An Evaluation of Effective Atomic Number in Compton Scattering System Using Low-Energy Gamma Rays

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1. Introduction

The attenuation coefficient, the electron density and the effective atomic number are basic quantities in determining the penetration of gamma rays in matter. There are many applications in the medical radiation dosimetry, medical imaging and inspection systems using gamma rays where it is required to determine the effective atomic number, Z_{eff}, of target materials. [1, 2] However, it is not easy to find the effective atomic number in a transmission gamma radiation system. Scattered gamma rays provide useful additional information as they have interacted with the materials of interest. The effective atomic number is determined in several ways depending on the type of photon scattering processes considered in the scattering system. [3]

In this study, a new approach using the ratio of the total attenuation coefficient to the incoherent attenuation coefficient is suggested for evaluating the Z_{eff} number of compound materials in a Compton scattering system with a Co-57 gamma source. In order to achieve acceptable accuracy of the approach, the ratio is processed by using the least squares method with many reference compounds.

2. Method and Results

1. Theory

For the low energy range of gamma rays where the pair production effect does not exist, the mass attenuation coefficient μ_t of an individual element can be given as a sum of three terms: photoelectric absorption μ_{pe}, Rayleigh scattering μ_R and Compton scattering μ_C

$$\frac{\mu_t}{\rho} = \frac{1}{\rho}\left(\mu_C + \mu_{pe} + \mu_R\right) = \frac{N_A}{A}\left(Z\sigma_C + \sigma_\tau + \sigma_R\right) \quad (1)$$

where ρ, A and Z are the density, mass number and atomic number, respectively. N_A is the Avogadro's number. The Compton scattering cross section σ_C is given by the Klein-Nishina formula. The photoelectric effect and Rayleigh scattering cross section, σ_τ and σ_R, can be determined by the following empirical formula

$$\sigma_C = \sigma_{KN},\ \sigma_\tau = a\frac{Z^m}{E^k} \text{ and } \sigma_R = b\frac{Z^n}{E^l} \quad (2)$$

where the constants a, b, m, k, n and l are determined to give the best fit to actual attenuation coefficients.

For a chemical compound a single number cannot present the atomic number uniquely across the entire energy region. Thus, an effective atomic number is used for characterizing the compound and the partial process of gamma rays in the material. The number is defined as

$$Z_{C,\tau,R} = \left(\sum_i w_i Z_i^q\right)^{1/q} \quad (3)$$

where q is 1, m and n for the effective atomic number Z_C in Compton scattering, for Z_τ in photoelectric effect and for Z_R in Rayleigh scattering, respectively. w_i is the weight fraction of the i^{th} element in the compound. Hence the total mass attenuation coefficient can be expressed as

$$\frac{\mu_t}{\rho} = \frac{N_A}{A}\left(Z_C\sigma_{KN} + a\frac{Z_\tau^m}{E^k} + b\frac{Z_R^n}{E^l}\right) \quad (4)$$

Although all the effective numbers Z_C, Z_τ and Z_R are related each other, for an unknown compound it is not easy to determine each of them by solving Eq.(4) because they can be seen as independent unknowns.

2. Calculation of Effective Atomic Number for a Compton Scattering System

In practice it is actually good enough to know just one effective atomic number among the effective atomic numbers defined in Eq.(3). To obtain an effective atomic number of compounds in a scattering system in which the total and Compton scattering attenuation coefficients can be obtained with measurements,[4] a new approach is suggested. Making a ratio μ_t/μ_C, one obtains

$$\frac{\mu_t}{\mu_C} = \frac{1}{\sigma_{KN}}\left(\sigma_{KN} + a\frac{Z^{m-1}}{E^k} + b\frac{Z^{n-1}}{E^l}\right) \quad (5)$$

In Eq.(5), σ_{KN} is independent of the atomic number. The term of Z^{n-1}, which represents for Rayleigh scattering, contributes a valid value to the total attenuation coefficient only when the gamma rays are scattered in a small angle or with high energy. Hence it can be assumed that the effective atomic number Z_{eff} of the compound is defined by

$$Z_{eff} = \left(\sum w_i Z_i^{m-1}\right)^{1/m-1} \quad (6)$$

where the value of m can be determined by applying Eq.(2) for the attenuation coefficients of the elements.

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In order to obtain the Z_{eff} the ratio μ_t/μ_C is expressed in the form of a polynomial function. The coefficients of the polynomial function can be determined by using any cross section database for many compounds. Most biological tissues and compounds used in clinical dosimetry have Z_{eff} numbers for all photon processes in the range from 5 to 35. Z_{eff} numbers of drug and explosive materials which are the most interested objects of inspection systems are in this range. Hence for illustrating the method about 200 compounds having Z_{eff} in the range from 5 to 35 are chosen, the attenuation coefficients were obtained from XCOM Database[5]. The ratios μ_t/μ_C of these compounds and the fitting curves for gamma energy at 98keV and 122keV are shown in Fig. 2 as an example. The degree of the polynomial function was chosen to give satisfactory fit to the actual ratios μ_t/μ_C of the compounds.

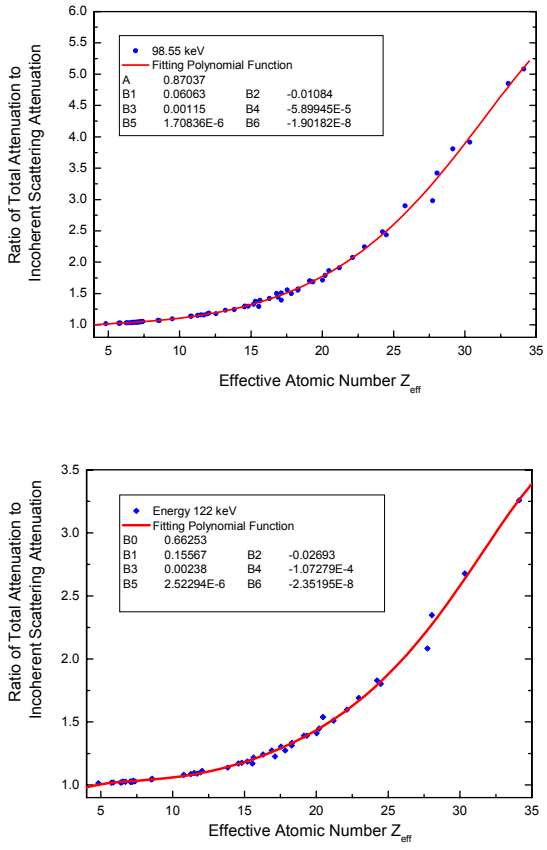

Fig. 2 Determination of the coefficients of the polynomial function of the ratio μ_t/μ_C for 98keV and 122keV energy

Figure 2 shows that the ratio μ_t/μ_C increases as Z_{eff} increases, therefore the polynomial functions of the ratio μ_t/μ_C can be easily solved by a numerical calculation. The calculation results for some common compounds which have the Z_{eff} in the interested range, from 5 to 35, at 98keV and 122keV energies are shown in Table 1 as an example. The error in the Table is the percentage error of the Z_{eff} calculated from the ratio μ_t/μ_C of the compounds by the bisect numerical method for both energies relative to the Z_{eff} obtained from Eq.(6). The errors about 2.5% demonstrate that the suggested method is a valid approach to evaluate the Z_{eff} of compounds by a scattering system.

Table 1 Calculated values of Z_{eff} for some common compounds

Material	Z_{eff}	Calculated Z_{eff}		Error (%)	
		98keV	122keV	98keV	122keV
Ethylene	5.740	5.747	5.725	0.11	0.26
Bakelite	6.372	6.422	6.416	0.78	0.69
TNT	7.198	7.288	7.274	1.24	1.06
Teflon	8.498	8.686	8.647	2.21	1.75
PVC	14.530	14.504	14.417	0.18	0.78
Steel	25.807	26.449	26.319	2.49	1.99
Brass	30.415	30.800	30.655	1.27	0.79

3. Conclusions

A new simple evaluation of the effective atomic number Z_{eff} of material in a Compton scattering system is performed by the expression of the ratio μ_t/μ_C as a polynomial function of Z_{eff}. The method can be applied to the low range of gamma energies where the Compton scattering does not dominate. For the compounds which have Z_{eff} from 5 to 35, the calculations have shown that the results are given with reasonable errors, less than 2.5%, good enough to be used in any further calculation of medical science or inspection purposes.

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