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Comparison of CMFD and p-CMFD Acceleration Methods for Neutron Transport Calculations

Nam Zin Cho and Gil Soo Lee

Korea Advanced Institute of Science and Technology
373-1 Kusong-dong, Yusong-gu
Taejon, Korea, 305-701

ABSTRACT

Partial current-based coarse mesh finite difference (p-CMFD) acceleration of the 3D whole-core transport calculation is described and its convergence/stability is compared with conventional CMFD for a varying degree of coarseness and mesh sizes. The results of the Fourier stability analysis and numerical tests show that p-CMFD is a significant improvement over CMFD in that p-CMFD is unconditionally stable except for the coarseness p=1 (i.e., fine-mesh acceleration case), while CMFD is divergent for all p's in the mesh size interval of ~0.5 < σh < ~10 when c is high and this interval becomes larger as p increases. A drawback of p-CMFD, however, is the loss of its acceleration effectivenss as the mesh size σh becomes large (as is CMFD).

I. INTRODUCTION

Although there is strong interest for 3D whole-core heterogeneous transport calculations using the method of characteristics (MOC), it is limited at present to assembly or super-assembly size problems due to the tremendous amount of memory and computing time. On the one hand, the 2D/1D fusion method[1,2] recently proposed by the authors, in which the MOC for radial 2D calculation is combined with the S_N-like transport method for axial 1D calculation, reduces this computational burden drastically and this may constitute a practical method for the 3D whole-core transport calculations.

On the other hand, it is important to use good acceleration methods in transport calculations. Among the many acceleration methods, the coarse mesh finite difference (CMFD) acceleration method that is popular in the fast solution of nodal diffusion equations[3-5], has been employed for the acceleration of the transport calculations with very good results[6-9], although the transport calculations were limited to the 2D problems. In the CMFD method, a current correction coefficient is introduced to preserve the interface net current (obtained from the transport sweep) between two coarse meshes. It is known that the convergence of CMFD is very fast in optically thin meshes but it becomes poor or divergent in optically thick meshes[10,11]. This may be due to the weak physical basis of the way the correction coefficient is introduced.

This paper provides (1) a partial current-based coarse mesh finite difference (p-CMFD) acceleration method, in which two correction coefficients are introduced at an interface between two coarse meshes such that partial currents are preserved, (2) its Fourier convergence analysis, and (3) its performance in the 2D/1D fusion method for two test problems in comparison with CMFD.

II. DESCRIPTION OF THE METHOD

To describe the p-CMFD acceleration method for which some preliminary results were presented recently in Ref. 12, let us consider coarse mesh i and coarse mesh $i+1$. Outgoing partial current at the right interface $i+1/2$ of coarse mesh i is corrected with a correction coefficient $\hat{D}^{+}_{i+1/2}$ by

$$J^{+,l+1}_{i+1/2} = \frac{-\widetilde{D}_{i+1/2}(\phi^{l+1}_{i+1} - \phi^{l+1}_{i}) + 2\hat{D}^{+}_{i+1/2}\phi^{l+1}_{i}}{2}, \tag{1a}$$

and similarly incoming partial current by

$$J^{-,l+1}_{i+1/2} = \frac{\widetilde{D}_{i+1/2}(\phi^{l+1}_{i+1} - \phi^{l+1}_{i}) + 2\hat{D}^{-}_{i+1/2}\phi^{l+1}_{i+1}}{2}, \tag{1b}$$

where l is the iteration index and $\widetilde{D}_{i+1/2}$ is the usual coupling coefficient determined by the finite difference method. The two correction coefficients are defined to preserve the respective partial currents as

$$\hat{D}^{+}_{i+1/2} = \frac{2J^{+,l+1/2}_{i+1/2} + \widetilde{D}_{i+1/2}(\phi^{l+1/2}_{i+1} - \phi^{l+1/2}_{i})}{2\phi^{l+1/2}_{i}}, \tag{2a}$$

$$\hat{D}^{-}_{i+1/2} = \frac{2J^{-,l+1/2}_{i+1/2} - \widetilde{D}_{i+1/2}(\phi^{l+1/2}_{i+1} - \phi^{l+1/2}_{i})}{2\phi^{l+1/2}_{i+1}}, \tag{2b}$$

Then, the net current is obtained as

$$J_{i+1/2}^{l+1} = J_{i+1/2}^{+,l+1} - J_{i+1/2}^{-,l+1}$$

$$= -\tilde{D}_{i+1/2}(\phi_{i+1}^{l+1} - \phi_i^{l+1}) - (\hat{D}_{i+1/2}^{-}\phi_{i+1}^{l+1} - \hat{D}_{i+1/2}^{+}\phi_i^{l+1}). \qquad (3)$$

The computational procedures of the p-CMFD method are similar to the original CMFD method and do not require any extra computation. In the new method, net currents are naturally preserved and, since outgoing or incoming partial current is corrected to be preserved by its own coarse mesh flux, it is more physically based. This indicates that p-CMFD should perform better in problems with strong flux gradients. Furthermore, the availability of the partial currents as part of the solution may be utilized for other uses.

III. RESULTS

To predict convergence behavior of the p-CMFD method, linearized Fourier analysis was performed for an infinite homogeneous one-dimensional model problem with the diamond difference (DD) discretization scheme for various *p*'s, the number of fine meshes per coarse mesh. Fig. 1 shows the improved behavior of p-CMFD over CMFD.

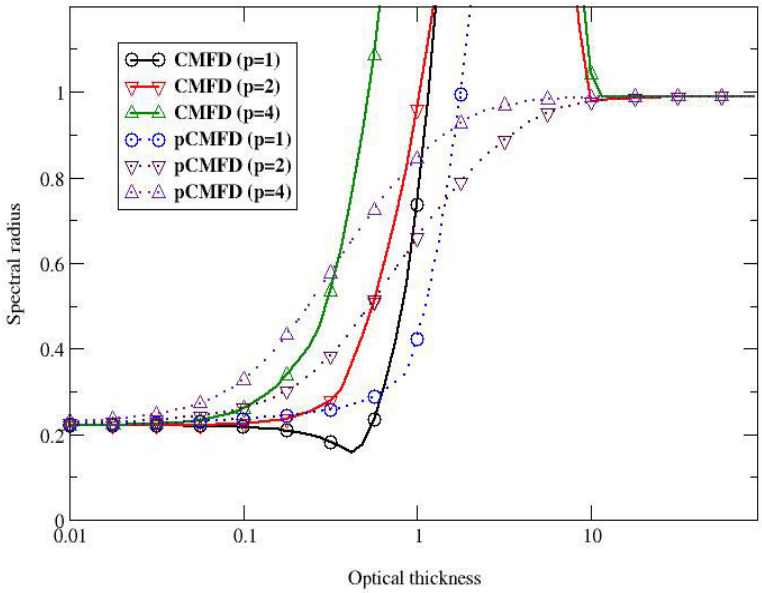

Fig. 1. Results of Fourier analysis of CMFD and p-CMFD with DD, c=0.99, and S_{16}.

Although p-CMFD is a general 3D acceleration method, its use in the 2D/1D fusion methodology was tested on small two-group 3D problems that consists of a BWR

assembly with 7x7 fuel cells and axial height 50cm with all reflective boundary conditions except the top side which has vacuum boundary condition. The configuration of this fuel assembly is given in Ref. 13. Table I shows the results for the case in which the full height consists of fuel. Table II shows the case when a top portion (10cm) of the height is replaced by water reflector. In all the runs, 24 fine meshes (p_{xy}=24) were used in a fuel cell. In axial calculations, the coarseness p_z used is 1 or 2. The relative error criteria used were 10^{-6} and 10^{-4} for the multiplication factor and fission source, repectively. Note that p-CMFD performed better than CMFD and succeeded in providing converged solutions where CMFD failed. When the axial coarseness p_z is 2, p-CMFD converges in all the test cases and this agrees with the results of Fourier analysis. The non-convergence in the coarse axial mesh cases (p_z=1 for p-CMFD and all p_z's for CMFD) may be attributed to the mesh size used falling in the unstable region (optical thickness 0.5~10 mean free paths) as predicted in Fig. 1.

TABLE I. Numerical Results of p-CMFD and CMFD in a Problem with Full Height Fuel

Number of Axial Nodes		Iterations (k_{eff})	Time (sec)
8	p-CMFD (p_z=1)	Not Converged	-
	p-CMFD (p_z=2)	36 (0.96244)	47
	CMFD (p_z=1)	Not Converged	-
	SI*	345 (0.96238)	451
10	p-CMFD (p_z=1)	118 (0.96256)	208
	p-CMFD (p_z=2)	45 (0.96256)	79
	CMFD (p_z=1)	Not Converged	-
	SI	358 (0.96251)	603
16	p-CMFD (p_z=1)	16 (0.96270)	58
	p-CMFD (p_z=2)	59 (0.96269)	203
	CMFD (p_z=1)	Not Converged	-
	SI	393 (0.96263)	1360
20	p-CMFD (p_z=1)	16 (0.96273)	83
	p-CMFD (p_z=2)	59 (0.96272)	290
	CMFD (p_z=1)	15 (0.96272)	81
	SI	416 (0.96266)	2060

* source iteration (no acceleration)

TABLE II. Numerical Results of p-CMFD and CMFD in a Problem with Reflector Top

Number of Axial Nodes (Fuel+Reflector)		Iterations (k_{eff})	Time (sec)
10+2	p-CMFD (p_z=1)	Not Converged	-
	p-CMFD (p_z=2)	50 (0.97497)	116
	CMFD (p_z=1)	Not Converged	-
	SI	180 (0.97479)	432
10+4	p-CMFD (p_z=1)	Not Converged	-
	p-CMFD (p_z=2)	52 (0.97453)	149
	CMFD (p_z=1)	Not Converged	-
	SI	264 (0.97450)	779
10+10	p-CMFD (p_z=1)	20 (0.97441)	101
	p-CMFD (p_z=2)	34 (0.97441)	181
	CMFD (p_z=1)	Not Converged	-
	SI	282 (0.97436)	1413
16+10	p-CMFD (p_z=1)	20 (0.97445)	153
	p-CMFD (p_z=2)	30 (0.97445)	242
	CMFD (p_z=1)	Not Converged	-
	SI	297 (0.97441)	2296

IV. CONCLUSIONS

The partial current-based CMFD (p-CMFD) acceleration method using two correction coefficients instead of one at an interface that preserve outgoing and incoming partial currents respectively has been described and its performance was tested by Fourier analysis and two test problems including a problem with reflector top. The numerical results agrees well with Fourier analysis, and the results show that p-

CMFD exhibits significantly increased stability regions compared to CMFD. p-CMFD with p≥2 is unconditionally stable.The improved performance of p-CMFD over CMFD indicates that the p-CMFD method warrants its wide use in the whole-core transport calculations.

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