Two-Group CMFD Accelerated Multigroup Calculation with Semi-Analytic Nodal Kernel

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1. Introduction

The Coarse Mesh Finite Difference (CMFD) formulation or the nonlinear nodal method [1] has been widely used in the solution of two-group problems providing high solution efficiency. In this paper, the two-group formulation is applied to multigroup problems as a means of acceleration. First, a new form of the semi-analytic nodal method (SANM) [2] is derived to solve a multi-group one-node problem, and the SANM kernel is then implemented within the framework of the two-group CMFD formulation involving dynamic group condensation [3]. The efficiency of this scheme is investigated with a set of fast reactor problems.

2. One-Node Semi-Analytic Kernel

With the source iteration scheme, the SANM can be applied easily to multi-group problems given the fission and scattering sources approximated by high order polynomials. For a one-node problem, a new SANM formulation is derived below with an emphasis on the simultaneous solution of the three directional solutions.

The transverse-integrated multi-group diffusion equation can be written as:

$$-\mathbf{D}\frac{d^2}{du^2}\boldsymbol{\phi}_u(u)+\mathbf{R}\boldsymbol{\phi}_u(u)=\frac{1}{k_{eff}}\mathbf{F}\boldsymbol{\phi}_u(u)+\mathbf{S}\boldsymbol{\phi}_u(u)-\mathbf{L}_u(u) \quad (1)$$

where

$$\mathbf{R}=\begin{bmatrix}\Sigma_{r1} & & 0\\ & \ddots & \\ 0 & & \Sigma_{rG}\end{bmatrix},\ \mathbf{F}=\begin{bmatrix}\chi_1\Sigma_{f1} & \cdots & \chi_1\Sigma_{fG}\\ \vdots & \ddots & \vdots\\ \chi_G\Sigma_{f1} & \cdots & \chi_G\Sigma_{fG}\end{bmatrix}, \text{ and}$$

$$\mathbf{S}=\begin{bmatrix}0 & \Sigma_{s1g} & \Sigma_{s1G}\\ \Sigma_{sg1} & \ddots & \Sigma_{sgG}\\ \Sigma_{sG1} & \Sigma_{sGg} & 0\end{bmatrix}.$$

Here, the right hand side, defined as the effective source term, is approximated by a quadratic polynomial in the following form:

$$\tilde{Q}(u)=\sum_{i=0}^{4}g_i\left(\frac{u}{a_u}\right)\tilde{Q}_{ig} \quad (2)$$

$$g_0(\tau)=1,\ g_1(\tau)=2\tau,\ g_2(\tau)=6\tau^2-1/2,$$
$$g_3(\tau)=2\tau(\tau-1/2)(\tau+1/2),$$
$$g_4(\tau)=(5\tau^2-1/4)(\tau-1/2)(\tau+1/2)$$

Given this approximation, one can obtain the analytic solution Eq. (1) which consists of the homogeneous and particular solutions of the following form:

$$\phi_{ug}(u)=\sum_{i=0}^{6}C_{ig}g_i(u/a_u)\ , \quad (3)$$
$$g_5(\tau)=\sinh(2\tilde{\kappa}_g\tau),\ g_6(\tau)=\cosh(2\tilde{\kappa}_g\tau),$$
$$\tilde{\kappa}_g=\frac{a_u\sqrt{\Sigma_{rg}/D_g}}{2}$$

The seven coefficients of the transverse integrated one-dimensional (TI1D) flux, Eq.(3), can be expressed in terms of the node average flux, two interface incoming currents, and four Weighted Residual Method (WRM) equations.

Once the coefficients are determined, the node average flux and outgoing partial currents can be obtained from the following relations:

$$j^+_{urg}-T_0\bar{\phi}=T_1C_{1g}+T_2C_{2g}+T_3C_{3g}+T_4C_{4g}+T_5j^+_{ulg}+T_6j^-_{urg}$$
$$j^-_{ulg}-T_0\bar{\phi}=-T_1C_{1g}+T_2C_{2g}-T_3C_{3g}+T_4C_{4g}+T_5j^-_{urg}+T_6j^+_{ulg}$$
$$\sum_{u=x,y,z}\frac{1}{a_u}\left(j^+_{urg}+j^-_{ulg}\right)+\Sigma_{rg}\bar{\phi}_g=\tilde{H}_g+\sum_{u=x,y,z}\frac{1}{a_u}\left(j^+_{ulg}+j^-_{urg}\right)$$
$$\tilde{H}_g=\frac{\chi_g}{k_{eff}}\sum_{g'=1}^{G}\nu\Sigma_{fg'}\bar{\phi}_{g'}+\sum_{\substack{g'=1\\ g'\neq g}}^{G}\Sigma_{sgg'}\bar{\phi}_{g'}+\bar{S}_g \quad (4)$$

Here, in the three dimensional problem, the linear system consisting of each directional outgoing currents and the node average flux are reduced to the following single matrix form:

$$\begin{bmatrix}1&0&0&0&0&0&a_{17}\\0&1&0&0&0&0&a_{27}\\0&0&1&0&0&0&a_{37}\\0&0&0&1&0&0&a_{47}\\0&0&0&0&1&0&a_{57}\\0&0&0&0&0&1&a_{67}\\a_{71}&a_{72}&a_{72}&a_{72}&a_{72}&a_{72}&a_{77}\end{bmatrix}\begin{bmatrix}j^+_{xrg}\\j^-_{xlg}\\j^+_{yrg}\\j^-_{ylg}\\j^+_{zrg}\\j^-_{zlg}\\\bar{\phi}_g\end{bmatrix}=\begin{bmatrix}b_1\\b_2\\b_3\\b_4\\b_5\\b_6\\b_7\end{bmatrix} \quad (5)$$

The unknowns of Eq.(5) can be determined uniquely through the backward substitution. The traditional method is to obtain the approximate solution of Eq. (5) by first determining the node average flux using the incoming currents and then updating outgoing currents using the new node average flux. We shall denote the simultaneously solved solution by SANM1 and the conventional solution by SANM2 for the comparison of the efficiency of these methods in Section 4.

3. Two-Group CMFD Formulation

The multi-group outgoing currents and node average flux determined by the one multi-group nodal kernel are then condensed to two-group ones by using them as the outgoing current and flux spectra. The two-group cross sections for CMFD equation are generated by spectrum weighting as follows:

$$\varphi_g = \frac{\bar{\phi}_g}{\bar{\phi}_G},\ \bar{\phi}_G = \sum_{g\in G}\bar{\phi}_g \quad (G=1,2), \tag{6}$$

$$\varsigma_{sg} = \frac{j_{sg}^{out}}{j_{sG}^{out}},\ j_{sG}^{out} = \sum_{g\in G} j_{sg}^{out} \quad (G=1,2), \text{ and}$$

$$\Sigma_{XG} = \sum_{g\in G}\Sigma_{Xg}\varphi_g$$

In addition, the two-group corrective nodal coupling coefficients are determined by the following equation:

$$\tilde{D}_{sG} = -\frac{\hat{D}_{sG}(\bar{\phi}_G^R - \bar{\phi}_G^L) + J_{sG}}{\bar{\phi}_G^R + \bar{\phi}_G^L} \quad \text{and} \tag{7}$$

$$\Gamma_{sG} = \frac{\dfrac{\beta_G^R\bar{\phi}_G^R + \beta_G^L\bar{\phi}_G^L}{\beta_G^R + \beta_G^L} - \phi_{sG}}{\bar{\phi}_G^R + \bar{\phi}_G^L}$$

In the CMFD problem, the two-group interface currents are represented using the corrective coefficients as the following form:

$$J_{sG} = -\hat{D}_{sG}(\bar{\phi}_G^R - \bar{\phi}_G^L) - \tilde{D}_{sG}(\bar{\phi}_G^R + \bar{\phi}_G^L) \quad . \tag{8}$$

Once a two-group CMFD solution is obtained, the multi-group flux and currents are computed from previously stored flux and outgoing current spectra, and then multi-group nodal iterations are performed.

$$\bar{\phi}_g = \varphi_g\bar{\phi}_G \ (g\in G),\ j_{sg}^{out} = \varsigma_{sg} j_{sG}^{out} \tag{9}$$

The following flowchart represents the entire process of the alternating two-group CMFD and multigroup nodal calculations.

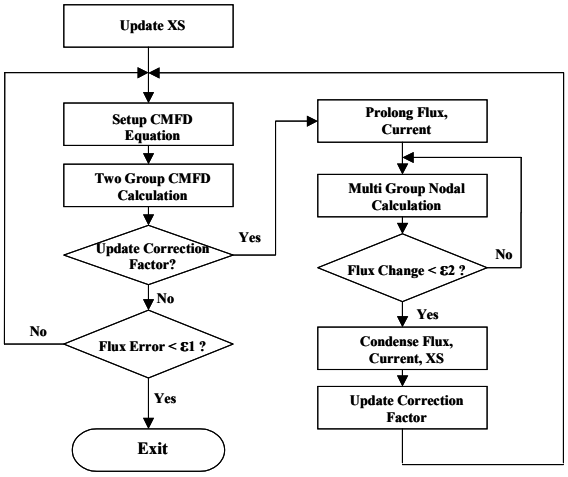

Figure.1 Flowchart of the nonlinear iteration scheme

4. Numerical Results and Conclusion

In order to examine the efficiency of the presented multi-group nonlinear nodal method, 9-group eigenvalue problems has been analyzed and the convergence characteristics were evaluated. The model core has the same geometry as that of PEACER300, a fast-spectrum transmutation reactor characterized by a pan-cake type core shape, but the cross sections used were obtained from the BFS75 9-group problem.

The eigenvalue, the number of outer iterations and computing time are listed in Table 1. First, it is noted that the number of outer iterations and the number of nodal updates are reduced significantly and the calculation time as well. Another point noted is that SANM1 spends more computing time than SANM2 in the nodal calculation due to the matrix solution whereas the time is less in the nonlinear nodal. This implies that the better convergence characteristics of SANM1 appears significant in the nonlinear nodal calculation involving much fewer outer iterations.

9G 3D Reactor 1 Node/FA				
	SANM1		SANM2	
	Nodal	Nonlinear Nodal	Nodal	Nonlinear Nodal
k_eff	1.05436	1.05436	1.05436	1.05436
Iterations	536	15	543	20
Nodal updates	536	28	543	37
Time (sec)	217.50	16.41	193.53	19.46

9G 3D Reactor 4 Node/FA				
	SANM1		SANM2	
	Nodal	Nonlinear Nodal	Nodal	Nonlinear Nodal
k_eff	1.05411	1.05411	1.05411	1.05411
Iterations	661	18	670	21
Nodal updates	661	32	670	40
Time (sec)	1053.48	74.91	947.40	84.27

Table.1 Comparison of nonlinear nodal calculation

In conclusion, a new SANM formulation for the one-node problem has been derived and a two-group CMFD formulation with the multi-group SANM kernel has been established. The multi-group calculations demonstrate that the computational efficiency increases significantly by the two-group CMFD formulation and the convergence behavior is superior with the new SANM formulation involving simultaneous solution of three directional outgoing fluxes and node average flux. This method proposed here will further be extended to multi-group transient calculations which would require excessive computing time.

REFERENCES

[1] K. S. Smith, "Nodal Method Storage Reduction by Nonlinear Iteration," *Trans. Am. Nucl. Soc.*, 44, 265 (1983)

[2] R. A. Shober, "A Nodal Method for Fast Reactor Analysis," *proc. Computational Methods in Nuclear Engineering*. ANS, Williamsburg, 1979.

[3] H. G. Joo, J. Y. Cho, J. S. Song, and S. Q. Zee, "Multi-Group Transient Calculation within The Framework of A Two-Group Hexagonal CMFD Formulation," *M&C 2001* Salt Lake City, 2001.