Proceedings of the Korean Nuclear Society Autumn Meeting
Yongpyong, Korea, October 2002

Angular Dependent Coarse-Mesh Rebalance Method for Acceleration of the Discrete Ordinates Neutron Transport Equation

Young Ryong Park and Nam Zin Cho

Korea Advanced Institute of Science and Technology
373-1 Kusong-dong, Yusong-gu,
Taejon, Korea 305-701
E-mail : nzcho@mail.kaist.ac.kr

Abstract

A new coarse-mesh rebalance method is developed and tested to accelerate one-dimensional discrete ordinates neutron transport equation. The method is based on the use of angular dependent rebalance factors. Unlike the original Coarse-Mesh Rebalance method, Fourier analysis and numerical results show that this Angular Dependent Coarse-Mesh Rebalance(ADCMR) method is unconditionally stable for any optical thickness and that the acceleration effect is significant.

1. Introduction

Since Kopp introduced the synthetic concept to accelerate source iteration of the transport calculation in early sixties[1], various acceleration methods were proposed such as conventional rebalance methods, Diffusion Synthetic Acceleration (DSA)[2,3,4], Transport Synthetic Acceleration (TSA)[5], Projected Discrete Ordinates (PDO)[6,7], and so on. Coarse-Mesh Rebalance (CMR) method[8,9] that was once the most popular acceleration methods has existed in the literature since the mid-1960s and was implemented in many commercial neutron transport codes. CMR method is versatile, which can be applied to a wide range of problems in various geometries with any S_N differencing scheme. However, people knew that CMR is unstable with scattering ratio c close to unity or with optically thick cells by experience and Cefus and Lasen showed this by analytically using Fourier analysis.[9] Due to this shortage and the development of unconditionally stable DSA method in late 1970s and early 1980s, CMR was replaced by DSA in many problems and codes. Although DSA shows better behavior in spectral radius, DSA still has a problem. For unconditional stability DSA needs consistency in spatial discretization of high and low order equations and DSA can be applied only to square meshes. These block up to extend DSA to two- or three-dimensional problem. Due to expansion of computer capacity, people tries to solve three-dimensional whole-core heterogeneous problems for accuracy. Even though computer technology is advancing, it needs many iterations and long computing time. Therefore we focus on CMR again. Because CMR is based on neutron balance over the coarse-mesh only, it will be possible to apply to other than S_N methods such as MOC and treat non-square meshes. These aspects are very attractive but CMR does not shows unconditional stability. For these reason, we will discuss an unconditionally stable coarse-mesh rebalance method by introducing angular dependent rebalance factors in coarse-meshes. The angular dependent rebalance factors are already introduced in fine-mesh cases by Hong and Cho[10,11] and Park and Cho[12,13] and it shows good results. So as a starting point we will describe Angular Dependent Coarse-Mesh Rebalance method, shortly ADCMR, for one-dimensional S_N equation with diamond-difference scheme and demonstrate unconditional stability by Fourier analysis.

2. Formulation

Let us consider slab geometry like Fig. 1. The whole problem consists of N coarse-mesh cells, each coarse-mesh containing p fine-meshes. The l-th source iteration for the S_N transport equation on a nonuniform mesh is described by

$$\frac{m_m}{h_i}(y_{m,i+1/2}^{l+1/2} - y_{m,i-1/2}^{l+1/2}) + s_{t,i} y_{m,i}^{l+1/2} = \frac{1}{2}(s_{s,i} f_i^l + Q_i), \qquad (1)$$

and

$$f_i^{l+1/2} = \sum_m w_m y_{m,i}^{l+1/2}, \qquad (2)$$

where

$$\sum_m w_m = 2. \qquad (3)$$

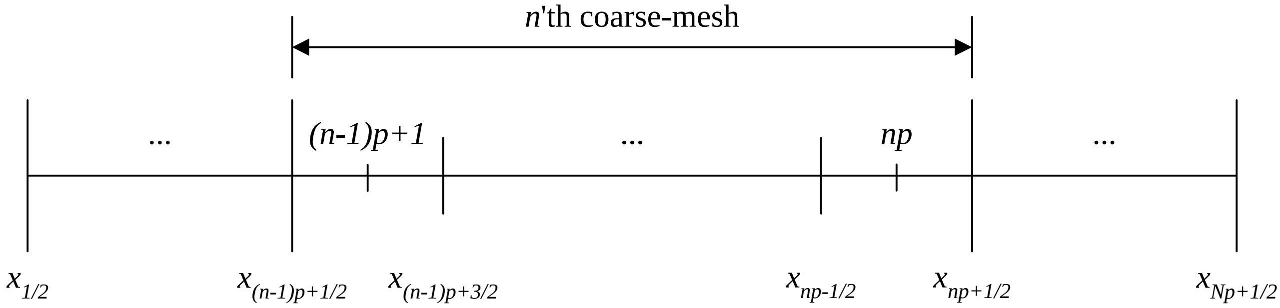

Fig. 1. Coarse-mesh divisions of slab geometry

The nonlinear rebalance factors *f* 's, which are defined on coarse-mesh boundaries only, are simply expressed as follows:

$$f^{+}_{n+1/2} = \frac{y^{\,l+1}_{m,np+1/2}}{y^{\,l+1/2}_{m,np+1/2}},\ m_m > 0, \tag{4a}$$

$$f^{-}_{n+1/2} = \frac{y^{\,l+1}_{m,np+1/2}}{y^{\,l+1/2}_{m,np+1/2}},\ m_m < 0. \tag{4b}$$

By an arbitrary spatial discretization scheme, the outgoing and mesh-centered angular fluxes can be expressed as follows using incoming angular flux in *i*-th fine-mesh:

$$\begin{aligned} y^{\,l+1/2}_{m,i+1/2} &= A^{+}_{m,i} y^{\,l+1/2}_{m,i-1/2} + B^{+}_{m,i}(s_{s,i} f^{\,l}_{i} + Q_i), \\ y^{\,l+1/2}_{m,i-1/2} &= A^{-}_{m,i} y^{\,l+1/2}_{m,i+1/2} + B^{-}_{m,i}(s_{s,i} f^{\,l}_{i} + Q_i), \end{aligned} \tag{5a}$$

$$\begin{aligned} y^{\,l+1/2}_{m,i} &= C^{+}_{m,i} y^{\,l+1/2}_{m,i-1/2} + D^{+}_{m,i}(s_{s,i} f^{\,l}_{i} + Q_i), \\ y^{\,l+1/2}_{m,i} &= C^{-}_{m,i} y^{\,l+1/2}_{m,i+1/2} + D^{-}_{m,i}(s_{s,i} f^{\,l}_{i} + Q_i). \end{aligned} \tag{5b}$$

After eliminating all interior angular fluxes successively, outgoing angular fluxes of the *n*-th coarse-mesh and all mesh-centered angular fluxes are expressed as

$$\begin{aligned} y^{\,l+1/2}_{m,np+1/2} &= \prod_{i=(n-1)p+1}^{np} A^{+}_{m,i} y^{\,l+1/2}_{m,(n-1)p+1/2} + \sum_{i=(n-1)p+1}^{np} \prod_{k=i+1}^{np} A^{+}_{m,k} B^{+}_{m,i}(s_{s,i} f^{\,l}_{i} + Q_i), \\ y^{\,l+1/2}_{m,(n-1)p+1/2} &= \prod_{i=(n-1)p+1}^{np} A^{-}_{m,i} y^{\,l+1/2}_{m,np+1/2} + \sum_{i=(n-1)p+1}^{np} \prod_{k=(n-1)p+1}^{i-1} A^{-}_{m,k} B^{-}_{m,i}(s_{s,i} f^{\,l}_{i} + Q_i), \end{aligned} \tag{6a}$$

$$\begin{aligned} y^{\,l+1/2}_{m,i} &= C^{+}_{m,i} \prod_{k=(n-1)p+1}^{i-1} A^{+}_{m,k} y^{\,l+1/2}_{m,(n-1)p+1/2} \\ &\quad + C^{+}_{m,i} \sum_{k=(n-1)p+1}^{i-1} \prod_{u=k+1}^{i-1} A^{+}_{m,u} B^{+}_{m,k}(s_{s,k} f^{\,l}_{k} + Q_k) + D^{+}_{m,i}(s_{s,i} f^{\,l}_{i} + Q_i), \\ y^{\,l+1/2}_{m,i} &= C^{-}_{m,i} \prod_{k=i+1}^{np} A^{-}_{m,k} y^{\,l+1/2}_{m,np+1/2} \\ &\quad + C^{-}_{m,i} \sum_{k=i+1}^{np} \prod_{u=i+1}^{k-1} A^{-}_{m,u} B^{-}_{m,k}(s_{s,k} f^{\,l}_{k} + Q_k) + D^{-}_{m,i}(s_{s,i} f^{\,l}_{i} + Q_i), \end{aligned} \tag{6b}$$

for $i = (n-1)p+1, (n-1)p+2, \Lambda\ , np.$

Now we change all the iteration indices to *l+1* and introduce rebalance factors given in Eqs. (4a) and (4b). Then multiplying Eq. (6a) by m_m and summing over half angle, we find the rebalance factors as

$$\left(\sum_{\mu_m>0}\mu_m\omega_m\psi^{l+1/2}_{m,np+1/2}\right)\phi^{+}_{n+1/2} = \left(\sum_{\mu_m>0}\mu_m\omega_m\prod_{i=(n-1)p+1}^{np}A^{+}_{m,i}\psi^{l+1/2}_{m,(n-1)p+1/2}\right)\phi^{+}_{n-1/2}$$
$$+\sum_{i=(n-1)p+1}^{np}\left(\sum_{\mu_m>0}\mu_m\omega_m\prod_{k=i+1}^{np}A^{+}_{m,k}B^{+}_{m,i}\right)(\sigma_{s,i}\phi_i^{l+1}+Q_i), \tag{7a}$$

$$\left(\sum_{\mu_m<0}\mu_m\omega_m\psi^{l+1/2}_{m,(n-1)p+1/2}\right)\phi^{-}_{n-1/2} = \left(\sum_{\mu_m<0}\mu_m\omega_m\prod_{i=(n-1)p+1}^{np}A^{-}_{m,i}\psi^{l+1/2}_{m,np+1/2}\right)\phi^{-}_{n+1/2}$$
$$+\sum_{i=(n-1)p+1}^{np}\left(\sum_{\mu_m<0}\mu_m\omega_m\prod_{k=(n-1)p+1}^{i-1}A^{-}_{m,k}B^{-}_{m,i}\right)(\sigma_{s,i}\phi_i^{l+1}+Q_i), \tag{7b}$$

and by summing over half angle of Eq. (6b), we find the mesh-centered scalar fluxes as follows:

$$\phi_i^{l+1}=\phi_i^{l+1,+}+\phi_i^{l+1,-},$$
$$\phi_i^{l+1,+}=\left(\sum_{\mu_m>0}\omega_m C^{+}_{m,i}\prod_{k=(n-1)p+1}^{i-1}A^{+}_{m,k}\psi^{l+1/2}_{m,(n-1)p+1/2}\right)\phi^{+}_{n-1/2} \tag{8a}$$
$$+\sum_{k=(n-1)p+1}^{i-1}\sum_{\mu_m>0}\omega_m C^{+}_{m,i}\prod_{u=k+1}^{i-1}A^{+}_{m,u}B^{+}_{m,k}(\sigma_{s,k}\phi_k^{l+1}+Q_k)+\sum_{\mu_m>0}\omega_m D^{+}_{m,i}(\sigma_{s,i}\phi_i^{l+1}+Q_i),$$
$$\phi_i^{l+1,-}=\left(\sum_{\mu_m<0}\omega_m C^{-}_{m,i}\prod_{k=i+1}^{np}A^{-}_{m,k}\psi^{l+1/2}_{m,np+1/2}\right)\phi^{-}_{n+1/2}$$
$$+\sum_{k=i+1}^{np}\sum_{\mu_m<0}\omega_m C^{-}_{m,i}\prod_{u=i+1}^{k-1}A^{-}_{m,u}B^{-}_{m,k}(\sigma_{s,k}\phi_k^{l+1}+Q_k)+\sum_{\mu_m<0}\omega_m D^{-}_{m,i}(\sigma_{s,i}\phi_i^{l+1}+Q_i), \tag{8b}$$

for $i=(n-1)p+1,(n-1)p+2,\ldots,np.$

If we use the Diamond-Difference (DD) spatial discretization scheme, constants A, B, C, and D are given by

$$A^{+}_{m,i}=\frac{2\mu_m-\sigma_{t,i}h_i}{2\mu_m+\sigma_{t,i}h_i},\qquad A^{-}_{m,i}=\frac{2\mu_m+\sigma_{t,i}h_i}{2\mu_m-\sigma_{t,i}h_i}, \tag{9a}$$

$$B^{+}_{m,i}=\frac{h_i}{2\mu_m+\sigma_{t,i}h_i},\qquad B^{-}_{m,i}=\frac{-h_i}{2\mu_m-\sigma_{t,i}h_i}, \tag{9b}$$

$$C^{+}_{m,i}=\frac{2\mu_m}{2\mu_m+\sigma_{t,i}h_i},\qquad C^{-}_{m,i}=\frac{2\mu_m}{2\mu_m-\sigma_{t,i}h_i}, \tag{9c}$$

$$D^{+}_{m,i}=\frac{h_i/2}{2\mu_m+\sigma_{t,i}h_i},\quad \text{and}\quad D^{-}_{m,i}=\frac{-h_i/2}{2\mu_m-\sigma_{t,i}h_i}. \tag{9d}$$

The overall procedure of our ADCMR method can be described as follows. First, the high-order transport equations are solved by transport sweep and the coefficients in Eqs. (7) and (8) are calculated. Second, the low-order equations (7a, 7b, 8a, 8b) are solved by iteration. Third, the final converged solutions of the low-order equations are used in the high-order equations. The procedure is repeated until the scalar flux converges in each mesh. This procedure is shown in Fig. 2.

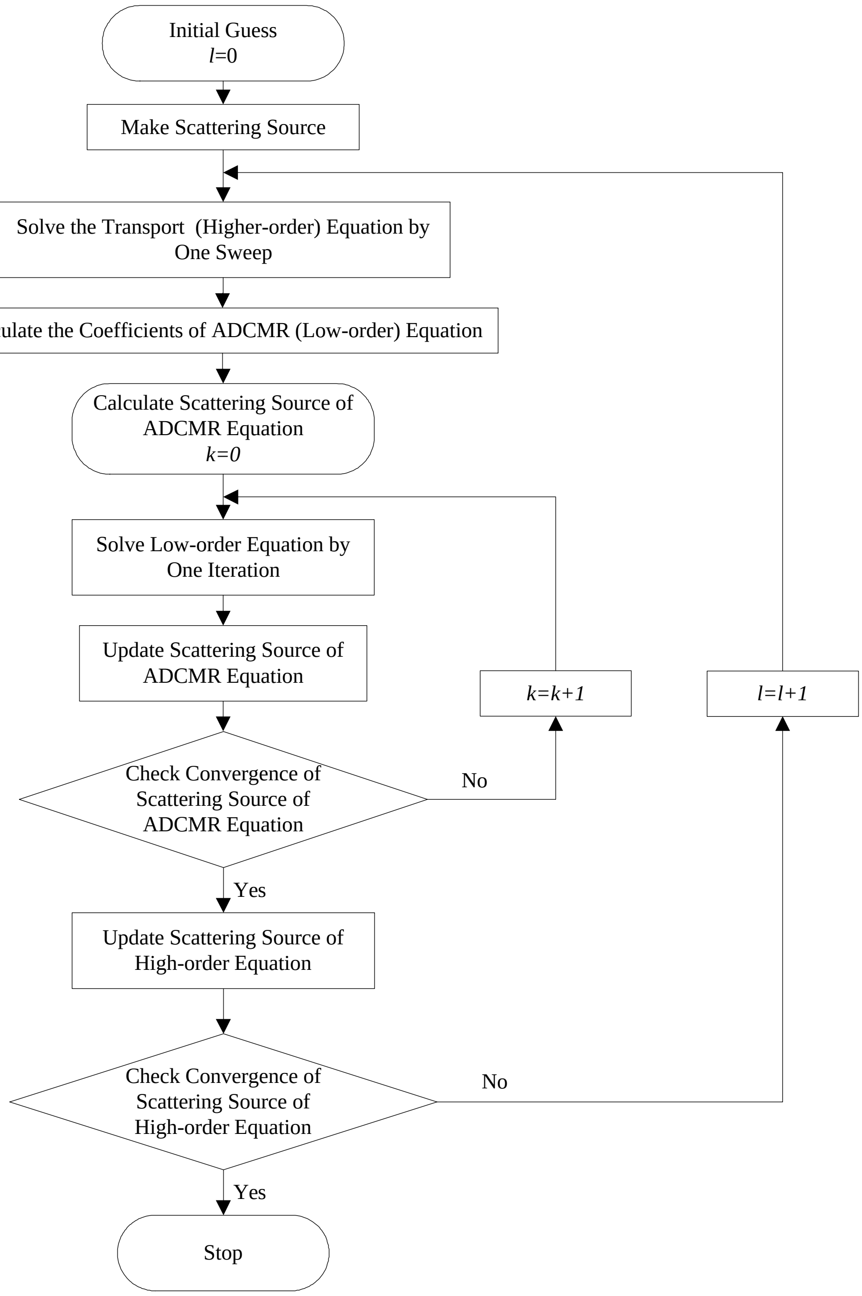

Fig. 2. The overall procedure of the ADCMR method

3. Fourier Analysis

Thus far the acceleration equations of the ADCMR method, which is a nonlinear scheme, have been derived in slab geometry. However, it is yet to be proved theoretically that the acceleration equations derived actually accelerate the transport equation. The most popular technique that analyzes iterative schemes is Fourier stability analysis that can apply only to linear methods. But Cefus and Larsen successfully applied this technique to the analysis of CMR and PDO iterative schemes through linearization. In this section we analyze theoretically the ADCMR method by the Cefus and Larsen's approach. To evade the complexity of ADCMR, a special class of infinite homogeneous medium problems with a flat source is considered. Therefore, the medium has the simple solution given by $f = Q / s_a$. Then, the ADCMR equations are linearized around this solution.

Let

$$
\begin{aligned}
y_{m,i+1/2} &= \frac{Q}{s_a}\left(\frac{1}{2} + e x_{m,i+1/2}\right), \\
f_i^{\pm} &= \frac{Q}{s_a}\left(\frac{1}{2} + e V_i^{\pm}\right), \\
f^{\pm} &= 1 + e F^{\pm}.
\end{aligned} \tag{10}
$$

Inserting Eq. (10) into the balance equation of the coarse-mesh (6a) and low-order equations (7a, 7b, 8a, and 8b) and dropping $O(e^2)$ terms, we find the system of linear equations as follows:

$$
\begin{aligned}
x_{m,p+1/2}^{l+1/2} &= (A_m^{+})^{p} x_{m,1/2}^{l+1/2} + s_s \sum_{i=1}^{p} (A_m^{+})^{p-i} B_m^{+} (V_i^{l+1,+} + V_i^{l+1,-}), \\
x_{m,1/2}^{l+1/2} &= (A_m^{-})^{p} x_{m,p+1/2}^{l+1/2} + s_s \sum_{i=1}^{p} (A_m^{-})^{i-1} B_m^{-} (V_i^{l+1,+} + V_i^{l+1,-}),
\end{aligned} \tag{11a}
$$

$$
\begin{aligned}
&\sum_{m_m>0} m_m w_m \left(x_{m,p+1/2}^{l+1/2} + \frac{1}{2} F_{3/2}^{+}\right) \\
&\quad = \sum_{m_m>0} m_m w_m (A_m^{+})^{p} \left(x_{m,1/2}^{l+1/2} + \frac{1}{2} F_{1/2}^{+}\right) + s_s \sum_{i=1}^{p} \left(\sum_{m_m>0} m_m w_m (A_m^{+})^{p-i} B_m^{+} \right) (V_i^{l+1,+} + V_i^{l+1,-}), \\
&\sum_{m_m<0} m_m w_m \left(x_{m,1/2}^{l+1/2} + \frac{1}{2} F_{1/2}^{-}\right) \\
&\quad = \sum_{m_m<0} m_m w_m (A_m^{-})^{p} \left(x_{m,p+1/2}^{l+1/2} + \frac{1}{2} F_{3/2}^{-}\right) + s_s \sum_{i=1}^{p} \left(\sum_{m_m<0} m_m w_m (A_m^{-})^{i-1} B_m^{-} \right) (V_i^{l+1,+} + V_i^{l+1,-}),
\end{aligned} \tag{11b}
$$

$$
\begin{aligned}
V_i^{l+1,+} &= \sum_{m_m>0} w_m C_m^+ (A_m^+)^{i-1} \left(x_{m,1/2}^{l+1/2} + \frac{1}{2} F_{1/2}^+\right) + s_s \sum_{k=1}^{i-1} \left(\sum_{m_m>0} w_m C_m^+ (A_m^+)^{i-1-k} B_m^+ \right) \left(V_k^{l+1,+} + V_k^{l+1,-}\right) \\
&\quad + s_s \left(\sum_{m_m>0} w_m D_m^+ \right) \left(V_i^{l+1,+} + V_i^{l+1,-}\right), \\
V_i^{l+1,-} &= \sum_{m_m<0} w_m C_m^- (A_m^-)^{p-i} \left(x_{m,p+1/2}^{l+1/2} + \frac{1}{2} F_{3/2}^-\right) + s_s \sum_{k=i+1}^{p} \left(\sum_{m_m<0} w_m C_m^- (A_m^-)^{k-(i+1)} B_m^- \right) \left(V_k^{l+1,+} + V_k^{l+1,-}\right) \\
&\quad + s_s \left(\sum_{m_m<0} w_m D_m^- \right) \left(V_i^{l+1,+} + V_i^{l+1,-}\right).
\end{aligned}
\tag{11c}
$$

In the equations above, the coarse-mesh index *n* and fine-mesh index *i* in constants A, B, C, and D disappear because the medium under consideration is infinite homogeneous.

Next, the following Fourier ansatz are chosen:

$$
\begin{aligned}
&x_{m,1/2}^{l+1/2} = b_m e^{j l x_{1/2}}, \qquad x_{m,p+1/2}^{l+1/2} = b_m e^{j l x_{p+1/2}}, \\
&V_i^{l+1/2,\pm} = a_i^\pm e^{j l x_i}, \qquad V_i^{l+1,\pm} = w a_i^\pm e^{j l x_i}, \\
&F_{1/2}^\pm = G^\pm e^{j l x_{1/2}}, \qquad F_{3/2}^\pm = G^\pm e^{j l x_{p+1/2}}.
\end{aligned}
\tag{12}
$$

Substituting the Fourier ansatz Eq. (12) into Eq. (11) gives following equations:

$$
\begin{aligned}
w a_i^+ &= s_s \sum_{k=1}^{p} \left[a_{ik}^+ + (w-1) b_{ik}^+\right] \left(a_k^+ + a_k^-\right) + s_s w \sum_{k=1}^{i-1} g_{ik}^+ \left(a_k^+ + a_k^-\right) + s_s w d^+, \\
w a_i^- &= s_s \sum_{k=1}^{p} \left[a_{ik}^- + (w-1) b_{ik}^-\right] \left(a_k^+ + a_k^-\right) + s_s w \sum_{k=1}^{i-1} g_{ik}^- \left(a_k^+ + a_k^-\right) + s_s w d^-,
\end{aligned}
\tag{13}
$$

where

$$
\begin{aligned}
a_{ik}^+ &= \exp[2(k-i)]tj \sum_{m_m>0} \frac{w_m (A_m^+)^{p+(i-1)-k} B_m^+ C_m^+}{\exp[2ptj] - (A_m^+)^p}, \\
a_{ik}^- &= \exp[2(k-i)]tj \sum_{m_m<0} \frac{w_m (A_m^-)^{p+k-(i+1)} B_m^- C_m^-}{\exp[-2ptj] - (A_m^-)^p},
\end{aligned}
\tag{14a}
$$

$$b_{ik}^{+} = \exp[2(k-i)]tj \frac{\sum_{m_m>0} w_m (A_m^{+})^{i-1} C_m^{+} \sum_{m_m>0} m_m w_m (A_m^{+})^{p-k} B_m^{+}}{\sum_{m_m>0} m_m w_m \left(\exp[2ptj] - (A_m^{+})^{p}\right)},$$

$$b_{ik}^{-} = \exp[2(k-i)]tj \frac{\sum_{m_m<0} w_m (A_m^{-})^{p-i} C_m^{-} \sum_{m_m<0} m_m w_m (A_m^{-})^{k-1} B_m^{-}}{\sum_{m_m<0} m_m w_m \left(\exp[-2ptj] - (A_m^{-})^{p}\right)}, \tag{14b}$$

$$g_{ik}^{+} = \exp[2(k-i)]tj \sum_{m_m>0} w_m (A_m^{+})^{i-1-k} B_m^{+},$$

$$g_{ik}^{-} = \exp[2(k-i)]tj \sum_{m_m<0} w_m (A_m^{-})^{k-(i+1)} B_m^{-}, \tag{14c}$$

$$d_{ik}^{+} = \sum_{m_m>0} w_m D_m^{+},$$

$$d_{ik}^{+} = \sum_{m_m<0} w_m D_m^{-}. \tag{14d}$$

Eq. (13) can be rewritten in matrix form as follows:

$$\begin{bmatrix} s\,{}_s\mathbf{A}_1^{+} + w(s\,{}_s\mathbf{A}_2^{+} - \mathbf{I}) & s\,{}_s\mathbf{A}_1^{+} + ws\,{}_s\mathbf{A}_2^{+} \\ s\,{}_s\mathbf{A}_1^{-} + ws\,{}_s\mathbf{A}_2^{-} & s\,{}_s\mathbf{A}_1^{-} + w(s\,{}_s\mathbf{A}_2^{-} - I) \end{bmatrix} \begin{bmatrix} \mathbf{a}^{+} \\ \mathbf{a}^{-} \end{bmatrix} = \mathbf{0} \tag{15}$$

where $\mathbf{I}$ is $p \times p$ identity matrix and

$$\mathbf{a}^{+} = \left[a_1^{+} \quad \Lambda \quad a_p^{+}\right]^T, \qquad \mathbf{a}^{-} = \left[a_1^{-} \quad \Lambda \quad a_p^{-}\right]^T, \tag{16a}$$

$$\mathbf{A}_1^{+} = \begin{bmatrix} a_{11}^{+} - b_{11}^{+} & K & a_{1p}^{+} - b_{1p}^{+} \\ M & O & M \\ a_{p1}^{+} - b_{p1}^{+} & \Lambda & a_{pp}^{+} - b_{pp}^{+} \end{bmatrix}, \qquad \mathbf{A}_1^{-} = \begin{bmatrix} a_{11}^{-} - b_{11}^{-} & K & a_{1p}^{-} - b_{1p}^{-} \\ M & O & M \\ a_{p1}^{-} - b_{p1}^{-} & \Lambda & a_{pp}^{-} - b_{pp}^{-} \end{bmatrix}, \tag{16b}$$

$$\mathbf{A}_2^{+} = \begin{bmatrix} b_{11}^{+} + d^{+} & b_{12}^{+} & & \Lambda & b_{1p}^{+} \\ b_{21}^{+} + g_{21}^{+} & b_{22}^{+} + d^{+} & b_{23}^{+} & & M \\ M & & & O & b_{p-1p}^{+} \\ b_{p1}^{+} + g_{p1}^{+} & b_{p2}^{+} + g_{p2}^{+} & & b_{pp-1}^{+} + g_{pp-1}^{+} & b_{pp}^{+} + d^{+} \end{bmatrix},$$

$$\mathbf{A}_2^{-} = \begin{bmatrix} b_{11}^{-} + d^{-} & b_{12}^{-} + g_{12}^{-} & & \Lambda & b_{1p}^{-} + g_{1p}^{-} \\ b_{21}^{-} & b_{22}^{-} + d^{-} & b_{23}^{-} + g_{23}^{-} & & M \\ M & & & O & b_{p-1p}^{-} + g_{p-1p}^{-} \\ b_{p1}^{-} & b_{p2}^{-} & & b_{pp-1}^{-} & b_{pp}^{-} + d^{-} \end{bmatrix}. \tag{16c}$$

Therefore, the eigenvalues w's of the iteration operator of ADCMR is obtained by equating the determinant of Eq. (15) to zero. The spectral radius r is then the largest absolute value of w’ s.

$$r = \sup_t |w(t)|. \tag{17}$$

For the DD discretization scheme, we can find the following properties:

$$\mathbf{A}_1^- = \overline{\mathbf{A}_1^+}^T, \qquad \mathbf{A}_2^- = \overline{\mathbf{A}_2^+}^T, \tag{18}$$

where $\overline{\mathbf{X}}^T$ denotes transpose matrix of $\mathbf{X}$ with complex elements conjugated.

4. Numerical Results

4.1 Benchmark Problems

Several benchmark problems are tested. The DD scheme is used for all problems. From the benchmark results we find that ADCMR is unconditionally stable in not only homogeneous but heterogeneous medium with optically thick mesh size. The acceleration effect is also significant.

i) Benchmark I (Reed’ s Problem)

- Infinite slab with homogeneous medium
- Constant mesh size
- Vacuum boundary condition
- S_6
- Error criteria ($\max_i \left|1 - f_i^{l+1} / f_i^l\right|$) : 10^{-4}

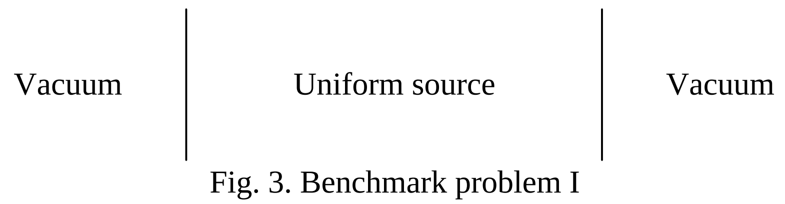

Fig. 3. Benchmark problem I

Table 1. Material data of benchmark problem I

Problem	c	s_t	h	Q	Total size
M1	1.0	1.0	1.0	1.0	30.0
M2	1.0	2.0	1.0	1.0	30.0
M3	1.0	1.0	1.0	1.0	60.0
M4	1.0	1.0	2.0	1.0	60.0

Table 2. Number of iterations of benchmark problem I

	SI	ADR ($p=1$)	ADCMR				
			$p=2$	$p=5$	$p=10$	$p=15$	$p=30$
M1	1072	4	5	4	4	3	2
M2	2641	5	5	4	4	4	2
M3	2644	5	5	4	4	4	4
M4	2642	5	5	4	4	4	2

ii) Benchmark II (Khalil’s Problem)

- Uniform isotropic scattering medium for various cross section with $c=0.98$
- Constant source in the left half of the slab
- $h=1cm$
- S_{16}
- Error criteria : 10^{-4}

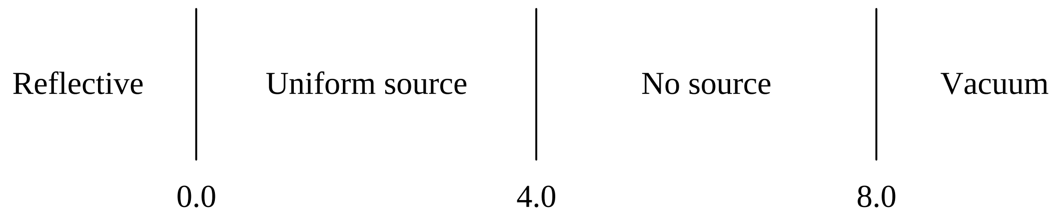

Fig. 4. Benchmark problem II

Table 3. Number of iteration of benchmark problem II

σ_t		1.0	2.0	4.0	6.0	10.0	20.0
SI		197	272	358	504	448	394
ADR($p=1$)		5	5	5	6	7	5
ADCMR	$p=2$	5	5	5	6	7	5
	$p=4$	4	4	4	6	6	5

iii) Benchmark III (Modified Adams and Martin’s Problem)

- Highly heterogeneous problem
- 10 meshes per material
- S_{16}
- Error criteria : 10^{-6}

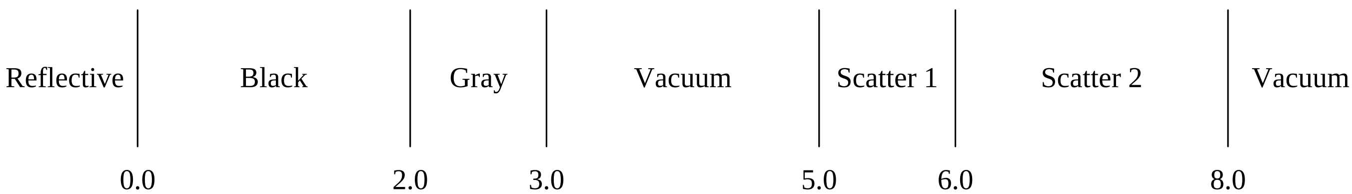

Fig. 5. Benchmark problem III

Table 4. Material data of benchmark problem III

	Black	Gray	Scatter 1	Scatter 2
Q	50.0	0.0	1.0	0.0
s_t	50.0	5.0	2.0	1.0
c	0.0	0.0	1.0	1.0

Table 5. Number of iterations of benchmark problem III

SI	ADR($p=1$)	ADCMR		
		$p=2$	$p=5$	$p=10$
112	6	6	6	6

4.2 Results of Fourier Analysis

In addition to the numerical results, we can confirm that ADCMR is unconditionally stable in one-dimensional problems with DD by the results of Fourier analysis, if desired. Fig. 6 shows the spectral radius for several scattering ratios when coarseness $p=2$. The smaller scattering ratio gives always better results. Fig. 7 shows that stability increases as coarseness increases. By comparison to the conventional CMR, ADCMR is more stable and effective. (Fig. 7 and Fig. 8)

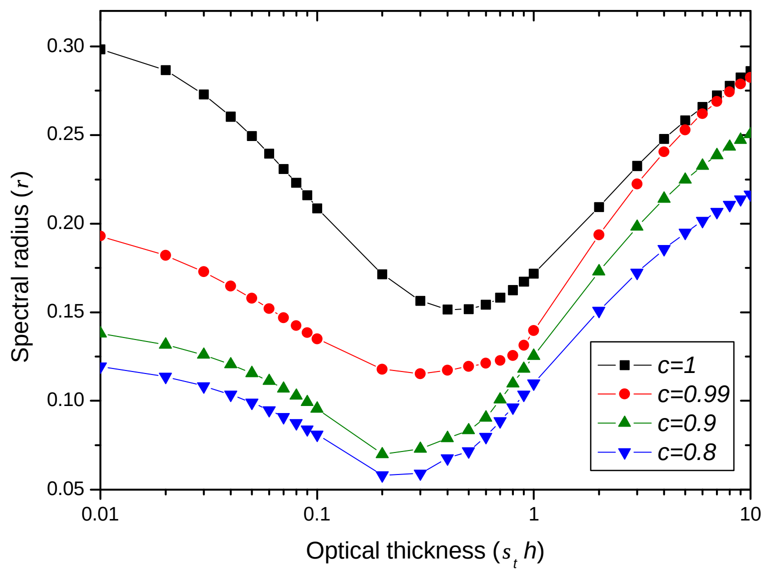

Figure 6. Spectral radius of ADCMR when $p=2$ for various scattering ratios

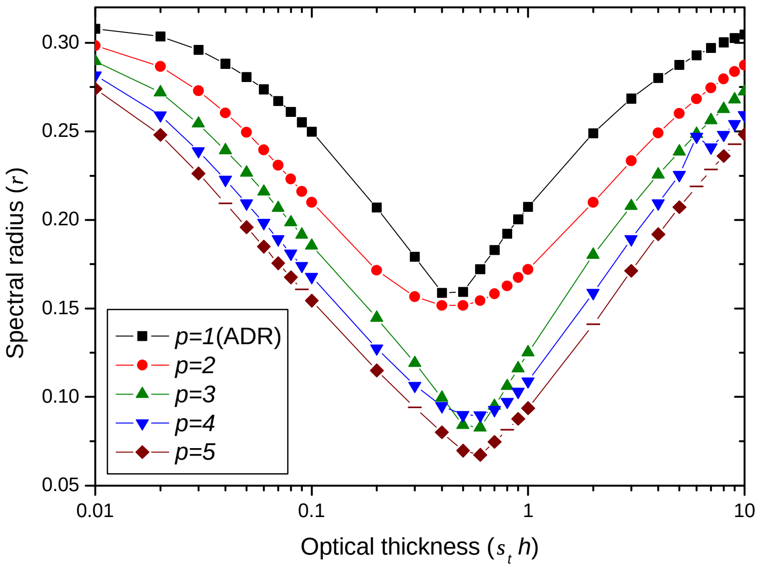

Figure 7. Spectral radius of ADCMR for various p with $c=1$

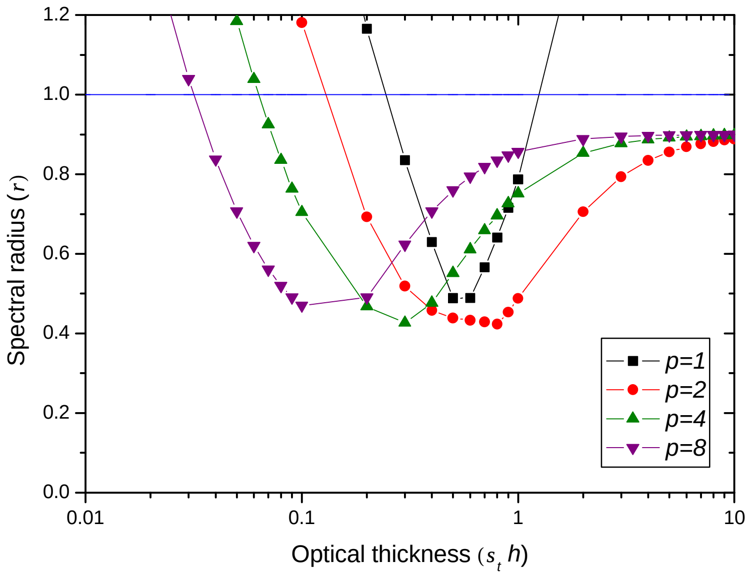

Figure 8. Spectral radius of CMR for various p with $c=1$

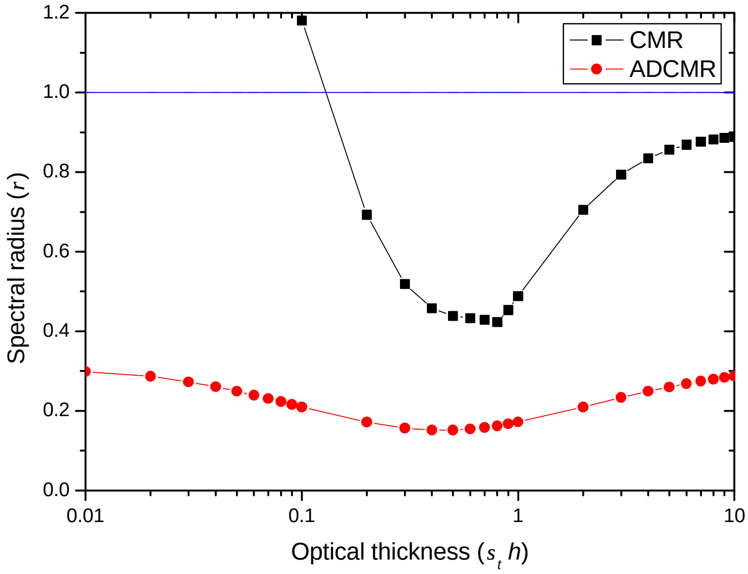

Figure 9. CMR vs. ADCMR (*p=2*, *c=1*)

5. Conclusions and Further Works

In this paper, a new nonlinear iteration method based on the angular dependent rebalance factor concept called the Angular Dependent Coarse-Mesh Rebalance (ADCMR) method was developed to accelerate one-dimensional discrete ordinates transport equation. The ADCMR method was successfully tested on several benchmark problems. Also, as a theoretical analysis of the ADCMR method, the Fourier analysis through linearization was used. The results show that ADCMR method is very effective and unconditionally stable in terms of the spectral radius and the number of iterations. From the above results we conclude that the ADCMR method can be used effectively in one-dimensional neutron transport calculations.

Finally the followings are our further work : first, extension to two- and three-dimensional problems, second, possibility of applications to other transport discretization schemes such as the Method Of Characteristics (MOC) and the Characteristic Direction Probabilities (CDP) method.

References

[1] H. J. Kopp, “Synthetic Method Solution of the Neutron Transport Equation,” PhD Thesis, University of California at Berkeley (1962).
[2] R. E. Alcouffe, “Diffusion Synthetic Acceleration Methods for the Diamond Differenced Discrete Ordinates Equations,” *Nucl. Sci. Eng.*, **64**, 344 (1977).
[3] E. W. Larsen, “ Unconditionally Stable Diffusion Synthetic Acceleration Methods for the Slab Geometry Discrete-Ordinates Equations, Part I : Theory,” *Nucl. Sci. Eng.*, **82**, 47

(1982).
[4] D. R. McCoy and E. W. Larsen, “Unconditionally Stable Diffusion Synthetic Acceleration Methods for the Slab Geometry Discrete-Ordinates Equations, Part II : Numerical Results,” *Nucl. Sci. Eng.*, **82**, 64 (1982).
[5] G. L. Ramone and M. L. Adams, “A Transport Synthetic Acceleration Method for Transport Iterations,” *Nucl. Sci. Eng.*, **125**, 257 (1997).
[6] V. Ya. Gol’din, “A Quasi-Diffusion Method of Solving the Kinetic Equation,” Zh. Vych. Mat., 4(1078), 1964, English Translation published in USSR Comp. Math. And Math. Phys., 4, 136 (1967).
[7] G. R. Cefus and E. W. Larsen, “Stability Analysis of the Quasidiffusion and Second Moment Methods for Iteratively Solving Discrete Ordinates Problems,” *Transport Theory Stat. Phys.*, **18**, 493 (1990).
[8] J. J. Duderstadt and W. R. Martin, Transport Theory, John Wiley & Sons, 1976.
[9] G. R. Cefus and E. W. Larsen, “Stability Analysis of Coarse-Mesh Rebalance,” *Nucl. Sci. Eng.*, **105**, 31 (1990).
[10] S. G. Hong and N. Z. Cho, “Rebalance Approach to Nonlinear Iteration for Solving the Neutron Transport Equations,” *Ann. Nucl. Energy*, **24**, 147 (1997).
[11] S. G. Hong and N. Z. Cho, “Angle-Dependent Rebalance Factor Method for Nodal Transport Equations,” *Trans. Am. Nucl. Soc.*, **79**, 139 (1998).
[12] C. J. Park and N. Z. Cho, “Additive Angular Dependent Acceleration Method of the Discrete Ordinates Transport Calculations,” *ANS International Meeting on Mathematical Methods for Nuclear Applications (M&C 2001)*, September 2001, Salt Lake City, U.S.A. (CD-ROM)
[13] C. J. Park and N. Z. Cho, “Convergence Analysis of Additive Angular Dependent Rebalance Acceleration for the Discrete Ordinates Transport Calculations,” *Nucl. Sci. Eng.,* to appear, September 2002.