

Evaluation of Truncation Error in Minimal Cut Set-Based Fault Tree Analysis

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1. Introduction

Truncation neglecting low-probability cut sets is usually applied to determine the minimal cut sets of a manageable size in probabilistic safety assessment (PSA). The truncation error has been of great concern in PSAs because risk and important measures are only computed from an incomplete cut set equation. A truncation error evaluation method, which is based on Monte Carlo techniques and the characteristics of coherent reliability systems, is proposed in this paper. The applicability and accuracy of the proposed method are described.

2. Method and Results

This section presents the mathematical formulation of truncation errors and the proposed algorithm for quantifying truncation errors in MCS-based fault tree analyses.

2.1 Mathematical Formulation of Truncation Errors

We consider a coherent fault tree consisting of k basic events $\mathbf{B} = (b_1, \dots, b_k)$. The structure function $\phi(\mathbf{X})$ of the fault tree is a deterministic binary function of the basic event state vector $\mathbf{X} = (x_1, \dots, x_k)$. When the top event is occurring, $\phi(\mathbf{X})$ is equal to 1. We denote the probability of the top event by $h(\mathbf{p}) = E[\phi(\mathbf{X})]$ where $\mathbf{p} = (p_1, \dots, p_k)$ and $p_i = \Pr\{x_i = 1\}$. Similarly, the functions with the probability of a specific basic event b_i set to 0 and 1 are denoted by $h(0_i, \mathbf{p}) = E[\phi(0_i, \mathbf{X})]$ and $h(1_i, \mathbf{p}) = E[\phi(1_i, \mathbf{X})]$, respectively [1,2].

In evaluating fault trees with probability-based truncation, only the MCSs with probability above pre-established cut-off value V_c are developed and the other cut sets are discarded. The developed MCSs, $\mathbf{K}_1, \dots, \mathbf{K}_m$, have probability greater than V_c . We can define a new structure function in terms of only the developed MCSs as follows:

$$\phi^m(\mathbf{X}) \equiv \max_{1 \leq j \leq m} \min_{i \in \mathbf{K}_j} x_i. \quad (1)$$

Let us define a new binary function $\delta(\mathbf{X})$ associated with the truncation error as:

$$\delta(\mathbf{X}) \equiv \phi(\mathbf{X}) - \phi^m(\mathbf{X}). \quad (2)$$

Then, the actual truncation error can be written as:

$$d(\mathbf{p}) = h(\mathbf{p}) - h^m(\mathbf{p}) = E[\delta(\mathbf{X})]. \quad (3)$$

Let us define a new value $W(\mathbf{X})$ related to the occurrence probability of $\mathbf{B}_1(\mathbf{X}) = \{b_i | x_i = 1\}$ for a sample state vector \mathbf{X} as:

$$W(\mathbf{X}) \equiv \prod_{b_i \in \mathbf{B}_1(\mathbf{X})} p_i = \prod_{i=1}^k p_i^{x_i}. \quad (4)$$

The binary function $\delta(\mathbf{X})$ has the following properties:

P1: If $W(\mathbf{X}) > V_c$, then $\delta(\mathbf{X}) = 0$.

P2: If $\delta(\mathbf{X}) = 0$, then $\delta(0_i, \mathbf{X}) = 0$ for all i .

P3: If $\delta(\mathbf{X}) = 1$ and $b_i \notin \mathbf{B}_1(\mathbf{X})$, then $\delta(0_i, \mathbf{X}) = 1$.

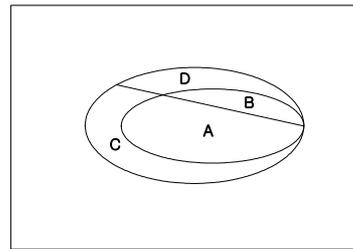


Figure 1. Relations among the binary functions: $A \cup B \cup C \cup D = \{\mathbf{X} | \phi(\mathbf{X}) = 1\}$, $A \cup B = \{\mathbf{X} | \phi^m(\mathbf{X}) = 1\}$, $A \cup C = \{\mathbf{X} | \phi(0_i, \mathbf{X}) = 1\}$, $A = \{\mathbf{X} | \phi^m(0_i, \mathbf{X}) = 1\}$, $C \cup D = \{\mathbf{X} | \delta(\mathbf{X}) = 1\}$ and $C = \{\mathbf{X} | \delta(0_i, \mathbf{X}) = 1\}$.

2.2 Evaluation of Truncation Errors

We developed a Monte Carlo algorithm to quantify the truncation errors for coherent fault trees. The proposed algorithm determines a random state vector $\mathbf{X}_s = (x_1, \dots, x_k)$ by the dagger sampling and its binary values $\delta(\mathbf{X})$ and $\delta(0_i, \mathbf{X})$ using their properties.

Kumamoto *et al.* [3] proposed the dagger sampling method, which appreciably reduces computing time to use a large number of trials for rare-event problems.

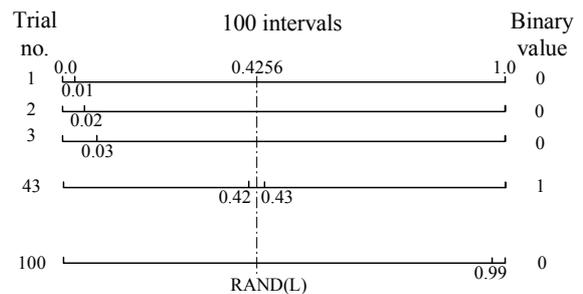


Figure 2. Generation of 100 samples for event with probability 0.01 by the dagger sampling method.

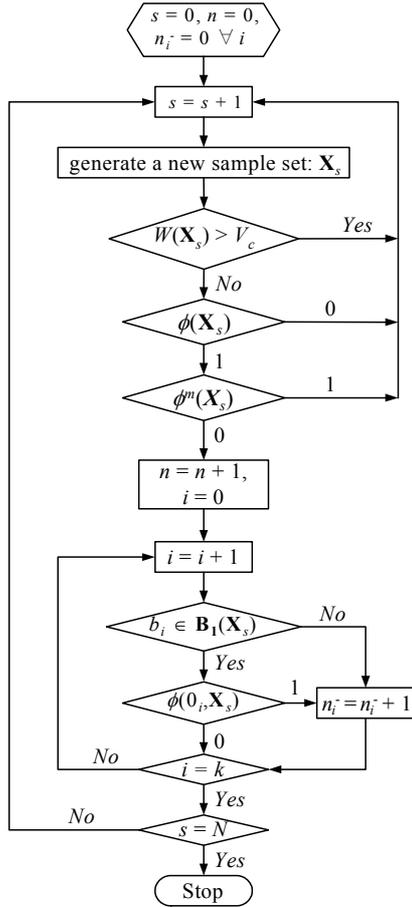


Figure 3. Flow chart for the proposed algorithm.

We have $n = \sum_{s=1}^N \delta(\mathbf{X}_s)$ and $n_i^- = \sum_{s=1}^N \delta(0_i, \mathbf{X}_s) \forall i$

from a proposed Monte Carlo simulation with sample size N . Then the truncation errors can be estimated by the unbiased binominal estimators, $\hat{d}(\mathbf{p}) = \frac{n}{N}$ and

$\hat{d}(0_i, \mathbf{p}) = \frac{n_i^-}{N} \forall i$. The variances of $\hat{d}(\mathbf{p})$ and $\hat{d}(0_i, \mathbf{p}) \forall i$

can be estimated by the following equations:

$$S^2(\hat{d}(\mathbf{p})) = \frac{1}{N-1} \hat{d}(\mathbf{p})(1-\hat{d}(\mathbf{p})), \quad (5)$$

$$S^2(\hat{d}(0_i, \mathbf{p})) = \frac{1}{N-1} \hat{d}(0_i, \mathbf{p})(1-\hat{d}(0_i, \mathbf{p})). \quad (6)$$

Inferring from the characteristics of coherent reliability systems, we get for all i

$$\hat{d}(1_i, \mathbf{p}) = \frac{\hat{d}(\mathbf{p}) - \hat{d}(0_i, \mathbf{p})}{p_i} + \hat{d}(0_i, \mathbf{p}). \quad (7)$$

2.3 Numerical Example

The example is a binary coherent fault tree, which contains 613 logic gates and 677 independent basic events. All runs were performed on a 2 GHz Pentium

IV using the FORTRAN program based on the proposed algorithm.

Table 1 gives simulation results of the truncation errors for 5 different cut-off value cases with a sample size $N = 10^8$. Table 2 provides an example simulation result in terms of importance measures of some basic events that are selected in the order of change ratio of RRW. This result is obtained from one simulation with $N = 10^8$ and $V_c = 10^{-8}$.

Table 1. Truncation errors (with $N = 10^8$)

V_c	# of MCSs	$h^m(\mathbf{p})$	$\hat{d}(\mathbf{p})$	95% Limit of $\hat{d}(\mathbf{p})$	CPU time (sec.)
1E-8	230	2.4164E-4	2.01E-6	2.24E-06	404.03
1E-10	2245	2.4407E-4	6.00E-8	1.00E-07	179.72
1E-12	12201	2.4420E-4	2.00E-8	4.33E-08	98.92
1E-14	51321	2.4420E-4	1.00E-8	2.65E-08	71.95
1E-15	103170	2.4420E-4	0.0	0.0	65.97

Table 2. Importance measures considering truncation errors (with $N = 10^8$ and $V_c = 10^{-8}$)

i	p_i	RRW _m ^a	RRW ^b	RAW _m ^c	RAW ^d
4	1.0	1.0878	1.0943	1.0000	1.0000
93	7.41E-3	1.0623	1.0638	8.8585	9.0392
94	7.41E-3	1.0645	1.0660	9.1182	9.2914
7	1.0	1.0043	1.0056	1.0000	1.0000
242	1.20E-3	1.0087	1.0097	8.1892	8.9875
345	3.40E-4	1.0025	1.0034	8.4556	11.0534

^aRRW_m = $h^m(\mathbf{p})/h^m(0_i, \mathbf{p})$ ^bRRW = $\{h^m(\mathbf{p}) + \hat{d}(\mathbf{p})\} / \{h^m(0_i, \mathbf{p}) + \hat{d}(0_i, \mathbf{p})\}$

^cRAW_m = $h^m(1_i, \mathbf{p})/h^m(\mathbf{p})$ ^dRAW = $h^m(1_i, \mathbf{p}) + \hat{d}(1_i, \mathbf{p})\} / \{h^m(\mathbf{p}) + \hat{d}(\mathbf{p})\}$

3. Conclusions

This paper presents a practical tool to evaluate truncation errors in minimal cut set-based fault tree analyses. This method can be easily applied in PSAs and does not require much computation time. The method will be useful in reviewing PSA results, particularly for the regulatory purpose.

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