Monte Carlo Perturbation Method for Geometrical Uncertainty Analysis

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1. Introduction

Recently, Monte Carlo (MC) method is widely used for nuclear design analysis because the geometric information of a given nuclear system can be handled directly. However, MC method requires a lot of computation time for a proper accuracy in spite of advance of hardware and software technology. Moreover, it is very difficult to estimate a sensitivity coefficient for nuclear safety and reactor design by MC method because the statistical uncertainty tends to be large.

Therefore, MC perturbation method[1] is introduced in order to reduce computing time in various MC sensitivity and uncertainty (S&U) analysis[2]. Takahashi proposed the MC perturbation method[3] for geometrical perturbation with cross section perturbation. In this study, the MC perturbation method based on number density perturbation is applied to perform geometrical uncertainty analysis.

2. Methodology

2.1 MC perturbation Formulation[1]

Eq. (1) shows the collision density equation of MC simulation by the Neumann series solution. P denotes the position $(\mathbf{r}, E, \mathbf{\Omega})$ in the phase space and $\psi(P)$ means the collision density.

$$\psi(P) = \tilde{S}_f(P) + \int dP' K(P' \to P)\psi(P') \quad (1)$$

where

$K(P' \to P)$ is transport kernel

$\tilde{S}_f(P)$ is first collision density of fission neutron

The variation of a tally Q due to a deviation of a input parameters, α can be expressed by

$$Q(\alpha + \Delta\alpha) - Q(\alpha) \equiv \delta Q(\alpha) = \sum_{n=1}^{\infty} \frac{1}{n!} \frac{d^n Q}{d\alpha^n} (\Delta\alpha)^n = \sum_{n=1}^{\infty} U_n (\Delta\alpha)^n \quad (2)$$

Assuming $q(P)$ is a response function of tally Q, the first order sensitivity coefficient, U_1 can be written by

$$U_1 = \frac{dQ}{d\alpha} = \sum_j \frac{d}{d\alpha}\left[\int_P dP q(P)\psi_j(P)\right]$$
$$= \sum_j \int dP_j \cdots \int dP_0 \left\{ u^{1q}(P_j) + \sum_{k=0}^{j} u^{1K}(P_{k-1} \to P_k) + u^{1S}(P'') \right\} \otimes \left\{ q(P_j) K(P_{j-1} \to P_j) \cdots K(P_0 \to P_1) \tilde{S}_f(P_0) \right\} \quad (3)$$

where

$$u^{1q}(P_j) \equiv \frac{1}{q(P_j)} \frac{\partial q(P_j)}{\partial \alpha}$$

$$u^{1K}(P_{k-1} \to P_k) \equiv \begin{cases} \dfrac{1}{K(P_{k-1} \to P_k)} \dfrac{\partial K(P_{k-1} \to P_k)}{\partial \alpha} & (k = 1, 2, \cdots) \\ \dfrac{1}{T(P'' \to P_0)} \dfrac{\partial T(P'' \to P_0)}{\partial \alpha} & (k = 0) \end{cases}$$

$$u^{1S}(P'') \equiv \frac{1}{S_f(P'')} \frac{\partial S_f(P'')}{\partial \alpha}$$

$u^{1q}(P_j)$ is the first order sensitivity of response function while $u^{1K}(P_{k-1} \to P_k)$ is the first order sensitivity of transport kernel. $u^{1S}(P'')$ is the first order sensitivity of fission source distribution. All the term for sensitivity can be calculated by the determination of α and $q(P)$. From Eq.(3), U_1 can be estimated by

$$\langle U_1 \rangle = \frac{1}{N} \sum_{i=1}^{N} \sum_j \left(u_j^{1q} + \sum_{k=0}^{j} u_k^{1K} + u_i^{1S} \right) w_{i,j} q_{i,j} \quad (5)$$

where

N = number of neutron histories.

$w_{i,j}$ = weight of the *i*-th neutron after *j*-th collision.

$q_{i,j}$ = response of Q from the *i*-th neutron after *j*-th collision.

Using U_1 which is updated on MC simulation, $\delta Q(\alpha)$ can be calculated. All the mathematical details for the MC perturbation are available in References 1.

2.2 Geometric Perturbation Analysis by MC Number Density Perturbation

In order to perform geometric perturbation analysis by MC perturbation, all the regions of a given system are divided into two regions. One is the non-perturbed region and the other is the perturbed region. The non-perturbed regions are modeled using average geometric parameters while the perturbed regions are set using the

geometric parameters including the uncertainties excluding the non-perturbed regions. In the perturbed regions, all the isotopic number density of the non-perturbed region, which is related with the uncertainty of geometric parameter, is perturbed to 100%, while that of the original region is perturbed to -100%.

3. Numerical Results

Fig. 1 shows the *Godiva* problem with one non-perturbed region and one perturbed region. In this problem, the uncertainty of the radius, σ(R), is set to 0.001 cm while the radius of fuel sphere is 8.741 cm. In this problem, the number densities of ^{235}U and ^{238}U were perturbed to 100%.

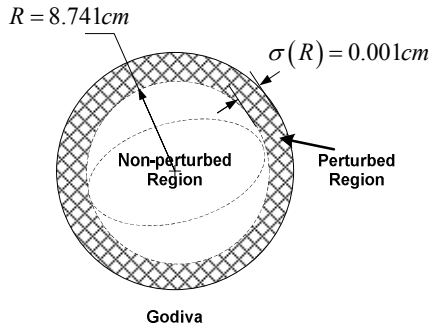

Fig. 1. *Govdia* problem.considering non-perturbed region and perturbed region for geometric perturbation analysis

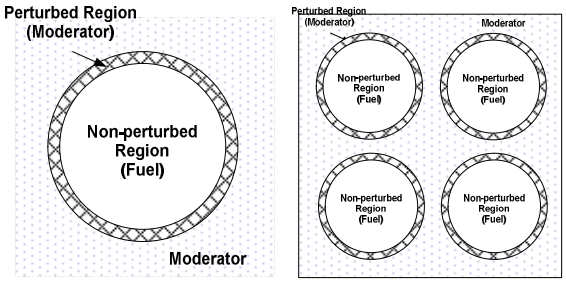

Fig. 2. *PWR fuel pin-cell problem*

Fig. 2 shows the configuration of *single pin-cell* and *2×2 pin-cell* problem. In the *single pin-cell* problem, the radius of the fuel pin is set to 0.46955±0.0050 cm. In the perturbed region, the number densities of ^{1}H and ^{16}O at the moderator region are perturbed to -100% and those of ^{235}U and ^{238}U at the fuel region were perturbed to 100%. In *2×2 pin-cell* problem, the configuration for each pin is equal to that of *single pin-cell* problem except the uncertainty of the radius of the fuel pin – 0.001cm.

σ(k_{eff})'s calculated by McCARD[4] Monte Carlo code for the three problems are listed in Table I. For the new MC perturbation method, the McCARD calculations using the ENDF/B-VI.8 libraries are performed using 10,000 neutron histories per cycle with 1,000 active cycles and 50 inactive cycles. To obtain reference solutions, 1,000 MC calculations by stochastic sampling method were performed. From the table, we can see that the results by the new MC method agree very well with the reference. Meanwhile, in the only *2x2 pin-cell* problem, the difference between the reference and direct subtraction method tends to be large for the statistical uncertainty.

Table I: σ(k_{eff})'s calculated by McCARD

Case	σ(k_{eff})		
	Direct Subtraction	***new MC perturbation***	***Reference (Sampling)***
Godiva	0.00559 ±0.00027	0.00588 ±0.00037	0.00585
Single Pin	0.00467 ±0.00020	0.00461 ±0.00085	0.00476
2×2 Pin	0.00059 ±0.00003	0.00038 ±0.00020	0.00039

4. Conclusions

In this study, a new approach to geometrical perturbation analysis is suggested by MC perturbation technique based on a nuclide number density. For three benchmarks, it is observed that the results by the new MC method agree very well with the reference by stochastic sampling method within 1 standard deviation. Using this new method, a small geometrical sensitivity coefficient can be estimated without a lot of computing time and a sophisticated processing.

In the near future, to compare with the existing MC approach[3] based on cross sections, a mathematical comparison study between two methods will be continued.

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