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Estimation of the Power Peaking Factor in a Nuclear Reactor Using Fuzzy Neural Networks

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Abstract

The local power density should be estimated accurately to prevent the fuel rod from being melted. The local power density at the hottest part of a hot fuel rod is more important information than the local power density at any other position in a reactor core, which is described by the power peaking factor. Therefore, in this work, the power peaking factor (Fq) that indicates the highest local power density in a reactor core is estimated by fuzzy neural networks using lots of measured signals of the reactor coolant system. The fuzzy neural network are trained using a training data set and are verified with another verification data set. The fuzzy neural networks are applied to the first cycle of the Yonggwang 3 nuclear power plant. The estimation accuracy of the power peaking factor is 1.02 % based on the relative 2σ error by using the fuzzy neural networks without in-core neutron flux sensors signal input and 0.38% with in-core neutron flux sensors signals, which is accurate enough to be used in LPD protection and monitoring.

1. Introduction

The calculation of the LPD and Departure from Nucleate Boiling Ratio (DNBR) is two major functions of CPC and COLSS that play each role in protection and monitoring systems. COLSS monitors the operating limits of a reactor core including LPD and DNBR and provides related information to operators. COLSS is a program that runs in the Plant Monitoring System (PMS) computer that helps plant operators to monitor Limiting Conditions for Operation (LCOs) specified in the technical specifications. However, COLSS carries out only a monitoring function about an operating limit of a core and does not provide nuclear reactor protection functions. On the other hand, CPC which provides the nuclear reactor protection functions calculates faster than COLSS but generates more conservative values. Therefore, CPC provides lower DNBR and higher LPD values than COLSS. COLSS periodically adjusts CPC based on operating variables that are accurately calculated by COLSS including power level, reactor coolant system flow, etc. LPD should be estimated accurately to prevent the fuel rod from being melted. LPD at the hottest part of a hot fuel rod is more important than the local power density at any other position in a reactor core, which can be explained by the power peaking factor (Fq). The DNBR studies have been extensively performed (Han, 1999, In, 2002, Kim, 1997, Lee, 1998, Lee, 2002, Na, 1999 and Na, 2000). In the meanwhile, the LPD research almost has not been performed using artificial intelligence methods which have been extensively used in a lot of engineering problems.

Therefore, the objective of this work is to predict the power peaking factor in a reactor core using the measured signals (in particular, including in-core neutron sensor signals) of the reactor coolant system by applying fuzzy neural networks according to operating conditions. The neural networks have extensively and successfully been applied to a variety of engineering problems. Fuzzy neural networks should be optimized to accomplish the good monitoring performance of the local power density.

The used output and input data are the power peaking factor values (Fq) in a reactor core and a lot of operating conditions which are characterized by reactor power, core inlet temperature, pressurizer pressure, coolant flowrate of a reactor core, axial offset, in-core neutron sensor signals, and a variety of control rod positions. The Fq value in a reactor core is predicted by the developed fuzzy neural networks using these various operating condition data as the inputs to the fuzzy neural networks. The proposed power peaking factor estimation algorithm is verified by using the nuclear and thermal data acquired from numerical simulations of the Yonggwang 3 nuclear power plant.

2. Fuzzy Neural Networks

In this work, the fuzzy neural networks that are most popular in the function approximation are used to predict the power peaking factor. A system that consists of a fuzzy inference system implemented in the framework of neural network is usually called an adaptive network-based fuzzy inference system (ANFIS) or fuzzy neural networks (Jang, 1993). The training of the fuzzy neural network is accomplished by a hybrid method combined with a backpropagation algorithm and a least-squares algorithm. Also, a first-order Sugeno-Takagi type (Takagi and Sugeno, 1985) fuzzy inference system is used where the i -th rule can be described as follows:

$$\text{If } x_1 \text{ is } A_{i1} \ AND \cdots AND \ x_m \text{ is } A_{im}, \text{ then } \hat{y}^i \text{ is } f^i(x_1, \cdots, x_m), \quad (1)$$

where x_j is the input variables to the fuzzy neural network (j =1, 2, ..., m ; m = number of input variables), A_{ij} the membership functions for the antecedent of the i -th rule and j -th input (i = 1, 2, ..., n ; n = the number of rules), and $\hat{y}^i$ the output of the i -th rule.

In Eq. (1), the *if* part is fuzzy linguistic, while the *then* part is crisp. Usually $f^i(x_1, \cdots, x_m)$ is a polynomial in the input variables but it can be any function as long as it can appropriately describe the output of the fuzzy inference system within the fuzzy region specified by the antecedent of the rule. In this work, the symmetric Gaussian membership function is used. The output of an arbitrary i -th rule, f^i , consists of the first-order polynomial of inputs as given in Eq. (2).

$$f^i(x_1, \cdots, x_m) = \sum_{j=1}^{m} q_{ij} x_j + r_i , \tag{2}$$

where

q_{ij} = the weighting value of the j -th input on the i -th rule output,

r_i = the bias of the i -th output.

The output of a fuzzy inference system with n rules is weighted sum of the consequent of all the fuzzy rules. The estimated signal from the fuzzy inference system is given by:

$$\hat{y} = \sum_{i=1}^{n} \overline{w}^i f^i = \mathbf{w}^T \mathbf{q} , \tag{3}$$

where

$$\overline{w}^i = \frac{w^i}{\sum_{i=1}^{n} w^i} ,$$

$$w^i = \prod_{j=1}^{m} A_{ij}(x_j) ,$$

$$\mathbf{q} = \left[q_{11} \cdots q_{n1} \cdots \cdots q_{1m} \cdots q_{nm} \;\; r_1 \cdots r_n \right]^T ,$$

$$\mathbf{w} = \left[\overline{w}^1 x_1 \cdots \overline{w}^n x_1 \cdots \cdots \overline{w}^1 x_m \cdots \overline{w}^n x_m \;\; \overline{w}^1 \cdots \overline{w}^n \right]^T .$$

The superscript i indicates that the parameters are related to the i -th rule. Figure 1 shows a fuzzy neural network.

The back-propagation algorithm is a general method for recursively training the fuzzy neural networks. It uses a gradient descent method. The gradient descent method tunes the antecedent parameters (center position and sharpness of membership functions) so that the predefined objective function E is minimized. In order to train an antecedent parameter a_{ij} , the following iterative calculation is used:

$$a_{ij}(t+1) = a_{ij}(t) - \eta_a \frac{\partial E}{\partial a_{ij}} \Big|_t , \tag{4}$$

where $E = \sum_{k=1}^{N} \left(y_k - \hat{y}_k \right)^2$, i = 1, 2, ..., n , j = 1, 2, ..., m , $t = 0, 1, 2, \cdots$, and η_a is a learning rate for a parameter a. The gradient descent method is very stable when the learning rate is small.

If we fix the antecedent parameters of the fuzzy inference system by the back-propagation algorithm, the resulting fuzzy neural networks is equivalent to a series of expansions of some basis functions. This basis function expansion is linear in its adjustable parameters. Therefore, we can use the least-squares method to determine the remaining parameters (consequent parameters q_{ij} and r_i). If a total number of N input-output training data are given, from Eq. (3) the consequent parameters are chosen to minimize the following cost function:

$$J = \frac{1}{2}\sum_{k=1}^{N}\left(y_k - \hat{y}_k\right)^2 = \frac{1}{2}(\mathbf{y}-\hat{\mathbf{y}})^2 = \frac{1}{2}(\mathbf{y}-\mathbf{Wq})^2, \tag{5}$$

where

$$\mathbf{y} = \left[y_1\ y_2\ \cdots\ y_N\right]^T,$$
$$\hat{\mathbf{y}} = \left[\hat{y}_1\ \hat{y}_2\ \cdots\ \hat{y}_N\right]^T,$$
$$\mathbf{q} = \left[q_{11}\cdots q_{n1}\cdots\cdots q_{1m}\cdots q_{nm}\ r_1\cdots r_n\right]^T,$$
$$\mathbf{W} = \left[\mathbf{w}_1\ \mathbf{w}_2\ \cdots\ \mathbf{w}_N\right]^T,$$
$$\mathbf{w} = \left[\overline{w}^1 x_1\cdots\overline{w}^n x_1\cdots\cdots\overline{w}^1 x_m\cdots\overline{w}^n x_m\ \overline{w}^1\cdots\overline{w}^n\right]^T.$$

$\mathbf{y}$ is the output data vector, $\mathbf{q}$ is the parameter vector, and the matrix $\mathbf{W}$ includes the input data. The equation for minimizing the cost function is as follows:

$$\mathbf{y} = \mathbf{Wq}. \tag{6}$$

The fuzzy neural network output is represented by the $N\times(m+1)n$-dimensional matrix $\mathbf{W}$ and the $(m+1)n$-dimensional parameter vector $\mathbf{q}$. The parameter vector $\mathbf{q}$ in Eq. (6) is solved by using the pseudo-inverse of the matrix $\mathbf{W}$ as follows:

$$\mathbf{q} = \left(\mathbf{W}^T\mathbf{W}\right)^{-1}\mathbf{W}^T\mathbf{y}. \tag{7}$$

3. Application to the Power Peaking Factor Estimation

The proposed algorithm was applied to the first fuel cycle of the Yonggwang unit 3 PWR plant. The used data were obtained by running the MASTER (Cho, 1999) and COBRA (Wheeler, 1976) codes based on some assumptions. The data comprise a total of 21875 input-output data pairs $\left(x_1, x_2, \cdots, x_{23}, y_r\right)$. The data are divided into both the training and verification data sets and also, these data sets are divided into two kinds of data with positive axial offset (AO) and negative AO. The training data set comprise one third of the acquired input-output data pairs and the verification data set comprises two thirds of the total data. $x_1, x_2, \cdots, x_{23}$ represent the reactor power, core inlet temperature, coolant pressure, mass flowrate, axial offset, 12 in-core neutron sensor signals, R1, R2, R3, R4, R5 and P control rod positions, and y_r is a power peaking factor (Fq) in the reactor core. The used in-core detector signals are ones located on the central part of the core (16, 20, 23, 26 instrument number locations as indicated on Figure 2 and 3 axial positions). The R1, R2 and so on are the names of the control rod groups. The ranges of the input signals that are used for training, in this work, are described in Table 1. The fuzzy neural networks are trained for both data sets divided into two positive (relatively high power at a top part of a reactor core) and negative AOs, which results in smaller errors compared with that of only one summed data set.

The number of rules of fuzzy neural networks is 4 for negative AO cases and 5 for positive AO cases. The antecedent parameters such as membership function parameters are optimized by the back-propagation method and the consequent parameters q_{ij} and r are optimized by the least-squares method.

Figure 3 shows the power peaking factors for ~7300 verification cases and their estimation error histogram (without and with in-core detector signals) for *negative* AO values. If we do not use the in-core neutron flux sensor signals, the relative two-sigma error is 0.99 percent relatively based on the difference between the maximum and minimum values of the used data and its maximum error is 26.78 percent (see Table 2). The relative two-sigma error was calculated based on the normal distribution since the error histograms resemble the normal distribution (refer to Figure 3 (b) and (c)). If we use the in-core neutron flux sensor signals, the relative two-sigma error is 0.35 percent and its maximum error is 5.44 percent. Figure 4 shows the power peaking factors and their estimation error histogram (without and with in-core detector signals) for *positive* AO values. If we do not use the in-core neutron flux sensor signals, the relative two-sigma error is 2.45 percent and its maximum error is 17.41 percent. If we use the in-core neutron flux sensor signals, the relative two-sigma error is 0.97 percent and its maximum error is 6.47 percent. In case we consider the relative two-sigma error for negative and positive AOs data together (see Figure 5 and Table 2), the relative two-sigma error is 1.02 percent without in-core sensor signals and 0.38 percent with in-core sensor signals. It is known that the use of SPND signals reduces the estimation error over about two times compared to that not using the SPND signals.

It is important to verify the fuzzy neural networks for verification data that had not been used in the training stage. It is known that the two-sigma error calculated by the fuzzy neural networks for the verification data is similar to the two-sigma error for the training data (see Table 2). Therefore, if the fuzzy neural networks are trained first using data at a variety of operating conditions, they can accurately estimate power peaking factors for any other operating data.

4. Concluding Remarks

In this work, fuzzy neural networks has been developed and applied to the estimation of the power peaking factor in the reactor core. The fuzzy neural networks are trained by using the data set prepared for training (training data) and verified by using another data set different (independent) from the training data. And also, two fuzzy neural networks are trained for both data sets divided into two positive and negative Aos, respectively. The developed fuzzy neural networks were applied to the first fuel cycle of the Yonggwang unit 3 PWR plant. The relative two-sigma error of the estimated power peaking factor is 0.38 percent when in-core neutron flux detector signals are used and 1.02 percent when they are not used. The use of SPND signals as input signals to the fuzzy neural networks reduces the estimation error over about two times compared to that not using the SPND signals. In summary, it is known that the fuzzy neural network is sufficiently accurate to be used in a power peaking factor monitoring.

Acknowledgement

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Table 1. Input and output signal ranges.

Input signals	*Nominal values*	*Ranges*
Reactor power (%)	100%	80 ~ 110
Inlet temperature (^{o}C)	295.8	290.5 ~ 301.7
Pressure (bar)	155.17	131.0 ~ 160.0
Mass flowrate (kg/m^2-sec)	3565.0	2994.6 ~ 4135.4
Axial offset	-	-0.645 ~ 0.536
Simulated in-core detector signals (12 different positions)	-	8.0 ~ 342.0
R1 control rod positions (cm)	-	0 ~ 381
R2 control rod positions (cm)	-	0 ~ 381
R3 control rod positions (cm)	-	0 ~ 381
R4 control rod positions (cm)	-	0 ~ 381
R5 control rod positions (cm)	-	0 ~ 381
R12 control rod positions (cm)	-	0 ~ 381
P control rod positions (cm)	-	0 ~ 381
Output signal	*Nominal value*	*Range*
Fq	-	1.928 ~ 4.511

Table 2. Results of the fuzzy-neural networks.

<table>
<tr><th colspan="2" rowspan="2"></th><th colspan="2">Training data</th><th colspan="2">Verification data</th></tr>
<tr><th>Relative maximum error (%)</th><th>Relative 2σ error (%) [1)]</th><th>Relative maximum error (%)</th><th>Relative 2σ error (%)</th></tr>
<tr><td rowspan="3">Without in-core sensor signals</td><td>Negative axial offset</td><td>6.99</td><td>0.73</td><td>26.78</td><td>0.99</td></tr>
<tr><td>Positive axial offset</td><td>6.46</td><td>2.33</td><td>17.41</td><td>2.45</td></tr>
<tr><td>Total</td><td>6.99</td><td>0.87</td><td>26.78</td><td>1.02</td></tr>
<tr><td rowspan="3">With in-core sensor signals</td><td>Negative axial offset</td><td>0.71</td><td>0.28</td><td>5.44</td><td>0.35</td></tr>
<tr><td>Positive axial offset</td><td>2.80</td><td>0.91</td><td>6.47</td><td>0.97</td></tr>
<tr><td>Total</td><td>1.19</td><td>0.34</td><td>5.44</td><td>0.38</td></tr>
</table>

1) Relative values based on the difference between the maximum value and the minimum value of the used data

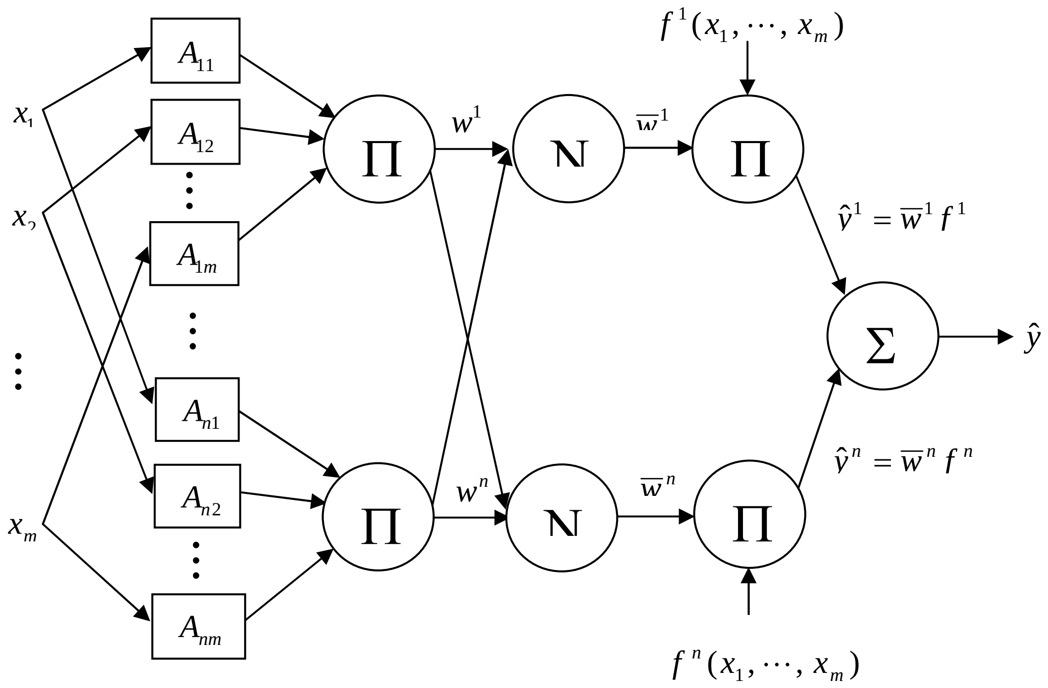


Fig. 1. Fuzzy neural networks.

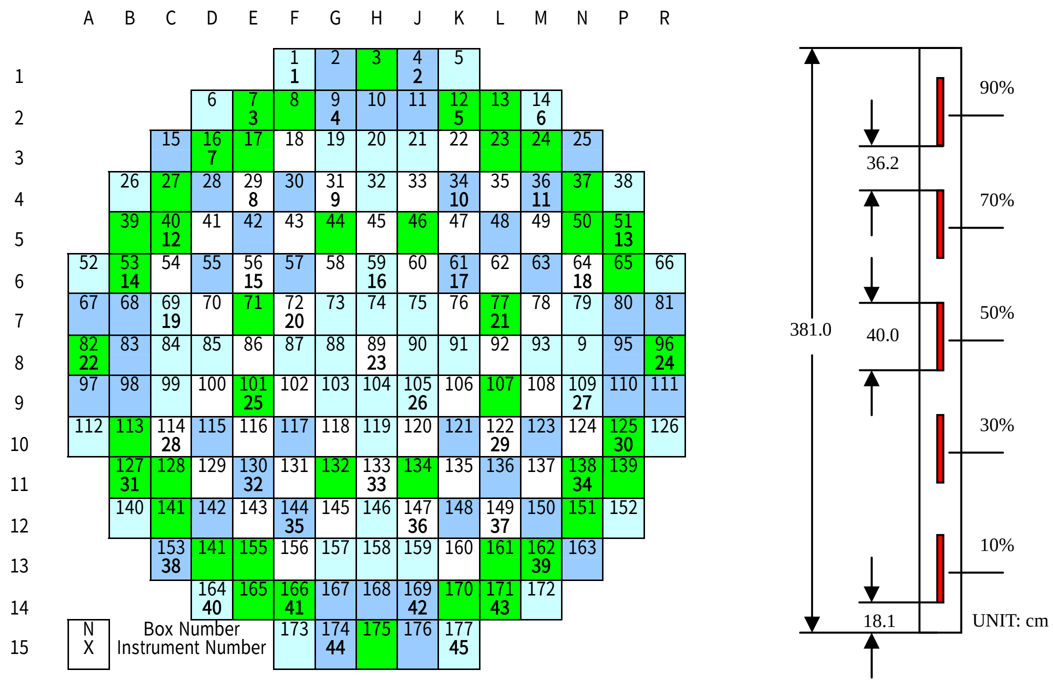
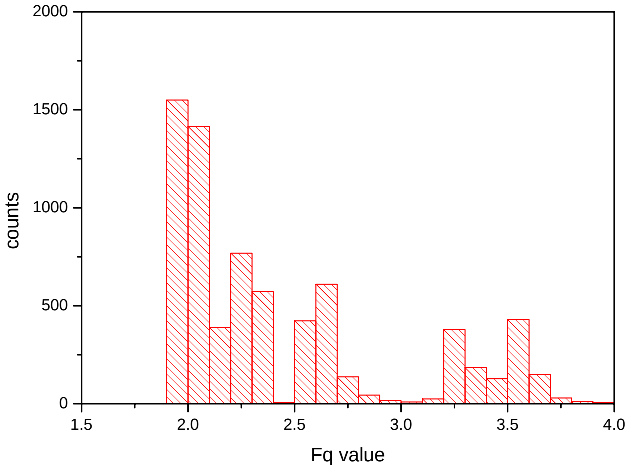
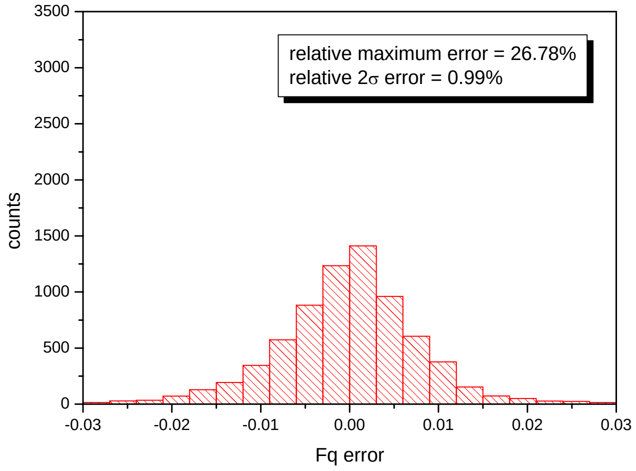


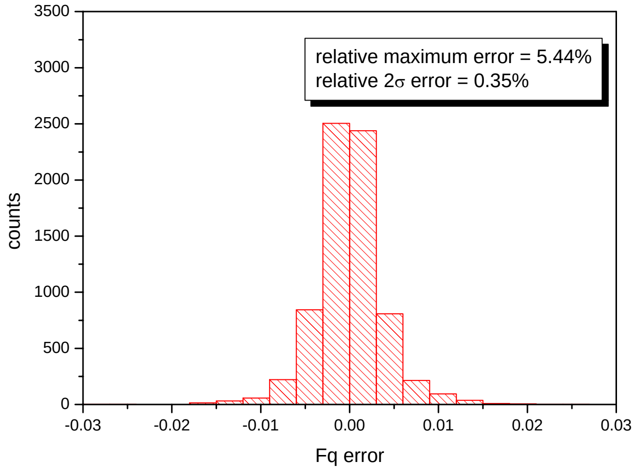
Fig 2. Fixed rhodium in-core detector location of YGN-3.



(a) Actual Fq histogram

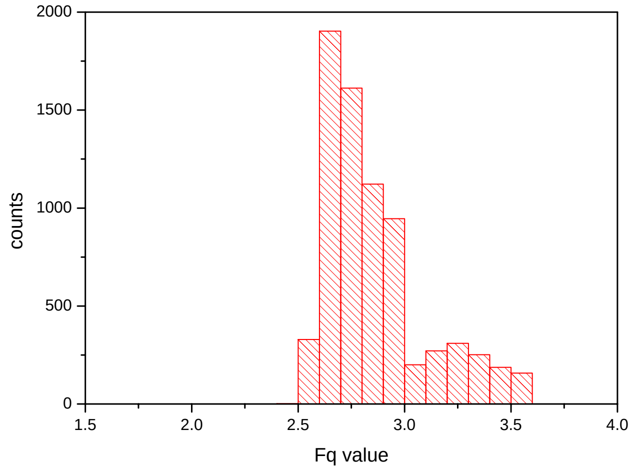


(b) Error histogram between actual Fq and estimated Fq (without SPND signals)

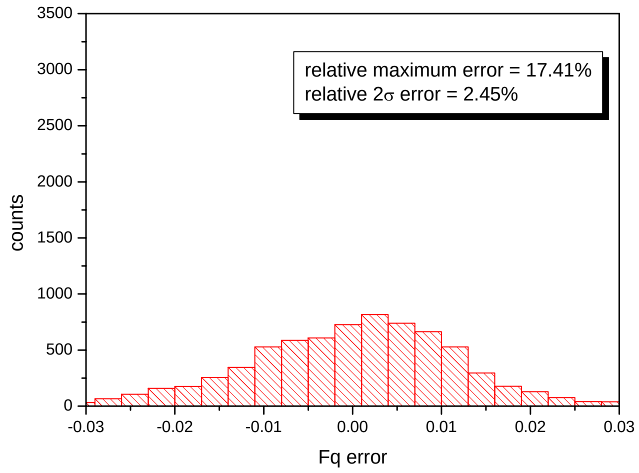


(c) Error histogram between actual Fq and estimated Fq (with SPND signals)

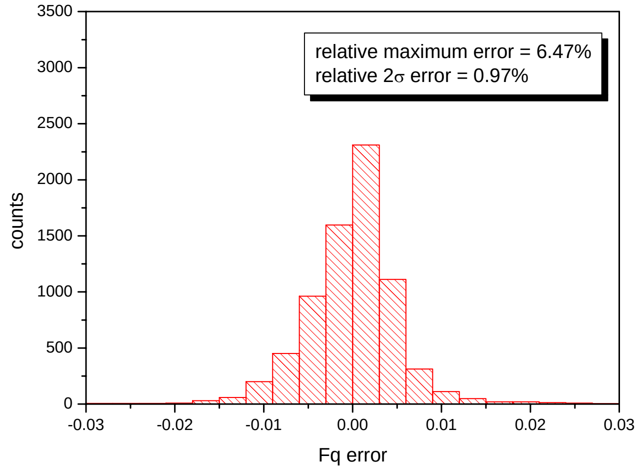
Fig. 3. Estimation performance of fuzzy neural networks for *negative* axial offset data.



(a) Actual Fq histogram

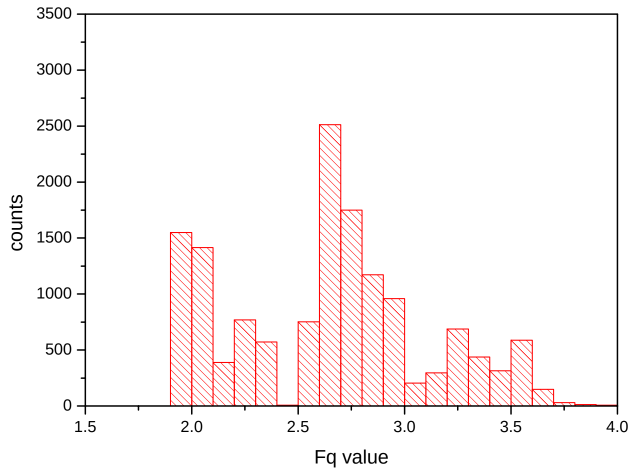


(b) Error histogram between actual Fq and estimated Fq (without SPND signals)

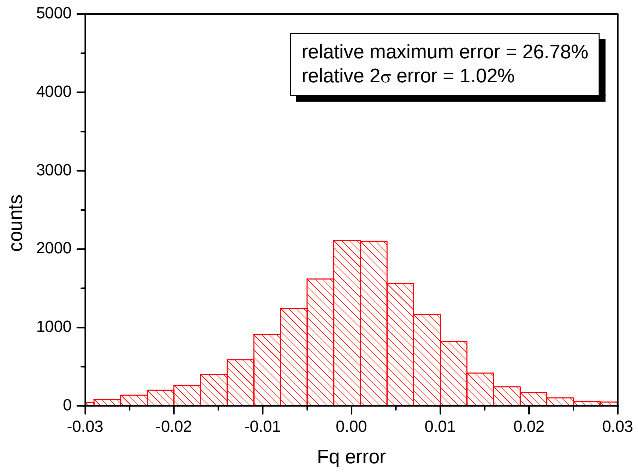


(c) Error histogram between actual Fq and estimated Fq (with SPND signals)

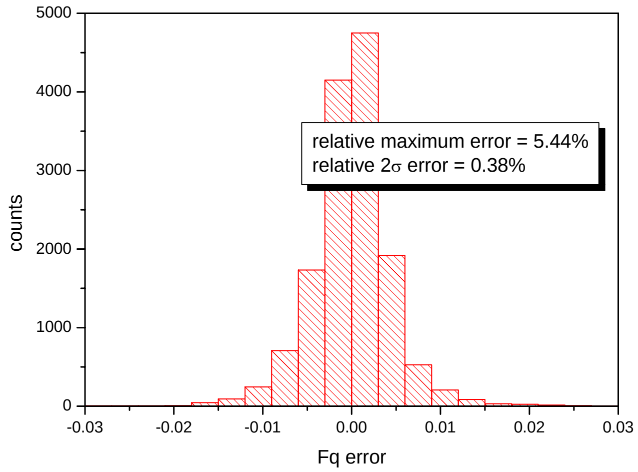
Fig. 4. Estimation performance of fuzzy neural networks for *positive* axial offset data.



(a) Actual Fq histogram



(b) Error histogram between actual Fq and estimated Fq (without SPND signals)



(c) Error histogram between actual Fq and estimated Fq (with SPND signals)

Fig. 5. Estimation performance of fuzzy neural networks (including *positive* and *negative* axial offset data).