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Adaptive under relaxation factor of MATRA code for the efficient whole core analysis

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1. Introduction

General governing equations of subchannel code applying the thermo-hydraulic design are nonlinear like other equations based on the Navier-Stokes equation. The nonlinear requires solving a system of nonlinear algebraic equations at each iteration step. Such nonlinearities are handled in MATRA code using outer-iteration with Picard scheme. The Picard scheme involves successive updating of the coefficient matrix based on the previously calculated values. The scheme is a simple and effective method for the nonlinear problem but the effectiveness greatly depends on the under-relaxing capability.

Accuracy and speed of calculation are very sensitively dependent on the under-relaxation factor in outer iteration updating the axial mass flow using the continuity equation. The under-relaxation factor in MATRA is generally utilized with a fixed value that is empirically determined.

Adapting the under-relaxation factor to the outer iteration is expected to improve the calculation effectiveness of MATRA code rather than calculation with the fixed under-relaxation factor. The present study describes the implementation of adaptive under-relaxation within the subchannel code MATRA.

2. Methods and Results

2.1 Adaptive under-relaxation

Subchannel codes such as VIPRE-01, COBRA-IIIC, MATRA, used the Picard iteration to solve the nonlinear system of equations. The iteration process needs the under-relaxation factor to stabilize a solution in iteration.

Using this modification of the under-relaxation factor, the coefficient matrix is updated based on the axial mass flow rate computed from the equation:

$$m^{n+1}{}_{i,j} = m^{n}{}_{i,j} + \omega_m \left(\tilde{m}^{n+1}_{i,j} - m^{n}{}_{i,j} \right) \qquad (1)$$

where $m^n_{i,j}$ is the axial mass flux at node i, subchannel j, at the iteration step n. ω_m is under-relaxation factor. $\tilde{m}$ is the mass flow rate at the previous iteration.

The under–relaxation factor ω_m depends on the progress of the iterations as shown on Fig. 1. In this figure, function A represents the under-relaxation factor ω_m. The method is originally developed by T. Durbin[1] and usually applied to the water head analysis in water resource field. There is a similar relation between water head equation and mass conservation in subchannel code. It is possible to apply Durbin's adaptive under-relaxation method to the MATRA code. According to his method, the relation between under-relaxation factor and convergence is given by the function[1]:

$$\omega = \omega_{initial} + \left(1 - \omega_{initial}\right)\exp^{-\alpha(\delta-\varepsilon)} \quad \text{for } \delta > \varepsilon \qquad (2)$$

and

$$\omega = 1.0 \quad \text{for } \delta \le \varepsilon \qquad (3)$$

where

$$\delta = \max\left| m^{n+1}_{i,j} - m^{n}_{i,j} \right| \qquad (4)$$

and α is the shape factor for under relaxation function, δ is the maximum change in mass flow rate between two iterations, and ε is the convergence or closure criterion.

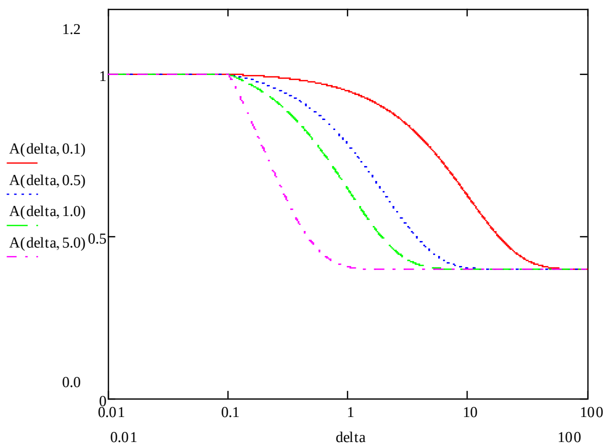

Fig.1. Under-relaxation function with varying shape factor

If the Picard iteration to update the mass flow rate diverges between two iterations, the under-relaxation is scaled downward using the shape factor.

2.2 Verification Problem

Verification of the adaptive under-relaxation factor(AURF) is performed with the SMART whole core and 1/8 lumping model. Verification cases to evaluate improvement of AURF are as shown on Table 1.

Table I: Verification problem case definition

Model	channel	Axial node	Operating Condition
SMT-1/8	21	40	SMART Normal operating condition
SMT-WC	16780	40	SMART Normal operating condition

2.3 Results of subchannel analysis

The performance of adaptive under-relaxation factor was tested using two different under relaxation factor, no under-relaxation and fixed under-relaxation. Fixed under-relaxation factor was used with the initial value of adaptive under-relaxation factor.

For each model, respective simulations were made with three difference under-relaxation factor model as shown on Table 2. In the comparisons with three models, iteration number and total CPU time was estimated. In case of lumping model, SMT-1/8, there is no significant difference in three models.

Table 2: Verification results with various under-relaxation

Model	CASE	Case Description	iterations	Total CPU time(sec)
SMT-1/8	1	No-URF	3	0.047
	2	Fixed URF	4	0.047
	3	AURF	4	0.047
SMT-WC	1	No-URF	100	Diverged
	2	Fixed URF	33	643
	3	AURF	15	287

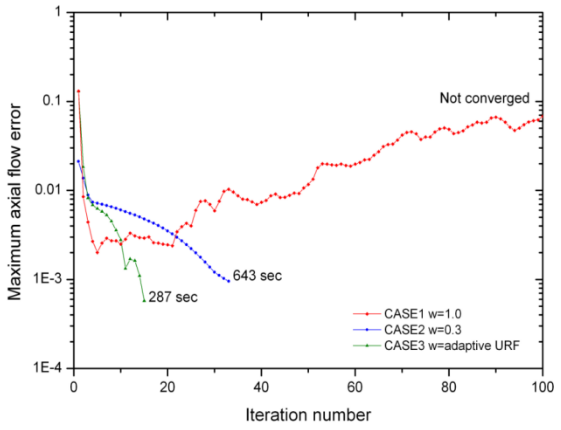

Fig.2. Comparisons for different under relaxation factor on the SMART whole core problem

The results for whole core model are shown on Fig. 2. For the case without under relaxation, convergence did not occur. For the simulation using a constant under-relaxation factor, optimal convergence occurred after 33 iterations. For the simulation using the proposed adaptive under-relaxation, convergence reached after 15 iterations. Compared with fixed relaxation factor, adaptive approach is the efficient as much as a factor 2. Additionally, the proposed approach has been applied to numerous other whole core problems with various operating condition. The experience has been that convergence always has been achieved. However, for some applications, a trial-and-error effort was required to find the under-relaxation parameter values like shape factor, updating duration, closure criteria that optimized convergence.

3. Conclusions

Picard iterations with adaptive under-relaxation can accelerate the convergence for mass conservation in subchannel code MATRA. The most efficient approach for adaptive under relaxation appears to be very problem dependent. However, for the present problem condition, the approach performed as well as or better than the other under-relaxation factors.

REFERENCES

[1] T. Durbin and D. Delemos, Adaptive underrelaxation of Picard iterations in ground water models, Ground Water, Vol. 45(5), pp. 648, (2007).