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Development of Measures for the Truncation Uncertainty in Fault Tree Analysis

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Abstract

The fault tree quantification uncertainty from the truncation error has been of great concern in the reliability evaluation of large fault trees and probabilistic safety analysis (PSA) in the nuclear field. This paper presents measures to estimate the amount of truncation error when quantifying fault trees with a truncation limit. The functions to calculate the measures are programmed into the new fault tree quantifier FTREX (Fault Tree Reliability Evaluation eXpert) and a Benchmark test was performed to show the efficiency of the measures. The developed measures are easily implemented into the existing fault tree solvers.

1. Introduction

Fault tree analysis is one of the most commonly used methods for the safety analysis of industrial systems. The traditional fault tree analysis is based on the minimal cut set (MCS) approach [1,2]. With the help of a Boolean algebra, a fault tree is converted into the OR combinations of numerous MCSs. The MCS is a minimum combination of failures that can lead to the top event.

In recent years, considerable progress has been made on improving the efficiency and accuracy of the fault tree methodology. The majority of fault trees can now be analyzed very quickly on personal computers. However, there can still be problems with very large fault trees for the nuclear and aerospace systems. The number of MCSs grows exponentially with the size of a fault tree, that is, the number of gates and events in the fault tree. Thus, if the fault tree consists of a large number of basic

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events and gates, then all the MCSs could not be calculated due to the limitations of the computational resources. A common practice in the evaluation of large fault trees is to truncate MCSs. During the computational process, the MCSs that have lower probability or frequency than the truncation limit are eliminated. The truncation limit is subjectively or heuristically selected by a reliability analyst. Thus, the top event probability might be significantly underestimated due to the truncation.

This source of fault tree quantification uncertainty from the truncation error has been of great concern in the reliability evaluation of large fault trees and PSA [3-5]. Several methods [6-8] have been investigated for the estimation of the truncation error. However, since these studies have overestimated the truncation error, there has is a great necessity for a more realistic method to estimate reasonably the truncation error.

Furthermore, importance measures, such as the risk achievement worth (RAW) [9] that is used for selecting risk-important structure, system, and component (SSC), can be significantly underestimated resulting from the truncation [5]. Thus, the truncation is the major source of a fault tree quantification uncertainty.

For the reliability evaluation of fault trees without the truncation, alternative ways have been investigated by using a binary decision diagram (BDD) [10,11]. However, large fault trees are still not efficiently solved since the size of a BDD structure exponentially increases according to the number of variables. Another problem is that the size of the final BDD structure is drastically dependent on the choice of the variable ordering for the BDD construction.

The main objective of this paper is to develop truncation uncertainty measures which can be used to estimate the amount of the truncation error. The developed measures are explained in Section 2. The functions to calculate the measures are programmed in the new fault tree quantifier FTREX [12,13]. The results of the Benchmark test are explained in Section 3.

2. Methodology

A typical fault tree quantification procedure is as follows:

1. Restructure a fault tree and identify independent modules,
2. Solve each module and assign the maximum MCS probability to the module,
3. Solve the fault tree where the modules are treated as basic events,
4. Substitute the modules in the final MCSs with their MCSs, and
5. Calculate the top event probability using the final MCSs.

Here the term "solve" in Steps 2 and 3 denotes "calculate the MCSs of the gates in a bottom-up way, that is, truncate the cut sets and subsume the subsets".

The full MCSs could be classified into three categories as follows:

C_i^k = MCSs that have probabilities larger than the truncation limit,

$\overline{C}_i^k$ = MCSs that are truncated when expanding the modules at Step 4,

$\hat{C}_i^k$ = MCSs that are truncated when solving a fault tree at Steps 2 and 3,

where the truncation limit is 1×10^{-k}. The exact top event probability is the sum of the three probabilities as

$$P_{Exact} = P_k + \overline{P}_k + \hat{P}_k \tag{1}$$

where

$$P_k = P\left(C_1^k + C_2^k + \ldots\right),\ \ \overline{P}_k = P\left(\overline{C}_1^k + \overline{C}_2^k + \ldots\right),\ \ \hat{P}_k = P\left(\hat{C}_1^k + \hat{C}_2^k + \ldots\right). \tag{2}$$

The MCSs C_i^k are final cut sets when the fault tree is solved with a truncation limit of 1×10^{-k}. Even though the MCSs $\hat{C}_i^k$ could not be calculated, the truncated MCSs $\overline{C}_i^k$ at Step 4 could be easily obtained with a little effort by a modification of the existing fault tree solvers.

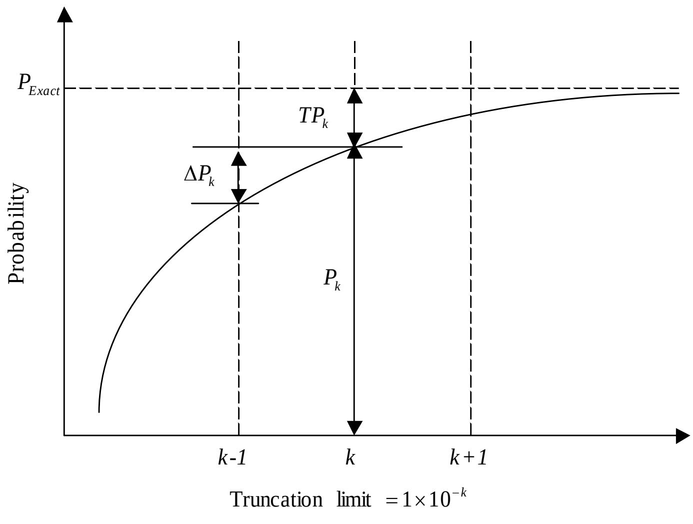


Fig.1 Typical top event probability curve according to the truncation limit

As a measure of the truncation error, the following TP (Truncated Probability) and TU (Truncation Uncertainty) are

$$TP_k = \overline{P}_k + \hat{P}_k \text{ and } TU_k = \frac{\overline{P}_k + \hat{P}_k}{P_k + \overline{P}_k + \hat{P}_k} \quad (3)$$

where $\lim_{k\to\infty} TP_k = 0$ and $\lim_{k\to\infty} TU_k = 0$.

If the truncation limit is not sufficiently low enough, TP and TU could not be calculated since the MCSs $\hat{C}_i^k$ and their probability $\hat{P}_k$ could not be obtained. Therefore, instead of TP and TU, their lower bounds LBTP (Lower Bound of Truncated Probability) and LBTU (Lower Bound of Truncation Uncertainty) are defined in this study as follows

$$LBTP_k = \overline{P}_k \text{ and } LBTU_k = \frac{\overline{P}_k}{P_k + \overline{P}_k}. \quad (4)$$

Their relations are

$$LBTP_k \le TP_k \text{ and } LBTU_k \le TU_k. \quad (5)$$

In a different way, TP_k in Eq. (3) could be estimated if $\Delta P_k = P_k - P_{k-1}$ is used instead of $\hat{P}_k$. As shown in Fig.1, ΔP_k is calculated by using the MCSs with the current truncation limit of 1×10^{-k}. In this case, ATP (Approximate Truncated Probability) and ATU (Approximate Truncation Uncertainty) could be defined as

$$ATP_k = \overline{P}_k + \Delta P_k \text{ and } ATU_k = \frac{\overline{P}_k + \Delta P_k}{P_k + \overline{P}_k + \Delta P_k}. \quad (6)$$

The ATP has the following inequality relation in the concave range of the top event probability.

$$P_{k+1} \le P_k + \Delta P_k \le P_k + ATP_k. \quad (7)$$

When the truncation limit is sufficiently low enough,

$$ATP_k \approx TP_k \text{ and } P_k + ATP_k \approx P_{Exact}. \quad (8)$$

For example, in the case of the surviving MCS at Step 3 $abc(d+e+f)(g+h+i+j)(k+l+m)$

1. the MCS has 3 modules $(d+e+f)$, $(g+h+i+j)$, and $(k+l+m)$,
2. the MCS $abcdgk$ survives at Step 4 in the extreme case, and
3. the remaining 12 MCSs $abc(e+f)(h+i+j)(l+m)$ are truncated at Step 4 and they are the MCSs for LBTP or LBTU in Eq. (4).

Here, probabilities of basic events have the alphabetical ordering.

3. Application

The functions to calculate LBTP, LBTU, ATP, and ATU are programmed into the new fault tree quantifier FTREX. The Benchmark test was performed for evaluating a single fault tree for the core damage frequency of the Ulchin Unit 3&4 [14].

Fig. 2a shows that the top event probability is converging to the exact frequency and the measures LBTP and ATP are dying out as lowering the truncation limit. Fig. 2b depicts the number of MCSs C_i^k and $\overline{C}_i^k$. As shown in Fig. 2b, a huge number of MCSs $\overline{C}_i^k$ are discarded at Step 4.

As shown in Fig. 2c and 2d, ATP behaves like the upper bound of TP. As shown in Fig. 2e, the importance measure RAW is significantly underestimated in the case of the higher truncation limit, which is used for selecting the risk-important SSCs in the nuclear field.

The truncation limit could be determined by suppressing the measures to be less than pre-determined limits. For example, if the reliability analyst or regulatory body wants to suppress the truncation errors, LBTU and ATU, to less than 1 percent, the truncation limit should be set at less than 1×10^{-11} and 1×10^{-13}, respectively. Thus, the four measures could be used as measures of the truncation error.

The calculation time of LBTP, LBTU, ATP, and ATU is less than 0.2 seconds. The developed measures are easily implemented into the existing fault tree solvers with a little modification.

4. Conclusions

The truncation error or uncertainty has been of great concern in PSA. This paper presents new measures to estimate the truncation error when quantifying fault trees with a truncation limit. The developed measures are easily implemented into the existing fault tree solver. The truncation limit could be determined by suppressing the measures to be less than the pre-determined limits. Thus, the measures could be used as an acceptability of the fault tree quantification results.

References

[1] Vesely WE, Goldberg FF, Roberts NH, and Haasl DF, Fault Tree Handbook, NUREG-0492, U.S.

Nuclear Regulatory Commission, Washington, DC, 1981.

[2] Barlow RE and Proschan F, Statistical Theory of Reliability and Life Testing, Holt, Rinehart and Winston, Inc., 1975.

[3] Nuclear Energy Institute Risk-based Applications Task Force, “Probabilistic Risk Assessment Peer Review Process Guidance,” 2000.

[4] ASME, “Standard for Probabilistic Risk Assessment for Nuclear Power Plant Applications,” ASME RS-S-2002, 2002.

[5] Fleming KN, "Issues and Recommendations for Advancement of PRA Technology in Risk-Informed Decision Making," U.S. Nuclear Regulatory Commission, NUREG/CR-6813, 2003.

[6] Modarres M and Dezfuli H. “A truncation methodology for evaluating large fault trees,” IEEE Transactions on Reliability, Vol. 33(4), pp.325-328, 1984.

[7] Brown KS, “Evaluating fault trees with repeated events, IEEE Transactions on Reliability, Vol. 39(2), pp.226-235, 1990.

[8] Contini S, “A new hybrid method for fault tree analysis,” Reliability Engineering and System Safety, Vol. 49, pp.13-21, 1995.

[9] Cheok MC, Parry GW, and Sherry RR, "Use of importance measures in risk-informed regulatory applications," Reliability Engineering and System Safety, Vol. 60, pp.213-226, 1998.

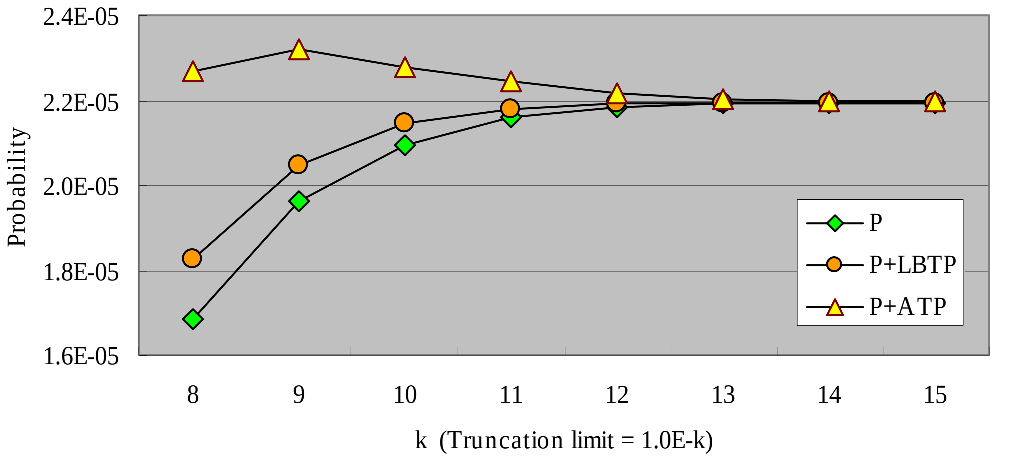
[10] Coudert O and Madre JC, "Implicit and Incremental Computation of Primes and Essential Primes of Boolean Functions," Proceedings of the 29th ACM/IEEE Design Automation Conference, DAC'92, June 1992.

[11] Rauzy A, "New Algorithms for Fault Trees Analysis," Reliability Engineering and System Safety, 40, pp. 203-211, 1993.

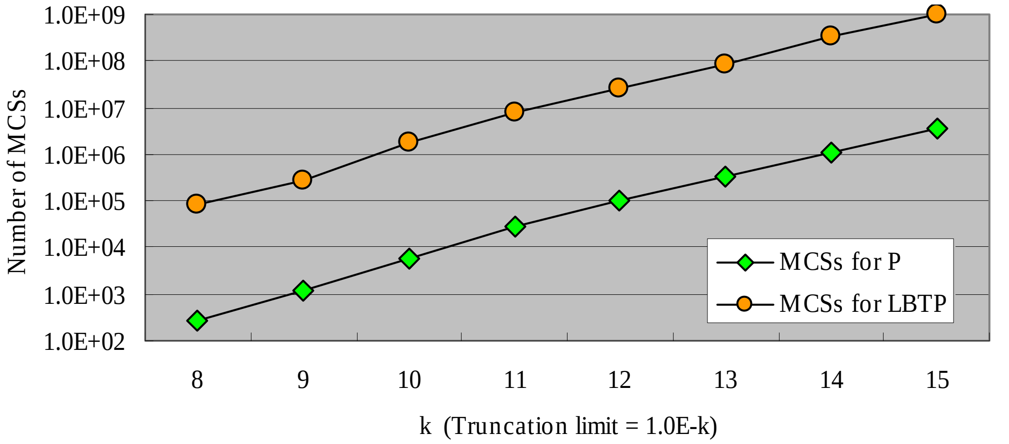
[12] Jung WS, Han SH, Ha JJ, "A Fast BDD Algorithm for Large Coherent Fault Trees Analysis," Reliability Engineering and System Safety, Vol. 83, pp. 369–374, 2004.

[13] Jung WS, Han SH, Ha JJ, "Development of an Efficient BDD Algorithm to Solve Large Fault Trees," Proceedings of the 7th International Conference on Probabilistic Safety Assessment and Management, June, Berlin, Germany, 2004.

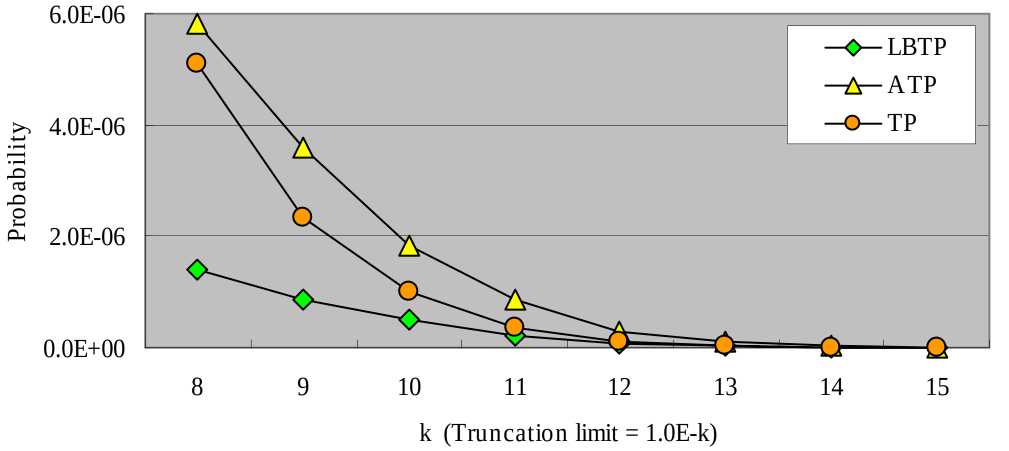
[14] Korea Electric Power Corporation. Ulchin Units 3&4 Final Probabilistic Safety Assessment Report, 1997.



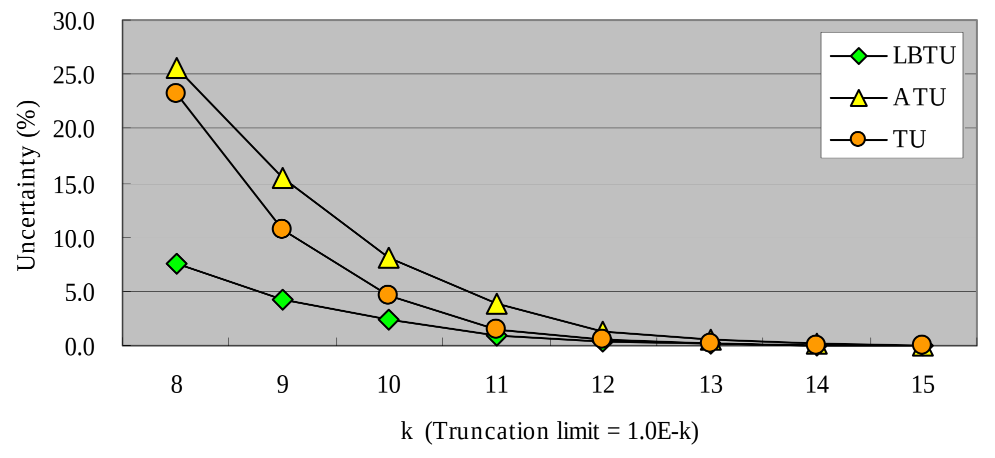
(a) Top event probability



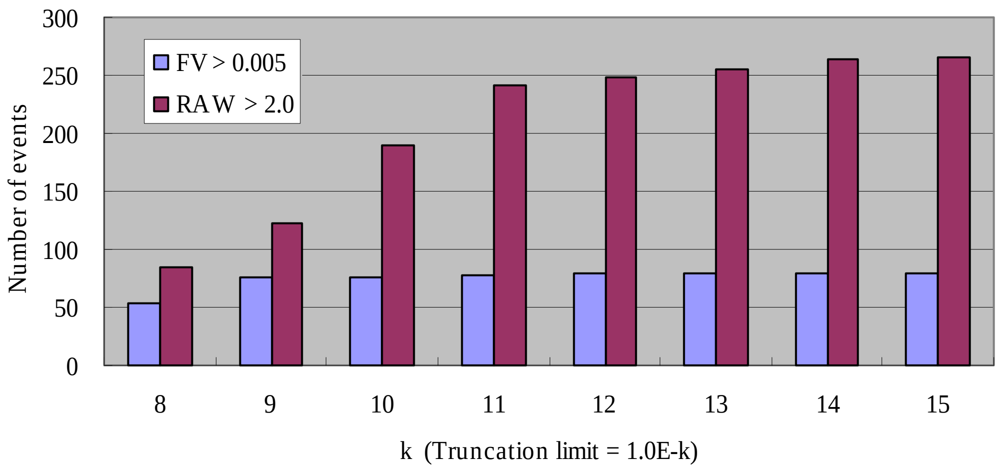
(b) Number of MCSs



(c) Measures for the truncation error



(d) Measures for the truncation uncertainty



(e) Important basic events

Fig. 2 Quantification results of the truncation error for Ulchin 3 NPP