

## Verification of the Analytic Function Expansion Nodal (AFEN) Method in Cylindrical (r,θ,z) Geometry for Pebble Bed Reactors

Nam Zin Cho, Joo Hee Lee, Jaejun Lee, and Do Sam Kim<sup>a)</sup>

Korea Advanced Institute of Science and Technology, Department of Nuclear and Quantum Engineering  
373-1 Kusong-dong, Yusong-gu, Daejeon, Korea; nzcho@kaist.ac.kr

### 1. Introduction

There is strong desire of making available high fidelity nodal codes in cylindrical (r,θ,z) geometry for pebble bed reactors[1]. The analytic function expansion nodal (AFEN) method developed quite extensively in Cartesian (x, y, z) geometry and in hexagonal-z geometry is a unique nodal method in that it does not use transverse integration[2]. Kim and Cho[3] developed an AFEN method in (r,z) coordinates. Recently, Cho et al[4, 5] extended the AFEN method to fully three-dimensional (r,θ,z) geometry. This paper provides further results, verifying the AFEN method in this unusual geometry for conventional nodal methods.

### 2. Basic Theory and Method

Assume that the core in (r,θ,z) geometry is discretized into  $N_r \times N_\theta \times N_z$  nodes, where  $N_r$ ,  $N_\theta$ , and  $N_z$  are the numbers of divisions in radial, azimuthal, and axial directions, respectively. The AFEN formulation in the (r,θ,z) coordinates system starts from the following two-group diffusion equations in a homogenized node :

$$-\nabla^2 \vec{\phi}(r, \theta, z) + [A] \vec{\phi}(r, \theta, z) = 0, \quad (1)$$

where

$$[A] = [D]^{-1} \left( [\Sigma] - \frac{1}{k_{eff}} [\chi][\nu\Sigma_f] \right).$$

All the notations are standard. The equations can be decoupled as follows:

$$\frac{\partial^2 \xi_\mu}{\partial r^2} + \frac{1}{r} \frac{\partial \xi_\mu}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \xi_\mu}{\partial \theta^2} + \frac{\partial^2 \xi_\mu}{\partial z^2} - \lambda_\mu \xi_\mu = 0, \quad (2)$$

where

$$\vec{\xi} = (\xi_1, \xi_2)^T = [R]^{-1} \vec{\phi}, \quad [R] = [e_1, e_2],$$

$\lambda_\mu, e_\mu (\mu = 1, 2)$  : eigenvalues and corresponding eigenvectors of [A]

A general solution to Eq. (2) can be represented in terms of analytic basis functions that can be obtained using the method of separation of variables. For practical implementation, we choose the modal solution  $\xi_\mu$  of a node expressed in a finite number of terms.

### 2.1. Outer Nodes

We use the following expression for an outer node:

$$\begin{aligned} \xi_\mu(r, \theta, z) = & a_{\mu 0} + a_{\mu 1} S(\kappa_\mu z) + a_{\mu 2} C(\kappa_\mu z) \\ & + a_{\mu 3} F_0(\kappa_\mu r) + a_{\mu 4} G_0(\kappa_\mu r) \\ & + [a_{\mu 5} \sin(\theta) + a_{\mu 6} \cos(\theta)] \\ & \times [a_{\mu 7} F_1(\kappa_\mu r) + a_{\mu 8} G_1(\kappa_\mu r)] \\ & + a_{\mu 9} z F_0(\kappa_\mu r) + a_{\mu 10} z G_0(\kappa_\mu r) \\ & + [a_{\mu 11} S(\kappa_\mu z) + a_{\mu 12} C(\kappa_\mu z)] \ln(r), \quad \mu = 1, 2, \end{aligned} \quad (3)$$

where  $\kappa_\mu = \sqrt{|\lambda_\mu|}$ , and other notations are defined in Ref.[4].

Note that each term in Eq. (3) is an analytic solution of Eq. (2). The thirteen coefficients in Eq. (3) are made to correspond to the thirteen nodal unknowns for a node : i) one node average flux, and ii) twelve half-interface average fluxes (two half-interface average fluxes for each of the six surfaces).

### 2.2. Innermost Nodes

In an innermost node, the inner radial surface degenerates (disappears) into the z-axis and thus less nodal unknowns may be necessary. In addition, usually the innermost nodes are smaller in size. Moreover, some of the terms in Eq. (3) render the solution unbounded at r=0. Therefore, the terms that involve  $G_0(\kappa_\mu r)$ ,  $G_1(\kappa_\mu r)$ , and  $\ln(r)$  are excluded. The remaining seven coefficients are then made to correspond to : i) one node average flux, ii) two half-interface average fluxes on the outer radial surface, and iii) four surface average fluxes on the other surfaces.

### 3. Implementation in the TOPS Code

After the coefficients in Eq. (3) are expressed in terms of the nodal unknowns, we build as many solvable nodal coupling equations as the number of these nodal unknowns to be determined. The nodal coupling equations in AFEN typically consist of the nodal balance equation, two half-interface current continuity equations. A computer code called TOPS has been developed implementing the method described above.

a) Present address : Korea Institute of Nuclear Safety,  
19 Kusong-dong, Yusong-gu, Daejeon, Korea

#### 4. Results and Discussion

The  $(r,\theta,z)$  geometry nodal code TOPS was tested on two test problems. The first test problem is the Dodd's Problem[6] and shown in Fig. 1. Although it is an  $(r,z)$  problem since the properties are constant in  $\theta$ -direction, it is solved by TOPS and by the method in Ref.[3]. Table I shows the results on  $k_{eff}$ . The results by TOPS are practically identical to the result obtained by the method in Ref.[3] (that was verified by VENTURE), indicating that the TOPS code is working correctly. We further confirmed that the nodal flux distributions are also in excellent agreement.

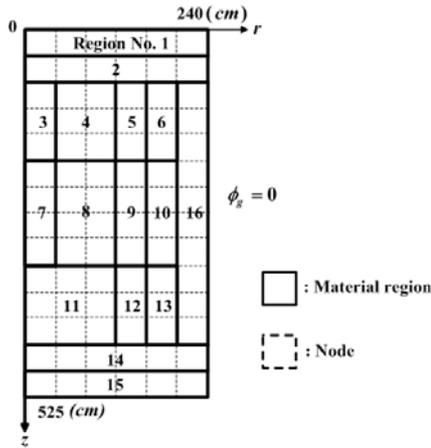


Fig. 1 Test Problem I

Table I. The results of  $k_{eff}$  for Test Problem I

	$k_{eff}$	Discrepancy (%)
Method of Ref. 3	0.8671886	Reference
$N_\theta = 4$	0.8671886	0.0000
$N_\theta = 8$	0.8671886	0.0000
$N_\theta = 12$	0.8671886	0.0000
$N_\theta = 16$	0.8671886	0.0000

The second test problems is a contrived one such that the region-wise properties are varied azimuthally, rendering the problem fully three-dimensional. Fig. 2 shows the configuration: Fig. 2(a) for a side view of quadrant I and III, and Fig. 2(b) for a side view of quadrant II and IV. Table II shows that the TOPS results are in excellent agreement with the VENTURE result.

Table II. The results of  $k_{eff}$  for Test Problem II

	$k_{eff}$	Discrepancy (%)
VENTURE <sup>a)</sup>	0.8449221	Reference
$N_\theta = 16$ <sup>b)</sup>	0.8454172	0.0585
$N_\theta = 24$ <sup>c)</sup>	0.8449914	0.0082

a)  $240 \times 48 \times 560 (N_r \times N_\theta \times N_z)$

b)  $6 \times 16 \times 14 (N_r \times N_\theta \times N_z)$

c)  $12 \times 24 \times 28 (N_r \times N_\theta \times N_z)$

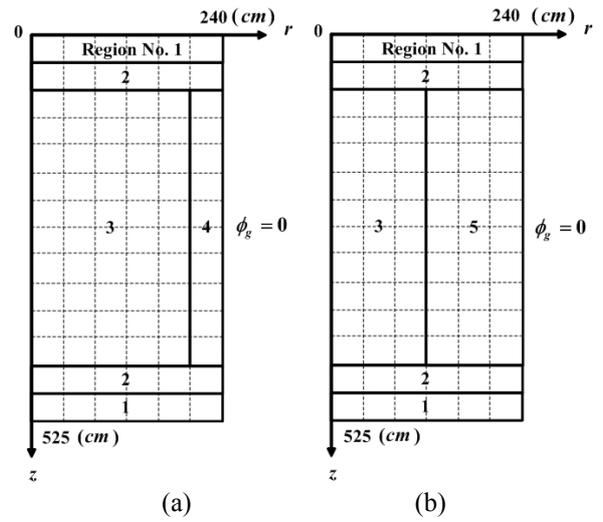


Fig. 2 Test Problem II

We conclude from the results of this paper that the AFEN method provides correct nodal solutions in  $(r,\theta,z)$  geometry. We note that this unusual geometry has defied the efficacy of the usual (transverse integrated) nodal methods[5].

#### ACKNOWLEDGMENT

This work was supported in part by the Ministry of Science and Technology (MOST) of Korea through the Nuclear Hydrogen Development and Demonstration (NHDD) Project coordinated by Korea Atomic Energy Research Institute.

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