Kernel Regression based Noise Smoothing of Reactivity Signal

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1. Introduction

Reactor deviation from the critical state is a parameter of specific interest defined by the reactivity (ρ). Reactivity ρ is an extremely important quantity used to define many of the reactor startup physics parameters. The time-dependent reactivity is normally determined by solving the using inverse neutron kinetics equation. The reactivity computer is a device to provide an on-line solution of the inverse kinetics equation. The DDRCS™ (Direct Digital Reactivity Computer System) is presently used in Korean nuclear power plants during initial plant startup and for recurrent physics tests. The input signal to DDRCS ™ is provided by the excore neutron detectors. The measurement signal of the neutron density is normally noise corrupted since the measurement system has no provision for a neutron distribution which neutrons are generated from fission or non-fission process. Therefore the neutron signal or the calculated reactivity should be filtered properly to give sufficiently large signal-to-noise ratio to prevent a high degree of interpretational uncertainty. This paper describes a kernel regression based noise smoother for that purpose.

Kernel based methods are most popular non-parametric estimators which can uncover structural features in the data which a parametric approach might not reveal.

In this paper, the performance of the kernel regression smoothing method is demonstrated for the measured reactivity signal contaminated with noise. The results show the developed smoother can be applied not only the noise smoothing but also bumpless follow of the signal with non-smooth edge.

2. Methods and Results

In this section some of the mathematical techniques in kernel regression are described.

2.1 Kernel Regression

Kernel regression is an old method for smoothing data still new work continues at a rapid pace. Kernel regression of statistics was derived independently by Nadaraya[1] and Watson[2]. Kernel regression[3] is the estimation of the functional relationship $y(t)$ between two variables y and t. Measurement produces a set of random variables $\{t_i, y_i; i=1,...,N\}$ on the interval $\{0 \le t_i \le T\}$. It is assumed that

$$y_i = y(t_i) + \varepsilon \tag{1}$$

where ε is a random noise variable with the mean equal to zero. The Nadaraya–Watson kernel regression estimate $y(t)$ of at $t=\tau$ from this random data is defined as the estimator $\hat{y}(\tau)$ as

$$\hat{y}(\tau) = \frac{\sum_{i=1}^{N} y_i k(\tau - t_i)}{\sum_{i=1}^{N} k(\tau - t_i)}. \tag{2}$$

The function $k(\tau - t_i)$ is the kernel function which can be chosen from a wide variety of symmetric functions. In this paper, the Gaussian density function is used, i.e.,

$$k(t) = \exp(-D(t_i, t_q)^2 / \sigma^2) \tag{3}$$

where D is the distance metric and Euclidean distance is used here defined by

$$D(t_i, t_q) = \|t_i - t_q\| = \sqrt{(t_i - t_q)^2}. \tag{4}$$

t_q is the query point where the smoothed signal is to be generated in the interval of time series data $\{0 \le t_i \le T\}$. σ^2 is the bandwidth of the kernel. σ^2 is a scaling factor which controls how wide the influencing measurements are spread around a query point. Bandwidth can also control the smoothness or roughness of a density estimate. Increasing the kernel width σ^2 means further away points get an opportunity to influence the query point. As $\sigma^2 \to \infty$, the smoothed signal tends to the global average.

2.2 Illustrative Example

Figure 1 shows an example of the smoothing performance. For $\sigma^2 = 10$, it is nice to see a smooth curve at last but rather bumpy. If σ^2 gets any higher, the smoothing is poor. As increasing σ^2, it is clearly not capturing the structure of the data. Note the smoothed curve becomes almost the average value of the raw data for $\sigma^2 = 500$. Large values of σ^2 produce high bias and low variance and vice versa. Experience shows what σ^2 we choose is more important than which kernel we choose.

Figure 2 gives the calculated weights for each values of σ^2. The weights shown are calculated for influencing the query point $t_i = 15.0$ for illustrative purpose. We can find only 2~3 neighboring points have non-zero weights for $\sigma^2 = 10$. For $\sigma^2 = 500$, all of the data points are influencing the query point $t_i = 15.0$

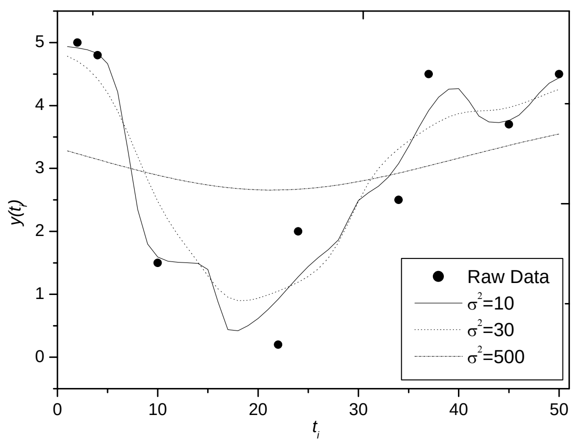

Figure 1. Effect of smoothing with σ^2 variation

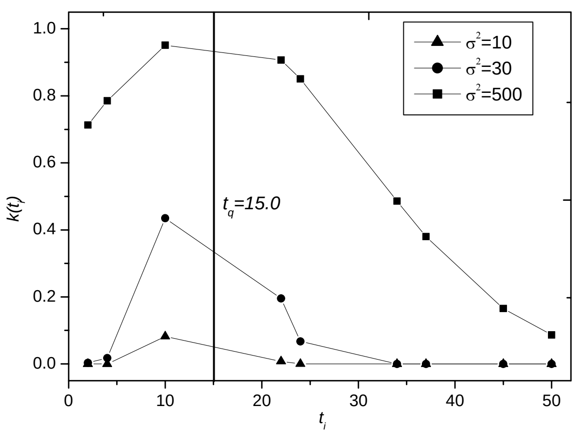

Figure 2. Variation of kernel $k(t_i)$ with σ^2

2.3 Reactivity Smoothing

Figure 3 shows the smoothing performance of the kernel smoother for noisy reactivity signal measured during external dynamic test at Kori unit 4 cycle 16. As shown in the figure, the reactivity estimation error can increase up to 4~5 pcm if ad hoc averaging is used. In some cases, this amount of erroneous estimate can affect the reactor physics test results. The kernel bandwidth $\sigma^2 = 10$ is used. The smoothing performance is definitely superior to Savitzky-Golay smoother or FFT filter.

Figure 4 gives the smoothing performance of the kernel smoother close to edge or corner point. Kernel smoother can reconstruct the original signal with minimum over- or undershot compared with well known smoothing algorithms. This kind of stepwise variation of reactivity is frequently found during reactor startup physics test from movement of control rods. Therefore the reconstruction capability of the edge points is an important characteristic of the reactivity smoother.

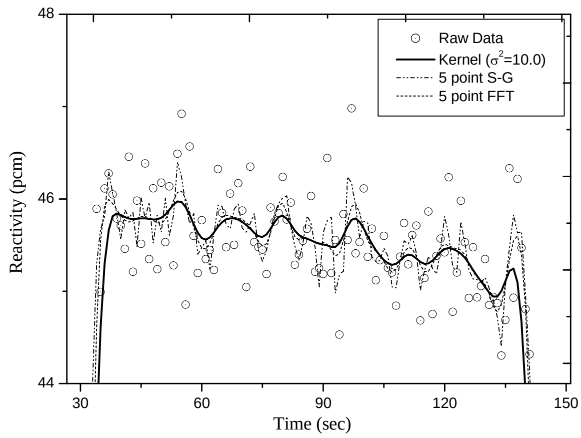

Figure 3. Noisy reactivity measurement during external dynamic test at Kori unit 4 cycle 16.

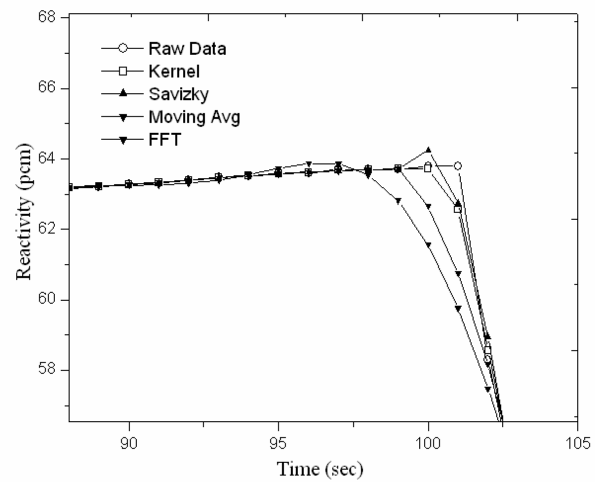

Figure 4. Performance of kernel smoother close to edge or corner point

3. Conclusion

Kernel based noise smoothing technique is developed and successfully applied to digital reactivity meter. Using the smoother, the reactivity estimation error can be minimized during the reactor startup physics tests. The performance of the algorithm is demonstrated comparing with well known smoothing methods for real measurement signal. The method can also be applied to edge or corner points important to reactivity variation.

REFERENCES

[1] E. A. Nadaraya, "On estimating regression," Theory Probab. Appl., vol. 10, pp. 186–190, 1964.

[2] G. S.Watson, "Smooth regression analysis," Sankhya Ser. A, vol. 26, pp. 101–116, 1964.

[3] A. Moore et. al., "Efficient Locally Weighted Polynomial Regression Predictions," Proceedings of the Fourteenth International Conference on Machine Learning, pp. 236-244, 1997.