Acoustic Scattering from the Cylindrical Structures by an Oblique Incident Plane Wave

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1. Introduction

Analytical and experimental researches of an acoustic wave scattering from cylindrical structures have been investigated extensively for the last several decades. Resonance scattering theory (RST) was one of the principal works in this area, and suggested that the acoustic pressure scattered from the scattering object consisted of a background and resonance pressure, and the resonance signal can be obtained by a rejection of background signal from the scattered acoustic pressure [1]. The research for the acoustic scattering based on the RST of a normally and obliquely incident plan acoustic wave by a cylindrical structure immersed in a fluid has been performed. When the wave is obliquely incident on the cylindrical rod or shell, additional types of waves are generated in the structure, and correspondingly additional resonance modes of the structure are excited. These additional waves have been called guided waves [2].

In this study, the problem of an acoustic scattering of a plane wave obliquely incident on a Zircaloy cylindrical shell of a infinite length has been considered. The form function and resonance spectrum according to the incident angle of the plane wave are calculated, and the behavior of the various modes generated in the shell is investigated theoretically.

2. Mathematical Analysis

A plane acoustic wave incident at an angle α on an elastic infinite shell, which has an outer radius a_1 and an inner radius a_0 , in a fluid is considered as in Fig. 1.



Figure 1. Geometry of an acoustic plane wave obliquely incident on a submerged cylindrical shell

The incident plane wave external to the scattering object is represented by

$$P_{inc} = \exp[i(k_z z - \omega t)] \sum_{n=0}^{\infty} \varepsilon_n i^n J_n(k_x r) \cos n\theta , \qquad (1)$$

and the outgoing scattered wave can be written as

$$P_{sc} = \exp[i(k_z z - \omega t)] \sum_{n=0}^{\infty} \varepsilon_n i^n R_n H_n^{(1)}(k_x r) \cos n\theta \,. \tag{2}$$

In the above equations, $k_x = k \cos \alpha$, $k_z = k \sin \alpha$ and R_n are the scattering coefficients to be determined from the boundary condition. $J_n(x)$ is the first kind Bessel function and $H_n^{(1)}(x)$ is first kind Henkel function of order n.

The far-field scattering pressure can be written as

$$P_{sc} \approx \exp[i(kr + kz - \omega t)] \sqrt{\frac{a_1}{2r} \sum_{n=0}^{\infty} \frac{2}{\sqrt{\pi i k a_1}}} \varepsilon_n R_n \cos(n\theta) .$$
(3)

The normalized far-field amplitude called the form function is given as

$$f_n(\theta,\eta) = \frac{2}{\sqrt{\pi i \eta}} \varepsilon_n R_n \cos(n\theta), \qquad (4)$$

where η is a normalized frequency ka_1 .

The resonant part of each mode can be obtained by rejecting the acoustical background $f_n^{(b)}(\theta,\eta)$ from normal mode components $f_n(\theta,\eta)$ according to the following equation [3]:

$$f_n^{(reso)}(\theta,\eta) = \frac{2}{\sqrt{\pi i \eta}} \varepsilon_n (R_n - R_n^{(b)}) \cos(n\theta) \,. \tag{5}$$

3. Results and Discussion

The resonance spectra and form function of a Zircaloy cylindrical shell are computed for various incident angles $\alpha = 0^{\circ}$, 3°, 5° and 45°. The outer diameter of the cylindrical shell is 10.7mm and its ratio ID/OD is 0.88. Physical properties of the shell are given in Table 1.

Table 1. Physical properties of Zircaloy material and water

	ρ	C_L	C_T
	(kg/m^3)	(m/s)	(m/s)
Zircaloy	6.55	4,600	2,360
Water	1	1,480	-

In case of an oblique incident wave, the travel paths of all waves propagated along the surface are helices, and the angle of each helix depends on the phase velocity of the corresponding surface wave [4].

Accordingly as the plane wave is obliquely incident on the cylindrical shell, the guided wave is created and propagates along helical paths in the cylindrical shell [5]. Figure 1 shows the resonance spectrum for some partial waves (n=15) with $\alpha = 0^{\circ}$, 3° and 5° . When compared with the normal incidence, the new peaks (or modes) observed on the spectrum are expected to be the guided wave. These additional modes are called guided T modes [6]. With on increase of α , all the resonance peaks are shifted up.



Figure 2. Resonance spectrum of the partial wave (n = 15)

The total form functions in the case of a normal ($\alpha = 0^{\circ}$) and oblique ($\alpha = 3^{\circ}, 45^{\circ}$) incidence in a frequency range of $ka = 0 \sim 25$ are shown in Fig. 3. In this result, certain peaks exist for the small incident angle. These newly created peaks are expected to be axially dependent T modes. Beyond the critical incident angle, no elastic resonance can be detected for the form function.



Figure 3. Total form function of Zr shells for various incident angle: (a) $\alpha = 0^{\circ}$; (b) $\alpha = 3^{\circ}$; (c) $\alpha = 45^{\circ}$

4. Conclusion

The acoustic scattering of a normally and obliquely incident plane acoustic wave by an isotropic cylindrical shell immersed in a fluid has been discussed. When the wave is obliquely incident on the cylindrical shell, all the resonance peaks are shifted up to higher frequencies with an increasing incident angle α . The new mode waves are observed in the resonance spectrum. These modes are expected to be axially guided T modes and can be used to measure the defects in the axial direction of the shell.

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