Numerical Simulation for the Improved Five-Sensor Probe Method by Introducing Correction Factors to Measure the Interfacial Area Concentration

Dong Jin Euh, Byong Jo Yun, Chul Hwa Song

Korea Atomic Energy Research Institute
P.O.Box 105, Yuseong, Daejeon, 305-600, KOREA
*E-mail address: djeuh@kaeri.re.kr

Abstract

Interfacial area concentration is an important parameter in the two phase flow models, which significantly affects the accuracy of the prediction. Currently, there are three probe methods to measure the interfacial area concentration, which are the double-, four- and five-sensor methods. Among them, the four- and five-sensor probe methods can be applied to various flow regimes where the shape of the bubbles is beyond the spherical shape. The five-sensor method was developed to measure the missing bubbles which bypass one or more of the rear sensors more accurately than the other methods. However, for the bubbles having a high lateral velocity, there can be measurement errors since the validity of the assumption for the missing interfaces in the five-sensor method is reduced. In this study, the measurement error is quantified and corrected with factors related to the turbulent intensity based on numerical simulations. The corrected five-sensor method also includes a modification for the small bubbles of which the shape can be considered to be spherical. The bubble parameters related to the bubble motion and geometry are determined by the Monte Carlo approach.

1. Introduction

In the formulation of the two-fluid model, appropriate constitutive relations for the interfacial transfer terms are needed to close the phasic balance equations, and their accuracy significantly affects the results of the calculation. In general, the interfacial transfer terms are proportional to the interfacial area concentration (IAC) which is defined as the interface area per unit of the two-phase mixture volume. Therefore, the IAC is one of the most important parameters in the two-fluid model. Since the IAC prediction depends on sound experimental data, it is important to develop an accurate instrumentation method for the IAC. The probe method is a useful measurement technique for the two-phase flow parameters such as the void fraction and bubble velocity. The measuring principle of the multi-sensor probe method in obtaining a local time-averaged IAC is based on the mathematical formula given by Ishii (1975). Currently, three types of probe methods have been used, which are the double-, four-,
and five-sensor methods. The double-sensor probe method is based on some statistical assumptions for the two-phase flow characteristics. (Kataoka et al., 1986; Kataoka et al., 1990; Revankar et al., 1992) This method is practical and useful in the dispersed flow regime where the bubbles have a spherical shape. However, in the other regimes where the particle shape is irregular, its applicability is rather restricted. The four-sensor probe method can predict the IAC without any assumptions for the bubble shape.(Kataoka et al., 1986; Ishii et al., 1991; Revankar et al., 1993) The local IAC can be obtained by measuring the three dimensional components of the velocity vector at the measuring points and the directional cosines of the sensors. However, due to the finite size of the probe, it is possible that one or more of the rear sensors do not detect the interface. The four-sensor method includes the method for the interfaces that pass all of the sensors and that for the missing interfaces. In this study, the former is called the four-point method. The missing interface has a very steep shape and large interfacial area concentration at the measuring point. The contribution of IAC from such interfaces may be so substantial that it must be appropriately considered. The four-sensor probe method can predict the IAC of such interfaces with a special mathematical formula, but it still has limitations due to a lack of information on the interfaces. The five-sensor method was developed to improve the defect of the four-sensor method.(Euh et al., 2001) It classifies the bubble passing type through the sensors into four categories and the appropriate measuring method is adopted for each category’s interface. The five-sensor method focuses on the missing bubbles which bypass one or more of the rear sensors. This method has an advantage that a more systematic approach for the missing bubbles can be made when compared with the four-sensor probe method.

Wu et al.(1999) performed sensitivity studies on the measurement of the local interfacial area concentration using a double-sensor probe by considering the influences of the bubble lateral motion and probe spacing. In their study, the statistical parameters related to the motion and geometry of the interface were determined by the Monte Carlo approach. Euh et al.(2001) proposed a five-sensor probe method and performed numerical analysis to evaluate the measuring method. In their study, various shapes of the bubbles, which were spherical, cap and ellipsoidal, were tested. As a result, the proposed method shows a good accuracy in measuring the local IAC. However, the study did not consider the bubble lateral fluctuate motion of the bubbles and the central rear sensor was assumed to be on the center line. Actually, the two sensor tips of the central front and rear sensors are difficult to place on the same vertical line without yielding a significant deformation to the interface. Therefore, more investigations for the actual bubble motion and probe design are needed. Le Corre et al.(2002) evaluated the double- and four-sensor probe methods for the measurement of the local bubble velocity and local interfacial area concentration. They insisted that the accuracy of the double-sensor probe method is fairly good for any flow condition with spherical bubbles, whereas the four-sensor probe method significantly underestimates the IAC due to a higher rate of the missing bubbles.

In this study, the revised measuring method for the interfacial area concentration by using the five-sensor probe is proposed concerning the bubble fluctuation and treatment for the small bubbles of which the shape can be regarded as spherical. For verification of the proposed method, numerical simulation techniques are used. The bubble parameters related to the bubble fluctuation and interface geometry are determined by the Monte Carlo approach.
2. Five-Sensor Probe Method

The configuration of the sensor tips in the probe is shown in figure 1. The probe consists of a central front sensor, a central rear sensor, and three peripheral rear sensors. The local measuring point is the location of the central front sensor. From the central rear sensor, one can additionally obtain the near axial velocity component of the interface. Thus the four velocity components of the interface at a measuring point can be obtained. By using the additional velocity vector, one can measure the bubble parameters effectively for the missing bubbles that bypass one or more of the rear sensors. The directional vector components of the rear sensors from the central front sensor are derived from the consideration of the geometry of the probe, and its derivation is described in Euh et al. (2001). The five-sensor method proposed in this study classifies the types of the interfaces passing through the sensors into four categories, and applies different mathematical formulations to each category. Category II and III interfaces are important missing interfaces that have a steep shape and large interfacial area concentration at the measuring point. The possibility that category IV interfaces are detected is very low according to Euh et al. (2001). However, for the proposed method in this study, very small bubbles of which the shape can be regarded as spherical are also included in category IV. The configuration of the sensor tips and bubble interface which is projected onto a plane is shown in figure 2.

![Figure 1. Configuration of The Sensor Tips](image1)

![Figure 2. Bubble Passing Type](image2)

2-1 Measuring Method for the Category I Interface

Category I interfaces pass all of the sensors. Euh et al. (2001) considered three sub-cells formed inside the measuring area and averaged the three IAC values referring to each sub-cell. This approach can reduce the curvature effect of the interface. However, for a small measuring area like the proposed design, the effect of considering the sub-cells is so small that the four-point method is used in this study. The four-point method can be summarized as follows:

\[
\tilde{a}_1 = \frac{1}{O} \sum_{j} \frac{\sqrt{|\mathbf{A}_1|^2 + |\mathbf{A}_2|^2 + |\mathbf{A}_3|^2}}{\sqrt{|\mathbf{A}_0|^2}}
\]

where
$|A_k|$ : determinant of the matrix including direction angle of rear sensors from the central front sensor tip and velocities (referred to Ishii et al. (1991))

$\Omega$: Problem time

$j$: interface number

2.2 Measuring Method for the Category II interfaces

Category II interfaces bypass one of the three peripheral rear sensors. Three sub-cells are considered for these category interfaces as shown in figure 3. The IAC is obtained from the weighted averaging for the angle defined in this figure.

$$a_{ij} = \frac{(a_{ij})_{Cell 1} a_1 + (a_{ij})_{Cell 2} a_2 + (a_{ij})_{Cell 3} a_3}{2p} \tag{2}$$

Figure 3 shows the configuration of one of the three cases included in this category. The IAC of sub-cell 1 can be obtained from the four-point method, but in the other sub-cells, some mathematical considerations are made since all three components of the velocity vector cannot be measured. The formula for the missing interface is somewhat different from that applied to the four-sensor method. The method for category II depends on the steepness of the interface. A sensor detects two interfaces per bubble, and one can expect that the upper side of the missing interface is steep at the measuring point, but the shape of the bottom interface can be flat or steep. The steepness of an interface can be judged by comparing the value referring to cell 1 with the value of a flat interface. For a steep interface, the interface is assumed to be parallel to the probe. For a flat type of interface, the value referring to cell 1 can be considered as the representative value for the measuring point. The mathematical considerations applied to cell 2 and 3 are as follows:

$$\left| \left( a_{ij} \right)_{Cell 1, bottom} - \frac{1}{\Omega} \frac{1}{v_{iz}} \right| < \varepsilon, \quad \text{(flat interface)} \tag{3a}$$

$$\left( a_{ij} \right)_{bottom} = \left( a_{ij} \right)_{Cell 1, bottom}$$

$$\left| \left( a_{ij} \right)_{Cell 1, bottom} - \frac{1}{\Omega} \frac{1}{v_{iz}} \right| > \varepsilon, \quad \text{(steep interface)} \tag{3b}$$

$$\left( a_{ib} \right)_{Cell 2} = \frac{t_b}{\Omega} \frac{f_{d2}}{s_{p2}}$$

$$\left( a_{ib} \right)_{Cell 3} = \frac{t_b}{\Omega} \frac{f_{d3}}{s_{p3}}$$

The condition shown in Eq. (3) is for checking the steepness of the interface. The second term in the absolute blanket, $1/(\Omega v_{iz})$, is the local IAC for a flat interface. In this study, the
steepness criteria, \( \varepsilon \), is set at 0.05. \( l_{dk} \) is the length scale that is occupied by a bubble in cell \( k \), and it is determined by using the statistical assumption as shown in figure 3. When the projected area of the probe is equally divided into three sub-cells, the length and area in each sub-cell is also given as in figure 3. The above relation can be simplified by the following equation.

For steep upper + steep bottom interface:

\[
(a_i)_{\text{bubble, II, 0}} = \frac{\left( a_{ij} \right)_{\text{Cell 1, upper}} + \left( a_{ij} \right)_{\text{Cell 1, bottom}} + \left( a_{ij} \right)_{\text{Cell 2}} + \left( a_{ij} \right)_{\text{Cell 3}}}{3}
\]

(4a)

For steep upper + flat bottom interface:

\[
(a_i)_{\text{bubble, II, 0}} = \frac{\left( a_{ij} \right)_{\text{Cell 1, upper}} + \left( a_{ij} \right)_{\text{Cell 2}} + \left( a_{ij} \right)_{\text{Cell 3}}}{3} + \left( a_{ij} \right)_{\text{Cell 1, bottom}}
\]

(4b)

However, for one of the three cases in this category like figure 4, a somewhat different formulation is applied. Although the four-point method can be applied to cell 2 theoretically, a large error can occur since the independency of the three velocity vector is low. The dependency of the velocity vectors occurs because the three rear sensors in sub-cell 2 are nearly on the same line. Therefore, for this case, the classical four-sensor method for the missing bubbles is applied.

\[
(a_i)_{\text{bubble, II, 0}} = \frac{t_b I_d}{O_s}
\]

(5)

As mentioned previously, the above model assumes that the interface is a steep shape and parallel to the probe. However, for the high lateral bubble motion case, the assumption can be invalid. Therefore, in the later section, the above model will be corrected using a certain factor by including the bubble turbulent intensity as follows:

\[
\left( \bar{a}_i \right)_{II} = \left( \bar{a}_i \right)_{II, 0} I_H
\]

(6)

2.3 Measuring Method for the Category III interfaces

Category III interfaces bypass two of three peripheral rear sensors. Figure 5 shows one of the three cases that belong to this category. For this case, an exact relation is considered to get the IAC. Since the x or y directional slope cannot be measured directly at the local measuring point “0”, some engineering considerations based on the measurable quantities are made. Let us consider a case such that an interface contacts sensors “0”, “1”, and “2”. Because one can statistically expect that the slopes of the interface in directions x and y at measuring point “0” would be larger than the linear slopes in each direction between sensors “0” and “2”, the interfacial slopes for each direction are assumed to be the elevation difference of the interface for the x and y distance respectively between sensors “0” and “2”. The IAC is then derived as
where the subscript number 0 and k are the identification numbers of the sensors. The lengths in the denominators of the terms in the root in Eq. (7), \( \Delta x_{0k} \) and \( \Delta y_{0k} \), are the distances between sensors “0” and “k” in the x and y direction respectively, which are determined by the geometry of the probe. The elevation difference of the surface at sensors 0 and k, \( \Delta z_{s0k} \), is obtained from the axial velocity component from the two central sensors and the delay time of the rear sensor “k” signal.

The category III model also assumes the interface to be a steep shape and parallel to the probe as in the category II model. Therefore, the model will be corrected using a certain factor by including the bubble turbulent intensity as follows:

\[
\left( \bar{a}_i \right)_{III,0} = \frac{1}{\Omega} \frac{1}{v_{iz}} \sqrt{1 + \left( \frac{\Delta z_{s0k}}{\Delta x_{0k}} \right)^2 + \left( \frac{\Delta z_{s0k}}{\Delta y_{0k}} \right)^2}, \quad (k = 2, 3, 4) 
\]

(7)

2.4 Measuring Method for the Category IV interfaces

Two kinds of interfaces are included in category IV. One is the bubble passing only the central front and rear sensors. However, since the radius formed by the three peripheral sensors is so small, the possibility of the occurrence of such interfaces is very low. The other kind of category IV interface is for small bubbles. Very small bubbles can be regarded as spherically shaped. The maximum spherical bubble size can be obtained by the following model.

\[
D_{ds} = 4 \sqrt{\frac{2\sigma}{g\Delta \rho} N_{1/3}^{1/3}} \quad \text{(Ishii, 1977)}
\]

(9)

where

\[
N_{\mu'} = \left( \frac{\mu_t}{\rho_t \sigma \sqrt{g\Delta \rho}} \right)^{1/2} \left( \rho \sigma \sqrt{\mu_t / g\Delta \rho} \right)^{1/2}
\]

In an actual measurement, the judgment for the bubbles satisfying the spherical size criteria is performed by the chord length of the individual bubbles. However, the chord length does not mean the exact bubble size. In other words, if the sensor penetrates the rim of the bubbles, the chord length would be smaller than the bubble diameter. For a sphere bubble, the maximum chord length is the bubble diameter. So, we set the criteria for the small bubbles at 2/3 times that of the bubble diameter by considering figure 6.
Interfacial area concentration for both kinds of bubbles is measured by the double sensor probe method. The double sensor method is expressed by the following formula.

\[
\left(\frac{\dot{m}}{m}\right)_{HV} = \left(\frac{\dot{m}}{m}\right)_{HV0} I_{IV} = \frac{4N_p}{O} \left(\frac{1}{v_b}\right) I_{IV}
\]

In the above formula, \(I_{IV}\) means the correction factor by considering the bubble turbulence effect, which will be analyzed in the later section.

3. Numerical Evaluation Method

The numerical evaluation to obtain the interfacial area concentration follows the flow chart as shown in figure 7. The key derived formulations are also summarized in the flow chart. Mathematical representation between the two sensors and the simulated interfaces follow that of Le Corre et al. (2002). Flow characteristics such as bubble size, bubble velocity, frequency and bubble shape are determined by the user input. In this study, 100,000 bubbles are sampled for the simulation per local measuring point. The directional vector of each rear sensor from the front sensor can be simply obtained by referring to the probe geometry. The interface normal vector and velocity vector are determined by two angles each, \(\mu\), \(\nu\), and \(\xi\), \(\phi\), which are chosen by a random number generator. Although the vectors can be easily derived for the sphere bubbles with a certain assumption, some more studies are needed for the other geometries of the bubbles. In this study, the evaluation is confined to the sphere bubble case and that for the other geometries is reserved for the next study. The study for the hemisphere and ellipsoidal shaped bubbles were performed in Euh et al. (2001) for the case of no lateral bubble movement. In this study, the lateral movement of the bubbles due to the turbulent fluctuation is considered where the fluctuation degree is determined by a random number generator given the maximum fluctuation ratio to the mean bubble velocity by the user input. In obtaining the delay time of the rear sensor signal, somewhat complicated formulas are needed. The derivation of the delay time follows that of Le Corre et al. (2002). In processing the time lags between the front and rear sensor signals, there can be no solution. This means that the interface passing the front sensor does not pass the rear sensor. The process is defined as “Check Effectiveness” in the flow chart. According to the missing characteristics, the category defined in the IAC measuring methodology is selected. The IAC can then be
obtained by using the time lags and mathematical formulations of the previous section.

\[
\begin{align*}
\delta_i - \delta_j & = S_{ij} \\
\Xi, \phi, \mu, \nu, \Omega(x) & \\
\end{align*}
\]

\[
\begin{align*}
\tilde{\eta}_i & = \sin \xi \cos \phi \hat{i} + \sin \eta \sin \varphi \hat{j} + \cos \xi \hat{k} \\
\tilde{\eta}_j & = \sin \mu \cos \varphi \hat{i} + \sin \mu \sin \varphi \hat{j} + \cos \mu \hat{k} \\
\tilde{\eta}_n & = ? \tilde{\eta}_i \\
\end{align*}
\]

\[
\begin{align*}
A & \begin{bmatrix}
\sin \phi & \cos \varphi & \sin \phi \\
\cos \phi & -\sin \varphi & \cos \phi \\
0 & -\sin \varphi & \cos \phi \\
\end{bmatrix}
\end{align*}
\]

**Figure 7. Flow chart to get the IAC**

4. Results and Discussion

4.1 Correction for the Missing Bubble Models According to the Bubble Lateral Motion

Category II, III and IV bubbles bypass one or more of the rear sensors after passing the front sensor. Category II and III models assume the bubble shape to be steep and we developed a measuring model based on an assumption for the shape of the interface. However, if the bubble moved laterally with a high intensity, the assumption can not be valid. In this section, the error of each model due to the bubble lateral motion is corrected with a factor by including the turbulent intensity. \( I_{II} \) and \( I_{III} \) are the correction factors for the category II and III bubbles respectively as defined in the previous section. Figure 8 shows the IAC correction results for the category II and III interfaces. In this figure, the x-axis means the simulated turbulent intensity ratio, named \( H' \), which is somewhat different from the actual turbulent intensity. Even if there is no fluctuation, the measured axial velocity experiences some fluctuation since the interface has a curvature and the central rear sensor measuring the axial velocity is not located at an exact central position but slightly away from the center line. Therefore, the turbulent intensity measured by the central front and rear sensors is slightly larger than the actual turbulence, in general. The correction factors for the category II and III bubbles are based on the simulated turbulent ratio since the turbulent intensity can only be obtained from a measurement in a real system. Although the maximum tested range of the turbulent intensity is 0.5, the simulated values are more than the actual value particularly for the large bubbles size case as shown in figure 8. The correction factors are adjusted to fit accurately for the 3mm average bubble size condition, which is a typical size in most of the interesting two-phase flow conditions, as follows:
\[ I_{II} = 1.12 + 1.28H' + 1.18H'^2 \]  
(11)

\[ I_{III} = 1.69 - 3.16H' + 2.04H'^2 \]  
(12)

Figure 8. IAC Correction for the Category II and III Models

Category IV interfaces have two types. One is for very small bubbles passing only the central front and rear sensors. The other is for the bubbles having a chord length less than that of the maximum spherical bubble. At atmospheric air/water conditions, the maximum spherical bubble diameter is 1.87mm by using equation (9). For the size of the spherical bubbles, the average chord length is 1.25mm based on figure 6. Therefore, if the bubble chord length is less than 1.25mm, the bubbles are included in category IV in this study. Figure 9 shows that the model for no fluctuation has a strong dependency on the bubble size. The small chord length can be measured if the sensor penetrates the rim region of the bubbles even if the bubble is large. For large bubbles, the rim area inducing a small penetrated bubble length would be small and for small bubbles, the rim area would be large. Therefore, the category IV model has a strong dependency on the bubble size. We adopted not only the turbulence effect but also the bubble size effect in the corrected model for the category IV interfaces as follows:

\[ I_{IV} = \left(1.14 + 1.33H' - 1.42H'^2\right)(0.6 + 0.4\%D_{av0}/D_{av0}) \]  
(13)

where \( D_{av0} \) is 3mm. Here, several iterations should be performed since the average bubble size can not be directly measured.
4.2 Corrected Results

Figures 10~13 show the corrected results for the simulated IAC. The IACs by using the five-sensor probe are compared with those of the four-sensor probe method and the theoretical values. In the comparison, the referred four-sensor configuration is a classical one which has one front sensor and three symmetrical rear sensors. Therefore, the four-sensor tips have the same configuration as those of the five-sensor probe if we don’t consider the central rear sensor. The compared four-sensor method is briefed as follows:

For the Category I interfaces: Four-Point Method
For the Category II and III interfaces: Missing bubble model
\[
(a_i)_{\text{bubble}} = \frac{t_b}{t_f} \frac{L_i}{S_p}
\]

For the Category IV interfaces: Double-Sensor Method (Wu et al., 1999)
\[
\tilde{a}_i = \frac{2N_b}{O} \left( \frac{1}{v_b} \right) \left( 2 + H^{2.25} \right) \]  

The lateral bubble motion effect is investigated to 50% of the turbulent intensity ratio. Figure 10 shows the results for 2.0mm of the average bubble size. Since the y-axis is the ratio of the simulated IAC to the exact IAC, the value is 1.0 if the two quantities coincide with each other. Total IACs simulated by the five-sensor method agree well with the exact ones as shown in figure 10. However, those by the four-sensor method somewhat underestimate the IACs. Each model for the various category interfaces in the five-sensor method also shows a good performance as shown in the figure. However, the four-sensor method has composition errors in the models. The model that is used in the four-sensor method for category IV interfaces underestimates the IAC although the model is well known to be accurate. The reason why equation (15) induces errors in our test is as follows. If all the sphere bubbles are measured by the double-sensor method of equation (15), the simulated IACs will
approximate the exact IAC well. However, the four- and five sensor methods apply the double-sensor method only to the bubbles having a small chord length. Although the small chord length would be measured from the small bubbles, it can also be measured if the sensor tip penetrates the rim of a bubble larger than the maximum spherical size criterion. The rim region of the bubble has a large IAC. Since the central region of the bubble may be easily grouped into category I, which has a small IAC, the average IAC of the category IV bubble is larger than that of the total bubbles. Therefore, equation (15) produces a lower value than the exact IAC for the category IV interface. The underestimation becomes larger as the bubble size increases as shown in figures 10-13, which illustrate well the large steepness of the rim region of the large bubble.

Figures 11-13 show the results for the other sizes of the bubbles, which are 3.0mm to 7.0mm of the average diameter, respectively. The characteristics of the measuring method are similar to those for 2.0mm of the average bubble sizes. Therefore, the five-sensor method shows a good performance for the various sizes of the bubbles. However, there are some errors in the category I model for the large bubble and the large fluctuation condition. For the category I interface, the four-point method is applied. Since the five-sensor method for the category I interfaces is the same as the four-sensor method, the problem occurs in both methods. The deviation is from the fact that the direction of the velocity components measured by using the probe sensor tips can be significantly different from the bubble direction moving with a high intensity of the lateral motion. Although the deviation occurs at an extreme flow condition and its magnitude is not significant, the problem should be solved in the next study.

Figure 14 shows the number frequency contribution of each category to the total frequency. A large bubble can pass all of the five sensors with a high probability since the cross section is large, however, a small bubble might frequently bypass one or more of the rear sensors. Figure 14 shows the trend well. For 2.0mm of the average bubble size, the missing bubble fraction is very large. However, as the bubble size goes up to 7.0mm, the frequency of the category I bubbles increases remarkably. Figure 15 is the IAC contribution results for each category of the bubbles to the total IAC. The trend of the number density contribution with an increasing bubble size is also shown in the IAC contribution results, in other words, for a large bubble condition, the IAC contribution of the category I bubbles is larger than that for the small bubble size condition. Another important point from figures 14 and 15 is that the contribution of the IAC from the missing bubbles is larger than that of the number frequency from them. For example, if one looks at figures 14(c) and 15(c), the IAC contribution ratio of category I is scattered for a level lower than the number frequency contribution of figure 14(c). The trend is shown in the results of all the other bubble sizes. This is reflected by the fact that the missing bubble has a steep shape and a large interfacial area concentration.

5. Conclusion

An improved five-sensor probe method considering the bubble fluctuation effect is proposed in this study. To evaluate the applicability of the proposed measuring method, sensitivity analysis was performed according to the various bubble sizes and the intensities of the bubble fluctuation under the spherical shaped bubble and fluctuation conditions. As a
result, the five-sensor method shows better features than the previous IAC measuring method. During the development of the measuring method, the interface characteristics of each category’s bubble were investigated and also evaluated with theoretical results. Each categorical model also shows a good correspondence with the theoretical values. The categorical frequency and IAC contribution illustrate well the characteristics of the missing bubbles.

Acknowledgement

This work has been financially supported by the Ministry of Science and Technology (MOST) of Korean government under the national nuclear mid- & long-term R&D program.

References

Figure 10. IAC Measurement Results for $D_{av}=2.0\text{mm}$

Figure 11. IAC Measurement Results for $D_{av}=3.0\text{mm}$
Figure 12. IAC Measurement Results for $D_{av}=5.0\text{mm}$

Figure 13. IAC Measurement Results for $D_{av}=7.0\text{mm}$
Figure 14. Number Frequency Contribution of Each Category of the Bubbles

Figure 15. IAC Contribution of Each Category of the Bubbles