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# A Superelement-Sweeping Method for Neutron Transport Calculations in Heterogeneous Geometry

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**Abstract**

In this paper, we propose a new $S_N$ sweeping method called superelement-sweeping for 2-D neutron transport calculation. Superelement is a unit of sweeping calculation and may consist of heterogeneous meshes. Unlike existing $S_N$ sweeping methods, superelement-sweeping method performs sweeping calculation with point value angular fluxes of a superelement. This superelement-sweeping method can treat heterogeneous geometries and mixed mesh shapes more easily than existing sweeping methods.

The paper provides a description of the new sweeping method and its numerical results of 2-D homogeneous and heterogeneous problems.

## I. INTRODUCTION

In the neutron transport calculation, $S_N$ method has been widely used with its simplicity. But its application areas are limited to the problems whose computational meshes are rectangular or triangular. For complex geometry problems including circular meshes, the method of characteristics(MOC)[1−3] and collision probability method (CPM)[4] are popular, but they cost much calculation time and have difficulties in direct extending to three dimensional problems.

In this paper, we propose a new sweeping method which can treat heterogeneous problems while preserving simplicity of $S_N$. This method is different from existing $S_N$ sweeping methods in the following viewpoints : 1) performs sweeping calculation with angular fluxes at corner points instead of edge average fluxes in existing methods, 2) performs sweeping calculation on a superelement which is allowed to consist of heterogeneous meshes, while one homogeneous mesh is treated in one sweeping in existing methods, 3) assumes flux distribution on a superelement instead of average angular flux on a computational mesh in existing methods.

The main difference is to assume flux distributions. Because we assume distributions on a superelement with heterogeneous geometry, accuracy of calculation may be lower than MOC and CPM. But calculation is much simpler than those methods and we can obtain continuous flux distributions in the entire of the problem.

In this paper, we assumed bi-linear flux distribution with basis functions of the finite element method(FEM)[5]. Thus it is expected that solution can be acceptable if flux shape is monotonic in a superelement. We also expect that use of small size of superelements and high order basis functions would provide better solution. To demonstrate efficiency and accuracy, we tested the method on a small test problem with homogeneous geometry and on a heterogeneous 2-D OECD benchmark problem C5G7 MOX[6].

## II. METHOD

The two-dimensional within-group transport equation is given as follows :

$$\mu_n \frac{\partial \psi_{g,n}(x,y)}{\partial x} + \eta_n \frac{\partial \psi_{g,n}(x,y)}{\partial y} + \sigma_g \psi_{g,n}(x,y) = q_{g,n}(x,y), \tag{1}$$

where standard notations[7] are used. Let us consider superelement in Fig. 1 which is a unit of sweeping calculation, that as an example, consists of heterogeneous three (mixed-shape) computational meshes.

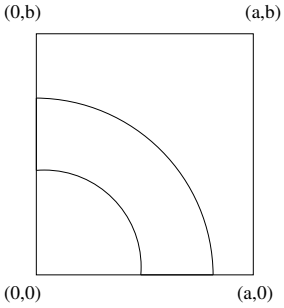


Figure 1: Rectangular superelement with heterogeneous geometry

In this superelement, we assume flux distribution as follows, dropping direction and energy group indices, $n$ and $g$ :

$$\psi(x,y) = \sum_{i=1}^{4} f_i(x,y) p_i, \tag{2}$$

where

$$\begin{aligned} p_1 &= \psi(0,b), \\ p_2 &= \psi(0,0), \\ p_3 &= \psi(a,0), \\ p_4 &= \psi(a,b), \end{aligned} \tag{3}$$

$$
\begin{aligned}
f_1(x,y) &= \frac{1}{ab}(a-x)y, \\
f_2(x,y) &= \frac{1}{ab}(a-x)(b-y), \\
f_3(x,y) &= \frac{1}{ab}x(b-y), \\
f_4(x,y) &= \frac{1}{ab}xy.
\end{aligned}
\tag{4}
$$

Eq.(2) consists of bi-linear basis functions of finite element method type in rectangular geometry. By integrating Eq.(1) over a superelement if there are $N$ computational meshes in a superelement, we obtain

$$
\begin{aligned}
&\int_0^b dy \int_0^a dx \left( \mu \frac{\partial \psi(x,y)}{\partial x} + \eta \frac{\partial \psi(x,y)}{\partial y} + \sigma(x,y)\psi(x,y) \right) = \int_0^b dy \int_0^a dx q(x,y), \\
&\int_0^b dy \int_0^a dx \left( \mu \frac{\partial \psi(x,y)}{\partial x} + \eta \frac{\partial \psi(x,y)}{\partial y} \right) + \sum_{m=1}^{N} \int_{V_m} dA \sigma_m \psi(x,y) = \sum_{m=1}^{N} \int_{V_m} dA q(x,y),
\end{aligned}
\tag{5}
$$

where material properties are constant in a computational mesh. Eq.(4) can be written as follows using Eq.(2) :

$$
\sum_{i=1}^{4} F_i p_i = Q, \tag{6}
$$

where

$$
F_i = \int_0^b dy \int_0^a dx \left( \mu \frac{\partial f_i(x,y)}{\partial x} + \eta \frac{\partial f_i(x,y)}{\partial y} \right) + \sum_{m=1}^{N} \int_{V_m} dA \sigma_m f_i(x,y), \tag{7}
$$

and

$$
Q = \sum_{m=1}^{N} \int_{V_m} dA q(x,y). \tag{8}
$$

These $F$ values can be pre-calculated before iteration. Now sweeping calculation is possible with Eq.(6). Fig. 2 shows the scheme of sweeping calculation for $\mu > 0, \eta > 0$.

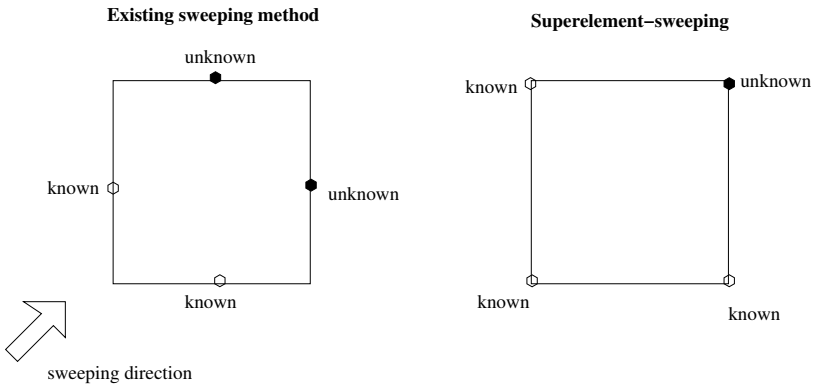


Figure 2: Sweeping schemes of existing and superelement-sweeping methods

In existing $S_N$ sweep methods, they calculate two unknowns with two knowns, using auxiliary equations(approximate) such as diamond difference scheme (DD). In the case of DD, unknowns

are obtained as follows :

$$\begin{aligned} \psi_{i+1/2,j} &= 2\psi_{ij} - \psi_{i-1/2,j}, \\ \psi_{i,j+1/2} &= 2\psi_{ij} - \psi_{i,j-1/2}, \end{aligned} \tag{9}$$

where

$$\psi_{ij} = \frac{\frac{2\mu}{\Delta x_i}\psi_{i-1/2,j} + \frac{2\eta}{\Delta y_j}\psi_{i,j-1/2} + q_{ij}}{\sigma_{ij} + \frac{2\mu}{\Delta x_i} + \frac{2\eta}{\Delta y_j}}. \tag{10}$$

In the superelement-sweeping method, there are one unknown ($p_4$) and three knowns ($p_1$, $p_2$ and $p_3$). We can calculate unknown $p_4$ with Eq.(6) :

$$p_4 = (Q - F_1 p_1 - F_2 p_2 - F_3 p_3)/F_4. \tag{11}$$

Average angular flux on a computational mesh can then be written as follows :

$$\psi_m = \sum_{i=1}^{4} \int_{V_m} dA f_i(x,y) p_i. \tag{12}$$

## III. SUPERELEMENT WITH MULTIPLE SHAPE FUNCTIONS

If superelement contains very heterogeneous geometry and material composition, bi-linear basis functions on rectangular geometry would not be enough for accurate calculations. Thus we may consider multiple shape functions on a superelement.

To implement multiple shape functions, we could consider more unknowns in a superelement. Let us consider one more unknown as in Fig. 3 when $\mu,\ \eta\ > 0$.

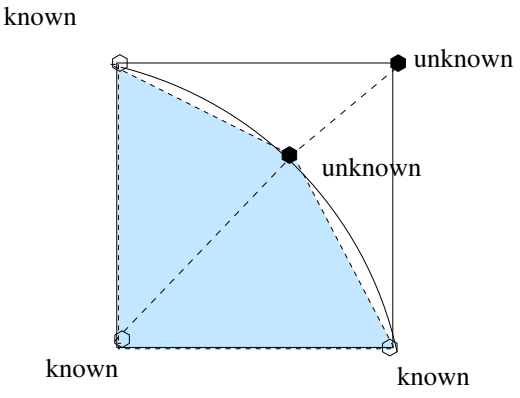


Figure 3: Superelement with two unknowns

There are four triangular regions and two unknowns. So we can consider basis functions of triangular geometry. We need two equations to obtain two unknowns. The two equations can be obtained by integrating Eq.(1) over shaded region and unshaded region, respectively. Once

we calcaulte two unknowns, we can also calculate average angular flux similarly to Eq.(12) and continue the sweeping calculation.

## IV. NUMERICAL RESULTS

First we tested the method on a small test problem with two energy groups described in Fig. 4. Each cell has size of 1.0cm × 1.0cm. There are two types of cells and material properties are homogeneous in a cell. The material properties are listed in Table 1.

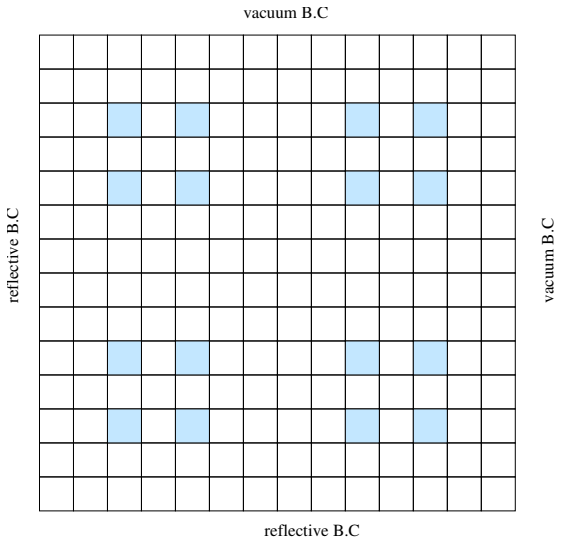


Figure 4: Configuration of the test problem

Table 1: Material properties

| | g | $\sigma_t$ | $\nu\sigma_f$ | $\sigma_{s,1\to g}$ | $\sigma_{s,2\to g}$ |
|---|---|---|---|---|---|
| 1 (white) | 1 | 0.2531 | 0.005925 | 0.233427 | 0.0000 |
| | 2 | 0.5732 | 0.098170 | 0.010690 | 0.514280 |
| 2 (shaded) | 1 | 0.2535 | 0.004820 | 0.233793 | 0.0000 |
| | 2 | 0.5797 | 0.08228 | 0.010950 | 0.524960 |

Results are compared with the TWODANT code. $S_{10}$ quadrature was selected for both calculations and each computational mesh size is 0.5cm × 0.5cm and iteration criteria were $10^{-6}$ for eigenvalue and scalar flux. The results are very similar. The eigenvalues are 0.4896751 from the TWODANT code and 0.4896750 from the superelement-sweeping method.

To test performance on heterogeneous problems, we chose the 2-D OECD benchmark problem C5G7 MOX which has seven energy group cross sections. $S_{10}$ quadrature was used and iteration criteria were $10^{-6}$ for eigenvalue and scalar fluxes. Table 2 shows the results.

It took 1216 outer iterations and 1100 sec with 2.8 GHz Pentium 4 machine. If we apply good acceleration in the calculation, it is expected that the computation time will be reduced to less than 1 minute, although its accuracy should be improved. To improve accuracy, high order shape

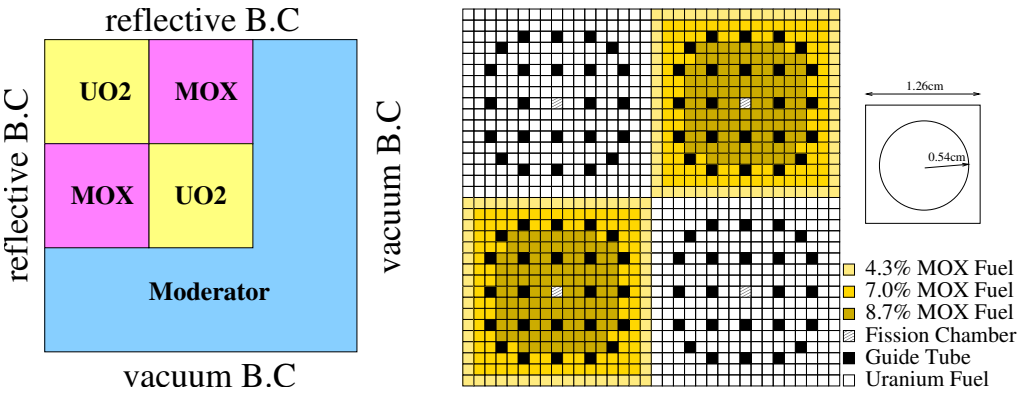


Figure 5: Configuration of the 2-D OECD benchmark problem

Table 2: Results of the OECD benchmark problem

| | $k_{eff}$ | Max./Min. pinpower | RMS error(%) |
|---|---|---|---|
| superelement-sweeping | 1.18858 | 2.579/0.230 | 1.99 |
| Reference | 1.18655 | 2.498/0.232 | - |

approximation can be considered.

## V. CONCLUSIONS

A new sweeping scheme of $S_N$ methods was developed and described in this paper. It performs sweeping calculation with angular fluxes at corner points and superelement which is allowed to consist of heterogeneous meshes as a unit of sweeping calculation. It assumes flux distribution in a superelement and we can obtain continuous flux distribution in the entire problem as a result of the sweeping calculation.

For the test problem with homogeneous geometry, superelement-sweeping provides almost the same results compared with DD calculation. For the OECD benchmark problem with heterogeneous geometry, there are 0.17% error for eigenvalue and 1.99% RMS error for power distribution compared with reference values provided by Monte Carlo calculation.

The superelement-sweeping method shows good performance in computation time, but it needs to improve accuracy. To improve accuracy, several methods such as high order basis functions being considered.

## ACKNOWLEDGMENT

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