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Coarse-Mesh Angular Dependent Rebalance Acceleration Method in *X-Y* Geometry

Young Ryong Park and Nam Zin Cho

Korea Advanced Institute of Science and Technology
373-1 Kusong-dong, Yusong-gu,
Taejon, Korea 305-701
E-mail : nzcho@mail.kaist.ac.kr

Abstract

We extend the coarse-mesh angular dependent rebalance acceleration method (CMADR) recently proposed by the authors to *x-y* geometry neutron transport calculations and apply it with diamond-difference and constant-constant nodal discretizations. Similarly to the one-dimensional case, the S_2-like rebalance factors are angular dependent and defined on coarse mesh boundaries only. The numerical tests show that CMADR method is very effective in reducing the number of iterations.

1. Introduction

Recently, the whole core heterogeneous transport calculation is possible due to advanced computer technology. But it still requires many iterations and long computing time so that the effective acceleration method is very important. Many acceleration methods were proposed and tested to accelerate source iteration such as diffusion synthetic acceleration (DSA)[1,2,3], coarse-mesh rebalance (CMR)[4,5], coarse-mesh finite difference (CMFD)[6,7,8,9,10], transport synthetic acceleration (TSA)[11], and so on.[12] Among these methods, the most popular DSA is unconditionally stable but it requires consistent discretizations. Therefore it is difficult to extend multi-dimensional problems. Also, performance degradation of DSA in highly heterogeneous media was reported in several papers. [13,14,15]
In these situations, the coarse mesh acceleration method is very attractive to accelerate whore core heterogeneous calculations. Since the number of unknowns decreases as the coarseness increases and low order equations are not dependent on spatial discretization of high order equations. But the conventional coarse mesh calculations such as CMR and CMFD are only conditionally stable. In our previous research, coarse-mesh angular dependent rebalance acceleration method (CMADR) [16,17] was proposed and tested for one-dimensional

problems. CMADR method is very effective in reducing the number of iterations and it provides unconditional stability for all mesh sizes, coarseness and scattering ratios, which are most desired features.
In this paper, we extend and test the CMADR method to *x-y* geometry problems. Like one-dimensional problems, angular dependent rebalance factors are defined only on the coarse mesh boundaries.

2. Formulation

Let us consider the *x-y* geometry like Figure 1. The whole problem consists of (*nx*, *ny*) coarse mesh cells, which contains (*px*, *py*) fine meshes per each coarse mesh.

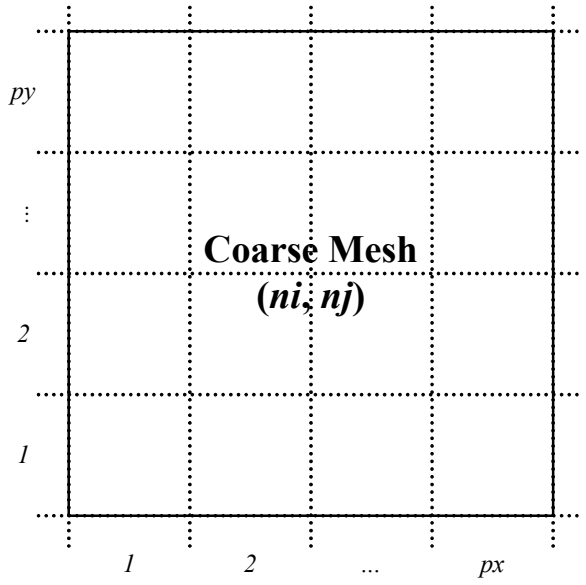

Figure 1. Geometry of the problem

To describe the CMADR method in *x-y* geometry, the general discretized form of the S_N transport calculation is written in the following form for fine mesh (*i*, *j*):

$$\vec{\psi}_{i,j,m}^{out,l+1/2} = \mathbf{A0}_{i,j,m}\vec{\psi}_{i,j,m}^{in,l+1/2} + \mathbf{B0}_{i,j,m}(\sigma_{s,i,j}\phi_{i,j}^{l} + s_{i,j}), \tag{1a}$$

$$\psi_{i,j,m}^{l+1/2} = \mathbf{C0}_{i,j,m}\vec{\psi}_{i,j,m}^{in,l+1/2} + \mathbf{D0}_{i,j,m}(\sigma_{s,i,j}\phi_{i,j}^{l} + s_{i,j}), \tag{1b}$$

where *l* is source iteration index, *m* is angle index, $\vec{\psi}_{i,j,m}^{out\text{ or in}}$ is outgoing or incoming angular flux vector, $\psi_{i,j,m}$ is interior angular flux, $\phi_{i,j}$ is interior scalar flux, and $s_{i,j}$ is interior source.
Similarly, the outgoing angular flux and all interior angular fluxes for the coarse mesh (*ni*, *nj*) can be written in the following form:

$$\vec{\psi}^{out,l+1/2}_{ni,nj,m} = \mathbf{A1}_{ni,nj,m}\vec{\psi}^{in,l+1/2}_{ni,nj,m} + \mathbf{B1}_{ni,nj,m}(\boldsymbol{\sigma}_{s,ni,nj}\vec{\phi}^{l}_{ni,nj} + \vec{s}_{ni,nj}), \tag{2a}$$

$$\vec{\psi}^{l+1/2}_{ni,nj,m} = \mathbf{C1}_{ni,nj,m}\vec{\psi}^{in,l+1/2}_{ni,nj,m} + \mathbf{D1}_{ni,nj,m}(\boldsymbol{\sigma}_{s,ni,nj}\vec{\phi}^{l}_{ni,nj} + \vec{s}_{ni,nj}), \tag{2b}$$

where $\vec{\psi}_{ni,nj,m}$ is the interior angular flux vector of the coarse mesh, $\boldsymbol{\sigma}_{s,ni,nj}$ is a diagonal matrix which contains the scattering cross sections, $\vec{\phi}_{ni,nj}$ is the scalar flux vector, and $\vec{s}_{ni,nj}$ is the interior source vector.

From now on, the indices, *ni* and *nj*, are omitted for simplicity. The next step is introduction of the nonlinear rebalance factor. The rebalance factor is the ratio of the new iterate to the previous scalar fluxes on the coarse mesh boundary and given by

$$f^{out\text{ or }in}_{\gamma,p}(\mu,\eta) = \frac{\psi^{out\text{ or }in,l+1}_{p}(\mu,\eta)}{\psi^{out\text{ or }in,l+1/2}_{p}(\mu,\eta)} = \begin{cases} f^{out\text{ or }in}_{ni,nj,\gamma}, & \text{for } \Omega(\mu,\eta) \in \text{quadrant } \gamma \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

where the index *p* represents a component of vector.

If all iteration indices in Eq. (2a) and (2b) are changed to *l+1* and inserting the rebalance factors into the equations, the following equations are obtained:

$$f^{out}_{\gamma,p}\psi^{out,l+1/2}_{m,p} = \sum_{q}\mathbf{A1}_{m,p,q}f^{in}_{\gamma,q}\psi^{in,l+1/2}_{m,q} + \sum_{q}\mathbf{B1}_{m,p,q}(\boldsymbol{\sigma}_{s,q,q}\phi^{l+1}_{q} + s_q), \tag{4a}$$

$$\psi^{l+1}_{m,p} = \sum_{q}\mathbf{C1}_{m,p,q}f^{in}_{\gamma,q}\psi^{in,l+1/2}_{m,q} + \sum_{q}\mathbf{D1}_{m,p,q}(\boldsymbol{\sigma}_{s,q,q}\phi^{l+1}_{q} + s_q), \tag{4b}$$

where *q* is a dummy index, which represent a component of the vector or matrix.

To obtain a low-order CMADR equation, Eq. (4a) is multiplied by weighting function $W(\mu,\eta)$ and integrated for angular variables over the each quadrant. Here, we use the $W(\mu,\eta) = \mu$ for *x*-direction edge and $W(\mu,\eta) = \eta$ for *y*-direction edge in this paper. The resulting equation is

$$\vec{f}^{out}_{\gamma} = \mathbf{U}_{\boldsymbol{\gamma}}\vec{f}^{in}_{\gamma} + \mathbf{V}_{\boldsymbol{\gamma}}(\boldsymbol{\sigma}_{\mathbf{s}}\vec{\phi}^{l+1} + \vec{s}), \tag{5}$$

where

$$\mathbf{U}_{\boldsymbol{\gamma}} = \frac{\sum_{m\in\text{quadrant }\gamma} w_m W(\mu_m,\eta_m)\mathbf{A1}_{m,p,q}\psi_{m,q}^{in,l+1/2}}{\sum_{m\in\text{quadrant }\gamma} w_m W(\mu_m,\eta_m)\psi_{m,p}^{in,l+1/2}}, \tag{6a}$$

$$\mathbf{V}_{\boldsymbol{\gamma}} = \frac{\sum_{m\in\text{quadrant }\gamma} w_m W(\mu_m,\eta_m)\mathbf{B1}_{m,p,q}}{\sum_{m\in\text{quadrant }\gamma} w_m W(\mu_m,\eta_m)\psi_{m,p}^{in,l+1/2}}. \tag{6b}$$

In this equation, the updated scalar flux vector $\vec{\phi}^{\,l+1}$ is still unknown and it is the sum of four directional scalar fluxes as given in following equation:

$$\vec{\phi}^{\,l+1} = \sum_{\gamma=1}^{4} \vec{\phi}_{\gamma}^{\,l+1}, \qquad \vec{\phi}_{\gamma}^{\,l+1} = \sum_{m\in\text{quadrant }\gamma} w_m \vec{\psi}_m^{\,l+1}. \tag{7}$$

Directional scalar fluxes are obtained from Eq. (4b) by integrating for angular variable for each quadrant.

$$\vec{\phi}_{\gamma}^{\,l+1} = \mathbf{X}_{\boldsymbol{\gamma}} \vec{f}_{\gamma}^{\,in} + \mathbf{Y}_{\boldsymbol{\gamma}}(\boldsymbol{\sigma}_{\mathbf{s}}\vec{\phi}^{\,l+1} + \vec{s}), \tag{8}$$

$$\mathbf{X}_{\boldsymbol{\gamma}} = \frac{\sum_{m\in\text{quadrant }\gamma} w_m W(\mu_m,\eta_m)\mathbf{C1}_{m,p,q}\psi_{m,q}^{in,l+1/2}}{\sum_{m\in\text{quadrant }\gamma} w_m W(\mu_m,\eta_m)\psi_{m,p}^{in,l+1/2}}, \tag{9a}$$

$$\mathbf{Y}_{\boldsymbol{\gamma}} = \frac{\sum_{m\in\text{quadrant }\gamma} w_m W(\mu_m,\eta_m)\mathbf{D1}_{m,p,q}}{\sum_{m\in\text{quadrant }\gamma} w_m W(\mu_m,\eta_m)\psi_{m,p}^{in,l+1/2}}. \tag{9b}$$

Equations (5), (7), and (8) are resembles the S2 transport equation so that we can solve these low order CMADR equation by sweeping like high order calculation.

3. Numerical Tests

To test CMADR method for *x-y* geometry, we selected three benchmark problems. First is a two-dimensional version of the McCoy-Larsen model problem.[18] Second is the iron-water benchmark problem.[19] Third is IAEA EIR-2 benchmark problem.[19] The finite difference method using diamond-difference (DD) scheme was used for McCoy-Larsen problem and

constant-constant (C-C) transport nodal method was used for iron-water and IAEA EIR-2 benchmark problem. In all calculations, uniform distribution was used for an initial guess and a pointwise relative maximum error was used as an stopping criterion of high and low order iteration:

$$\max_i \left| 1 - \frac{\phi_i^l}{\phi_i^{l+1}} \right|. \tag{10}$$

3.1. 2-D version of McCoy-Larsen problem

- Uniform, isotropically scattering 8cm × 8cm rectangular box
- $c=0.98$
- S_8
- Error criteria : 10^{-4}

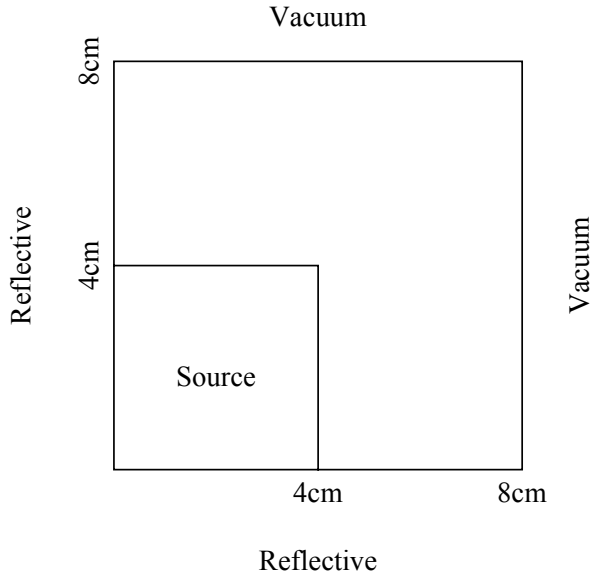

Figure 2. 2-D version of McCoy-Larsen problem

TABLE 1. Number of Iterations

σ_t	SI	CMADR(DD)
0.01	5	3
0.01	17	5
0.50	81	5
1.00	161	6
2.00	-[a]	-

[a] : stop due to negative fixup

3.2. Iron-water benchmark problem

- A diagonally symmetric and isotropically scattering medium
- 20 × 20 mesh divisions
- *c=0.994* for water *and c=0.831* for iron
- S_8
- Error criteria : 10^{-5}

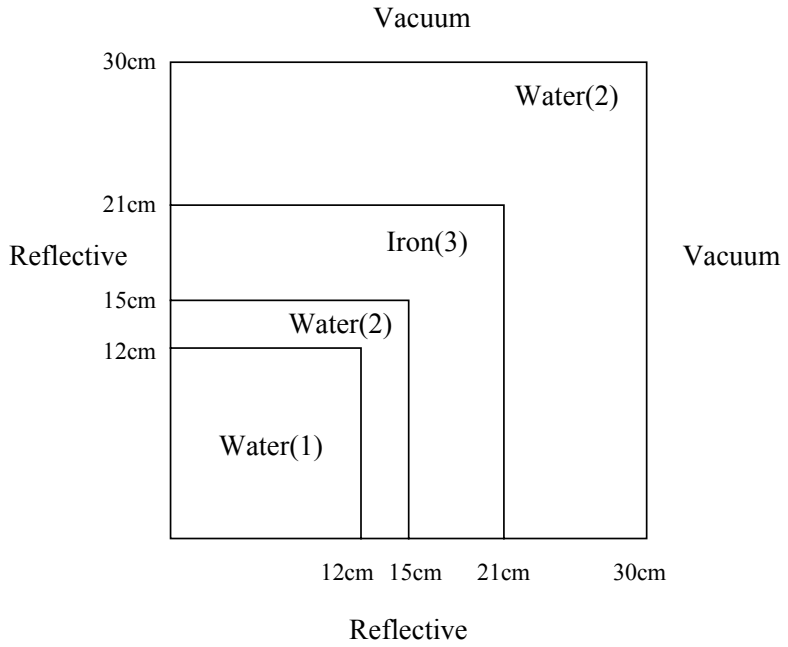

Figure 3. Iron-water benchmark problem

TABLE 2. Material properties for benchmark problem

Composition	σ(cm^{-1})	c	Source strength
1 (water)	3.33	0.994	1.0
2 (water)	3.33	0.994	0.0
3 (iron)	1.33	0.831	0.0

TABLE 3. Number of Iterations

	SI	CMADR(C-C)
Number of iterations	1061	4

3.3. IAEA IER-2 benchmark problem

- Two fueled regions, two absorbing regions
- 20 × 20 mesh divisions
- All vacuum boundaries
- S_8
- Error criteria : 10^{-4}

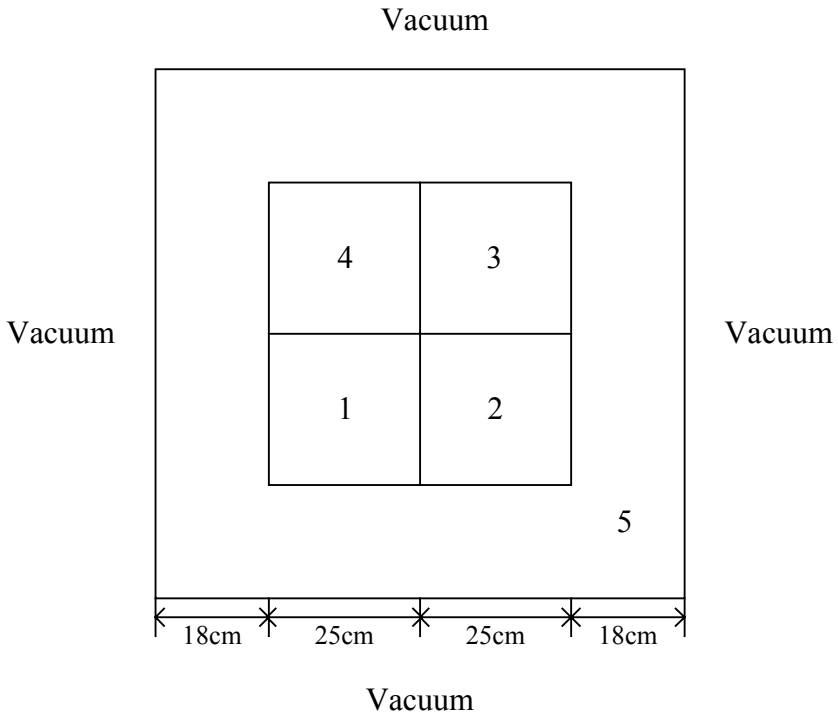

Figure 4. IAEA EIR-2 benchmark problem

TABLE 4. Material properties for benchmark problem

Composition	σ(cm^{-1})	σ_s(cm^{-1})	Source strength
1	0.60	0.53	1.0
2	0.48	0.20	0.0
3	0.70	0.66	1.0
4	0.65	0.50	0.0
5	0.90	0.89	0.0

TABLE 5. Number of Iterations

	SI	CMADR(C-C)
Number of Iterations	1724	4
Computing Time [a] (Sec)	351.05	16.01
Speed up	1	21.93

[a] : SUNBLADE2000 workstation

4. Conclusions and Further Works

In this paper, coarse-mesh angular dependent rebalance acceleration (CMADR) method was successfully implemented in *x-y* geometry problems. CMADR method was successfully applied to FDM calculation with DD scheme and C-C transport nodal calculations. The benchmark results show that CMADR method can be used effectively in two-dimensional neutron transport calculations.
Finally, the following further works are in progress: i) Fourier analysis for *x-y* geometry, ii) implementation of Krylov subspace method, and iii) increasing of coarseness.

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