment building but the vibration characteristics of containment building in nuclear power plant have not been precisely investigated in three dimensional sense. In order to deliver a thorough information of vibration characteristic of containment building, we try to implement the solid FE based on B bar method as a module of the NUCAS [2]. The several numerical tests are carried out to investigate the vibration characteristic of containment building under free vibration condition. The effect of FE mesh refinement on natural frequencies is examined to find out a reasonable FE mesh for vibration analysis of containment building. In addition, the allowable size of artificial opening at the apex of the containment building is proposed. In the numerical test, the subspace iteration method is used to calculate the lowest eigenvalues of containment building with lumped mass formulated with HRZ method [3].

2. Solid Finite Element Formulation

A brief description of the solid FE formulation based on B bar method [4] is provided here. In the formulation, the isoparametric approach is used and the geometry and the displacement field of the solid element are therefore defined by using the same shape function as follow

$$\mathbf{x} = \sum_{a=1}^{8} N_a \mathbf{x}^a \quad \text{and} \quad \mathbf{u} = \sum_{a=1}^{8} N_a \mathbf{u}^a \tag{1}$$

where $\mathbf{x}$ is the position vector of a generic point in the element domain, $\mathbf{x}^a$ denotes the position vector of node a, $\mathbf{u}$ is the displacement vector, $\mathbf{u}^a$ is nodal displacement vector at node a and the shape function has the following form

$$N_a = \frac{1}{8}(1+\xi_1\xi_1^a)(1+\xi_2\xi_2^a)(1+\xi_3\xi_3^a), \qquad (a = 1, 8) \tag{2}$$

The six strain terms in solid element are defined by global displacement components (u_i) as follows

$$\{\ \boldsymbol{\varepsilon}\ \} = \left\{ \begin{array}{c} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{31} \end{array} \right\} = \left\{ \begin{array}{c} \partial u_1/\partial x_1 \\ \partial u_2/\partial x_2 \\ \partial u_3/\partial x_3 \\ \partial u_2/\partial x_1 + \partial u_1/\partial x_2 \\ \partial u_3/\partial x_2 + \partial u_2/\partial x_3 \\ \partial u_1/\partial x_3 + \partial u_3/\partial x_1 \end{array} \right\} \tag{3}$$

where, u_1, u_2 and u_3 are the displacement components associated with the unit vector directions of the global coordinate system x_i.

In discritized FE domain, strain terms can be calculated by substituting the terms of (1) and (2) into (3). Hence, the strain terms can be written in terms of the nodal displacement vector

$$\boldsymbol{\varepsilon} = \sum_{a=1}^{8} \mathbf{B}^a \mathbf{u}^a \tag{4}$$

where the component of the matrix $\mathbf{B}_a$ can be written in terms of the gradient of shape function $N_{a,1}(= \partial N_a / \partial x_i)$ as follows

$$\mathbf{B}^a = \begin{bmatrix} N_{a,1} & 0 & 0 & N_{a,2} & 0 & N_{a,3} \\ 0 & N_{a,2} & 0 & N_{a,1} & N_{a,3} & 0 \\ 0 & 0 & N_{a,3} & 0 & N_{a,2} & N_{a,1} \end{bmatrix}^{\mathrm{T}}. \tag{5}$$

In this study, we adopted B bar method which can alleviate element deficiencies in incompressible material and the strain-displacement matrix is therefore separated into dilatational and deviatoric parts such as

$$\bar{\mathbf{B}}^a = \mathbf{B}^a_{dev} + \bar{\mathbf{B}}^a_{dil} \tag{6}$$

where the dilatational part $\bar{\mathbf{B}}^{dil}_a$ is defined by using the mean dilatation method in the following form

$$\bar{\mathbf{B}}^a_{dil} = \frac{1}{3} \begin{bmatrix} \bar{B}^a_1 & \bar{B}^a_1 & \bar{B}^a_1 & 0 & 0 & 0 \\ \bar{B}^a_2 & \bar{B}^a_2 & \bar{B}^a_2 & 0 & 0 & 0 \\ \bar{B}^a_3 & \bar{B}^a_3 & \bar{B}^a_3 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} \tag{7}$$

in which $\bar{B}^a_i$ is calculated by

$$\bar{B}^a_i = \frac{1}{V_e} \int_{V_e} N_{a,i} \, dV. \tag{8}$$

Therefore, we can define the following strain-displacement matrix as a substitute strain-displacement matrix for solid FE

$$\bar{\mathbf{B}}^a = \frac{1}{3} \begin{bmatrix} 2B^a_1 + \bar{B}^a_1 & \bar{B}^a_1 - B^a_1 & \bar{B}^a_1 - B^a_1 & 3B^a_2 & 0 & 3B^a_3 \\ \bar{B}^a_2 - B^a_2 & 2B^a_2 + B^a_2 & \bar{B}^a_2 - B^a_2 & 3B^a_1 & 3B^a_3 & 0 \\ \bar{B}^a_3 - B^a_3 & \bar{B}^a_3 - B^a_3 & 2B^a_3 + \bar{B}^a_3 & 0 & 3B^a_2 & 3B^a_1 \end{bmatrix}^{\mathrm{T}} \tag{9}$$

The standard strain-displacement matrix is replaced with the strain-displacement matrix of (9) in the formation of the stiffness matrix. Note that the rigidity matrix for isotropic material [1] is used in this study.

3. Equilibrium Equation

In the absence of external loads and damping effects, the dynamic equilibrium equation based on the principle of virtual work (or more precisely virtual power) can be written as

$$\int_{\Omega} \delta\boldsymbol{\varepsilon}^{\mathrm{T}}\, \mathbf{D}\, \boldsymbol{\varepsilon}\, d\Omega = \int_{\Omega} [\delta\mathbf{u}]^{\mathrm{T}}\, \rho\, \ddot{\mathbf{u}}\, d\Omega \tag{10}$$

For a discretized FE domain, the displacement field can be expressed in terms of the nodal displacements $\mathbf{u}^a$ and the global shape functions $\widehat{\mathbf{N}}_a$ [4] which are constructed from local shape functions N_a and the acceleration can be also written in the same way

$$\mathbf{u} = \sum_{a=1}^{np} \widehat{\mathbf{N}}_a(\xi_1, \xi_2, \xi_3)\mathbf{u}^a, \quad \ddot{\mathbf{u}} = \sum_{a=1}^{np} \widehat{\mathbf{N}}_a(\xi_1, \xi_2, \xi_3)\ddot{\mathbf{u}}^a \tag{11}$$

where np is the total number of the node in the discretized domain and the virtual terms associated with the displacement and acceleration are

$$\delta\mathbf{u} = \sum_{a=1}^{np} \widehat{\mathbf{N}}_a(\xi_1, \xi_2, \xi_3)\delta\mathbf{u}^a, \quad \delta\ddot{\mathbf{u}} = \sum_{a=1}^{np} \widehat{\mathbf{N}}_a(\xi_1, \xi_2, \xi_3)\delta\ddot{\mathbf{u}}^a \tag{12}$$

The strain-displacement and virtual strain-displacement relationships can then be written as

$$\boldsymbol{\varepsilon} = \sum_{a=1}^{np} \widehat{\mathbf{B}}^a\mathbf{u}^a \qquad \delta\boldsymbol{\varepsilon} = \sum_{a=1}^{np} \widehat{\mathbf{B}}^a\ \delta\mathbf{u}^a \tag{13}$$

where $\widehat{\mathbf{B}}^a$ is the global strain-displacement matrix which are constructed from local strain-displacement matrix $\mathbf{B}^a$. Substituting equations (11), (12) and (13) into (10) yields

$$\delta\mathbf{u}^T[\mathbf{K}\mathbf{u} - \mathbf{M}\ddot{\mathbf{u}}] = 0 \tag{14}$$

Since the virtual displacements $\delta\mathbf{u}$ are arbitrary, the above equation may be written as

$$\mathbf{K}\mathbf{u} - \mathbf{M}\ddot{\mathbf{u}} = 0 \tag{15}$$

A general solution of (15) may be written as

$$\mathbf{u} = \phi_k e^{i\omega_k t} \tag{16}$$

Substituting (16) into (15) yields

$$[\mathbf{K} - \omega_k^2 \mathbf{M}]\phi_k = 0 \tag{17}$$

where ϕ_k is a set of displacement-type amplitude at the nodes otherwise known as the modal vector and ω_k is the *natural frequency* associated with the k^{th} mode and $\mathbf{K} = \mathbf{K}^{(e)}$ and $\mathbf{M} = \mathbf{M}^{(e)}$ are global stiffness and mass matrices which contain contributions from the element stiffness and mass matrices, can be written as

$$\mathbf{K}^{ab(e)} = \int_{\Omega} [\bar{\mathbf{B}}^a]^{\mathrm{T}} \mathbf{D}\, \bar{\mathbf{B}}^b \, d\Omega\,, \quad \mathbf{M}^{ab(e)} = \int_{\Omega} \rho\, \mathbf{N}_a^{\mathrm{T}} \mathbf{N}_b d\Omega \tag{18}$$

where the constitutive matrix $\mathbf{D}$ for the isotropic material is described in Reference [1] and the matrices $\bar{\mathbf{B}}^a$ is the strain-displacement matrices obtained from (9). The ρ denotes the material density.

4. Numerical Test

A full scale containment building in the nuclear power plant is considered in the numerical test. The geometry and perspective view of the containment building are illustrated in Figure 1. The basemat of containment building is not considered in FE mesh modeling and is assumed to be clamped at the junction between the cylinder wall and the basemat. The material properties used in vibration analysis are the elastic modulus $E = 316088 kg/cm^2$ and Poisson's ratio $\nu = 0.17$. The material density of concrete is assumed to be $\rho = 0.0024 kg/cm^2$. The containment building is analyzed with all of eight FE meshes, which have 12, 16, 20, 24, 28, 32, 36 and 40 nodes in circumferential direction, to investigate the effect of the FE mesh refinement. From numerical result, the number of nodes in circumferential direction is a quite sensitive to higher frequencies of the containment building. However, the fundamental frequency is not much effected since a coarse FE mesh can produce a reasonable value of fundamental frequency as shown in Figure 2.

The artificial opening at the apex of containment building is introduced to facilitate a FE mesh generation process and also produce a regular mesh distribution in the containment building. The validity of artificial opening is investigated with various size of opening such as $\theta/90^\circ = 0.01, 0.02, 0.03, 0.04, 0.05, 0.1, 0.2$ and 0.3. The size of the opening is illustrated in Figure 3. From numerical test, the natural frequencies of containment building changes greatly in higher mode when the size of the opening is greater than $\theta/90^\circ = 0.1$. However, a stability had been preserved with the value of $\theta/90^\circ = 0.1$ in the previous study [5] where an assumed strain shell element [6] is used in the analysis. This discrepancy between different types of FE may be from the reason that the solid element shows inherently a little bit stiffer behaviour than the shell element and also the assumed strain method is recognized as more enhanced approach than B bar method to remove the deficiencies in FEs. The variance of the natural frequency of containment according to the size of the opening at apex is illustrated in Figure 4 and the numerical values are summarized in Table 1 and the mode shapes of the containment building are also illustrated in Figure 5 for the future reference solution.

Table 1: The natural frequencies of containment building (Hz.)

	The size of artificial opening at the apex ($\theta/90^\circ$)							
mode	0.01	0.02	0.03	0.04	0.05	0.1	0.2	0.3
1	4.7510	4.7515	4.7525	4.7541	4.7562	4.7749	4.8533	4.9883
2	8.4657	8.4645	8.4630	8.4612	8.4590	8.4365	8.1237	6.4742
3	9.7572	9.7560	9.7548	9.7536	9.7524	9.7465	9.7353	9.5736
4	9.9899	9.9894	9.9888	9.9883	9.9878	9.9860	9.9904	9.7227
-*	-	-	-	-	-	-	-	9.7227
5	13.206	13.207	13.209	13.213	13.219	13.266	11.401	10.024
-*	-	-	-	-	-	-	11.401	-
6	13.888	13.890	13.892	13.895	13.899	13.934	13.464	13.800
7	15.371	15.370	15.369	15.369	15.368	15.364	14.073	14.306
8	16.695	16.694	16.692	16.690	16.689	16.681	15.357	15.351
9	17.367	17.363	17.355	17.343	17.323	16.897	16.656	16.379
10	19.504	19.503	19.501	19.500	19.498	19.491	18.121	18.221
11	21.523	21.521	21.518	21.517	21.515	20.071	19.477	18.302
12	22.836	22.831	22.831	22.836	22.772	21.513	21.547	19.463
13	23.000	23.001	23.002	22.966	22.844	22.931	23.200	21.655
14	23.202	23.162	23.089	23.005	23.009	23.048	23.268	23.425
15	23.788	23.788	23.788	23.788	23.787	23.786	23.283	23.732

* : no multiple frequency

Many multiple frequencies are detected because of the symmetricity of containment building but a few single frequency associated with torsional mode also exist.

5. Concluding Remarks

The vibration characteristic of containment building is exploited by using a lower-order solid FE formulated with B bar method. The solid FE based on B bar method produces valid frequencies only with the value less than $\theta/90^\circ = 0.1$ of artificial opening at the apex of containment building. An appropriate solid FE mesh model for the vibration analysis of containment building is identified as the model having 32 nodes in circumferential direction. The present FE model in general produces multiple natural frequencies except a few single frequency. Finally, the present numerical results is proposed as a future reference solution for free vibration analysis.

Acknowledgement

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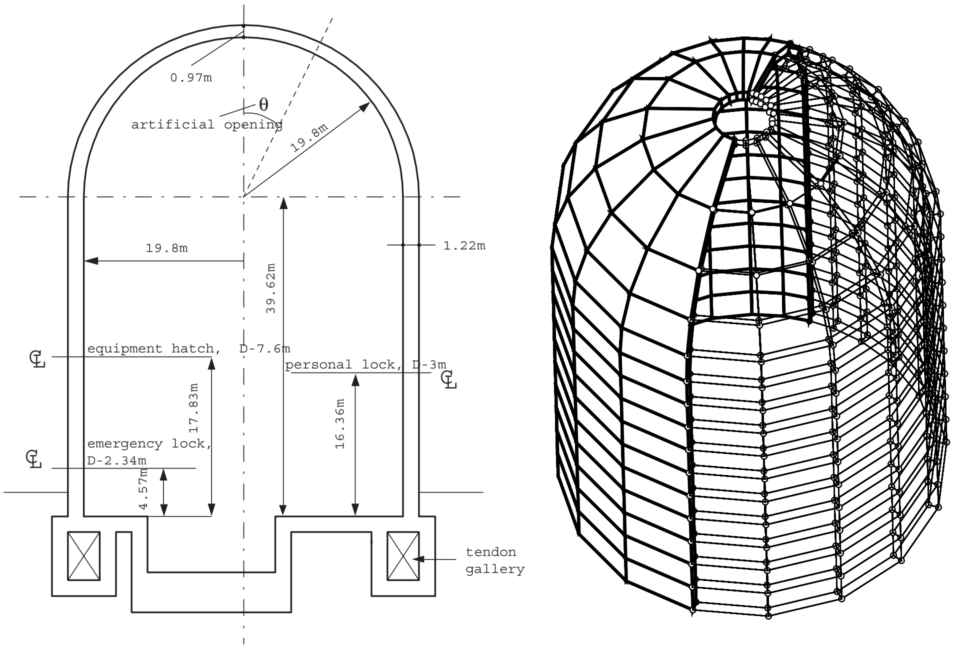

Figure 1: Containment building: (left) geometry and (right) perspective view

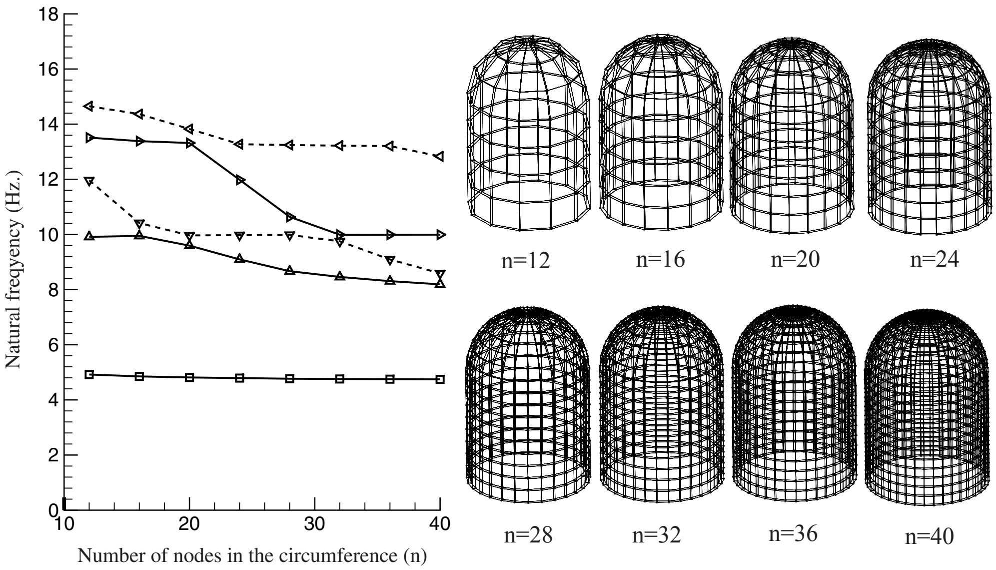

Figure 2: The frequencies with different FE meshes

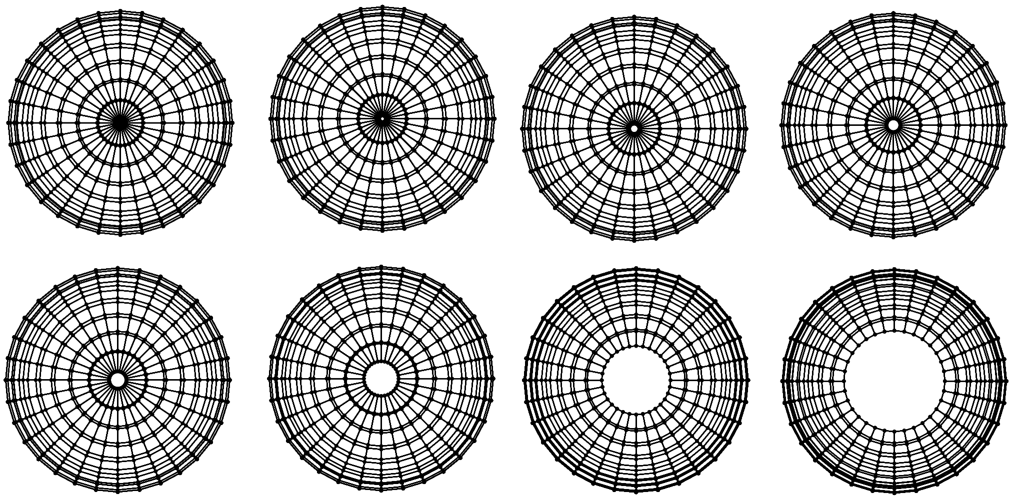

Figure 3: Different sizes of artificial opening

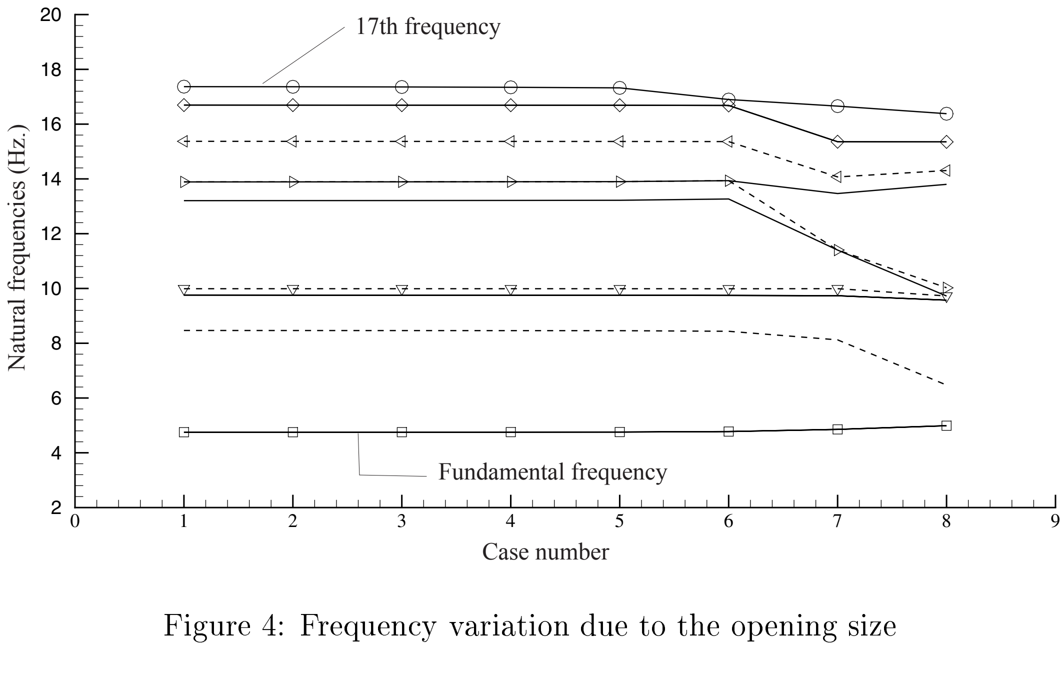

Figure 4: Frequency variation due to the opening size

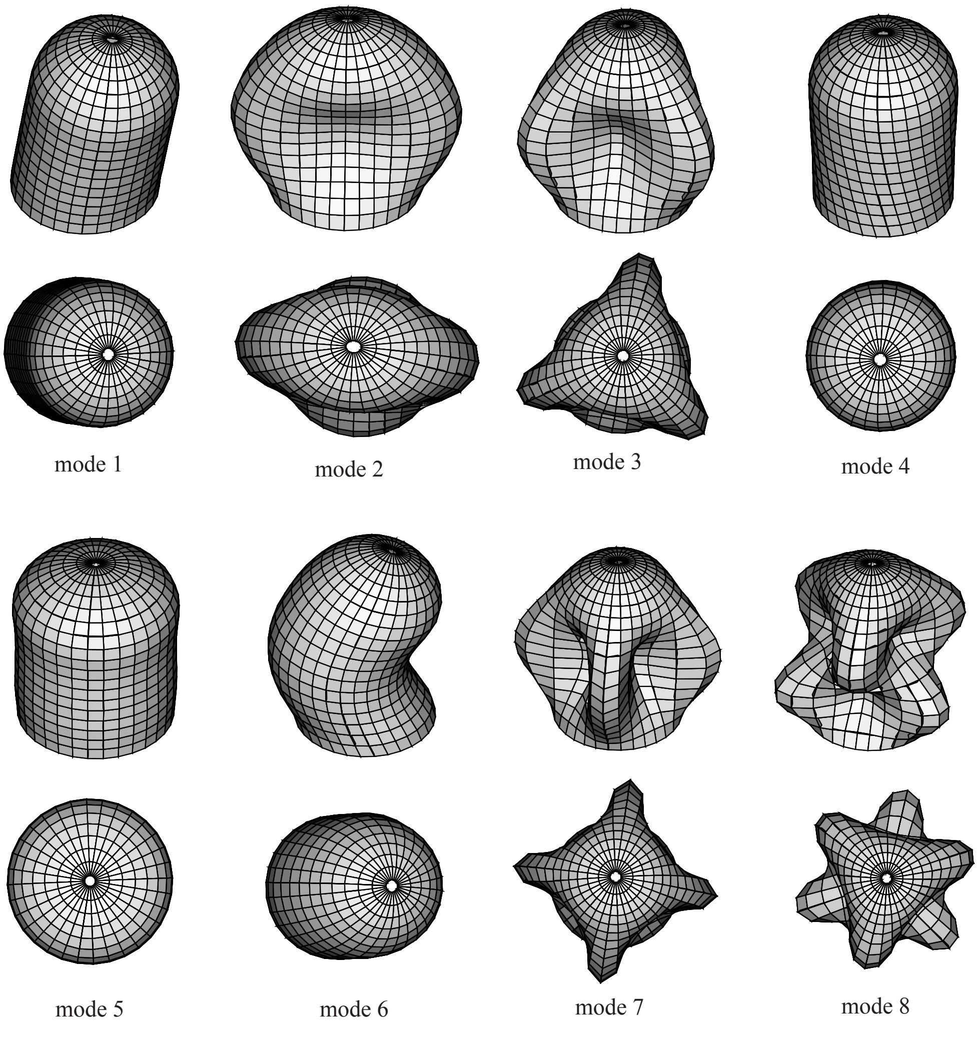

Figure 5: The mode shapes of containment building