

Plastic Limit Load Solution for Pipe Bends with Local Wall Thinning

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1. Introduction

Failure of a pipeline due to local wall thinning is getting increasing attention recently. Accordingly several assessment methods against local wall thinning have been proposed. However, existing methods tend to concentrate on straight pipes with local wall thinning, but not on pipe bends with local wall thinning. Due to its significance in structural integrity assessment, an assessment equation for pipe bend with local wall thinning is urgently needed. Noting that for a pipe made of a sufficiently ductile material, the limit load analysis is popularly used to estimate a maximum load-carrying capacity. In this context, this paper proposes plastic limit load solutions for pipe bends with local wall thinning, based on systematic three-dimensional (3-D) finite element (FE) limit analyses.

2. Finite Element Analysis

Figure 1 depicts a 90° pipe bends. The outer diameter, mean radius and thickness of the pipe are denoted by d_o , r and t , respectively, and the bend radius by R . The following non-dimensional variable (the bend characteristic) should be noted:

$$\lambda = \frac{Rt}{r^2} = \frac{(R/r)}{(r/t)} \quad (1)$$

The definition of ϕ is required. When the axial extent of local wall thinning is within the pipe bend, is simply defined as the half angle, as shown in Fig. 1d.

$$\phi = \phi_1 + \phi_2 = \frac{\pi}{4} + \frac{x}{R} \quad (2)$$

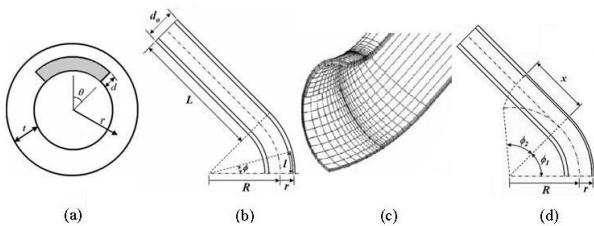


Figure 1. Schematic of elbow; (a) longitudinal section, (b) partial circumferential wall thinning (c) mesh of partial circumferential wall thinning, (d) wall thinning exist over the elbow end

3. Plastic Limit Loads for Pipe Bends with Local wall thinning Under In-Plane Bending

3.1 Plastic Limit Loads for Un-cracked pipe bends

Before presenting FE limit load solutions for pipe bends with local wall thinning, it is worth reviewing

plastic limit loads for un-cracked pipe bends, as they will be base-line solutions to normalize the FE solutions for pipe bends with local wall thinning.

Recently based on the FE results, Kim [2] suggested the following limit moment solution for un-cracked pipe bends under pure bending from Calladine[1] equation.

$$M_0 = 4\sigma_y r^2 t \cdot A (\lambda + k)^n \quad (3)$$

$$A = 0.6453 \left(\frac{r}{t}\right)^{0.0772}; k = 1.5398 \left(\frac{r}{t}\right)^{-0.6755}; n = 0.5157 \left(\frac{r}{t}\right)^{0.0601}$$

3.2 Limiting Cases: Sufficiently Long Wall Thinning & Circumferential Part-Through Surface Cracks

In the limiting case of local wall thinning with a sufficiently long longitudinal extent, it is found that plastic limit loads tend to approach asymptotic values, and thus they do not depend on the longitudinal length of local wall thinning. Equation (4) and figure 2a are lower bound values of normalizing moment.

$$\frac{M_L}{M_0} = f_\infty = 1 - C_1 \left(\frac{d}{t}\right) \quad (4)$$

$$C_1 = 0.09 + 0.8 \left(\frac{\theta}{\pi}\right) + 1.63 \left(\frac{\theta}{\pi}\right)^2$$

A circumferential part-through surface crack is the other limiting case of local wall thinning when the longitudinal length of the wall thinning approaches zero. Equation (5) and (6) are limit moment of surface crack on extrados and intrados wall-thinning. Figure 2b show a limiting case of limit moment.

$$\frac{M_L}{M_0} = f_0 = -4.4 \left(\frac{\theta}{\pi} - 0.1\right) \left(\frac{d}{t}\right) + \left(2.3 \frac{\theta}{\pi} + 1.01\right) \quad (5)$$

$$\frac{M_L}{M_0} = f_0 = -4.0 \left(\frac{\theta}{\pi} - 0.1\right) \left(\frac{d}{t}\right) + \left(2.22 \frac{\theta}{\pi} + 0.81\right) \quad (6)$$

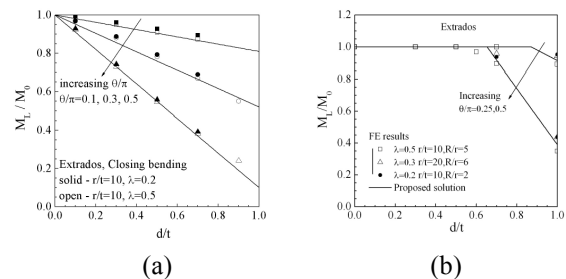


Figure 2. Limiting cases; (a) Lower bound value at large longitudinal wall thinning length under global bending (b) Solution of surface crack under global bending

3.3 Limit Loads for Pipe Bends with Local Wall Thinning in Plane bending.

In the previous two sub-sections, closed-form limit load solutions for pipe bends with local wall thinning under in-plane bending were proposed. The proposed solutions (7) cover two limiting cases of the longitudinal extent of local wall thinning.

$$\frac{M_L}{M_o} = \begin{cases} (f_\infty - f_o) \left(\frac{4\phi}{\pi} \right) + f_o & \text{for } 0 \leq \frac{4\phi}{\pi} \leq 1 \\ f_\infty & \text{for } \frac{4\phi}{\pi} \geq 1 \end{cases} \quad (7)$$

Where f_o and f_∞ are from Eq. (4), Eq. (5) and Eq. (6). Figures 3 show the proposed solution.

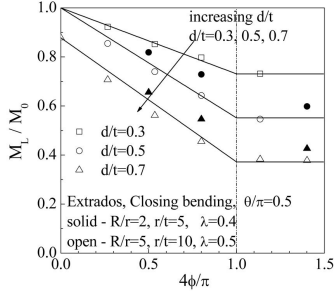


Figure 3. Comparison of proposed solution with FE solution under global bending

4. Plastic Limit Loads for Pipe Bends with Local wall thinning Under Internal Pressure

4.1 In Case of pipe bends Under Internal Pressure

Similarly under in-plane bending, it is worth reviewing plastic limit loads for un-cracked pipe bends under inner pressure. A theoretical limit pressure of a pipe bend, P_o , (based on the von Mises yield condition) is given by (8).

$$P_o = \frac{2}{\sqrt{3}} \frac{\sigma_y t}{r} \left(\frac{1-r/R}{1-r/2R} \right) \quad (8)$$

Based on the FE results, the following approximation is proposed for pipe bends with sufficiently long wall thinning, subject to internal pressure:

$$f_\infty = \frac{P_L}{P_o} = (1.6 - \lambda) \left(1 - \frac{d}{t} \right) \quad (9)$$

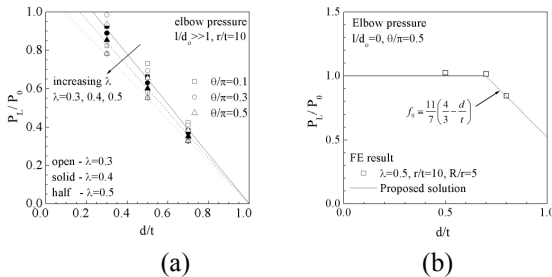


Figure 4. Solution for both ends of FE results ; (a) lower bound value at large longitudinal wall thinning length, $l/d_o \gg 1$, (b) assumed solution of surface crack, $l/d_o = 0$

A circumferential part-through surface crack is the other limiting case of local wall thinning when the longitudinal length of the wall thinning approaches zero. Equation (10) is limit moment of surface crack on wall-

thinning

$$f_o = \frac{P_L}{P_o} = \begin{cases} 1.0 & \text{for } d/t \leq 0.7 \\ \frac{11}{7} \left(\frac{4-d}{3} - \frac{d}{t} \right) & \text{for } d/t > 0.7 \end{cases} \quad (10)$$

4.2 Limit Loads for Pipe Bends with Local wall thinning Under Internal Pressure

In the previous a sub-section, closed-form limit load solutions for pipe bends with local wall thinning under internal pressure were proposed. The proposed solutions (11) cover two limiting cases. For internal pressure, it is found that l/d_o is the better parameter and thus is used in this section.

$$\frac{P_L}{P_o} = (f_o - f_\infty) \exp \left(-10\beta \cdot \frac{l}{d_o} \right) + f_\infty; \quad \beta = -0.276 + 1.83 \left(\frac{d}{t} \right) \quad (11)$$

Where f_o and f_∞ are from Eq. (9) and Eq. (10). Figures 5 show the proposed solution.

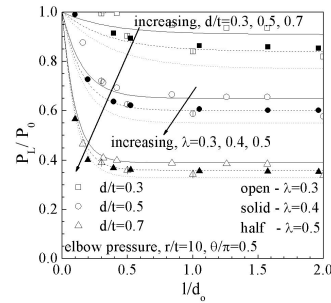


Figure 5. Comparison of proposed solution with FE solution under inner pressure

5. Conclusion

This paper presents limit loads for pipe bends with local wall-thinning based on detailed three dimensional finite element limit analyses. FE analyses are performed based on elastic-perfectly-plastic materials and the geometrically linear assumption. Local wall-thinning is considered, together with different wall-thinning location. Based on the FE results, closed-form approximations are proposed for plastic limit loads of pipe bends with local wall-thinning. The result in this paper would be useful to estimate limit loads collapse of pipe bends

REFERENCES

- [1] Calladine CR., "Limit analysis of curved tubes. Journal of Mechanical Engineering Science," 16: 85-87, 1974.
- [2] Kim, Y.j. and Oh, C.S., "Limit Loads for Pipe Bends under Combined Pressure and In-plane Bending based on Finite Limit Analysis," Transaction of the KSME(A), Vol.30, No.5, 2006.