Application of Influence Function Method to the Fretting Wear Problems

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1. Introduction

Numerical analysis by influence function method (IFM) is demonstrated in this study in order to investigate the fretting wear problems on the secondary side of the steam generator, caused by flow induced vibration. Two-dimensional numerical contact model in terms of Cauchy integral equation is developed. The distributions of normal pressures, shear stresses and displacement fields are derived between two contact bodies which have similar elastic properties. The work rate model is adopted to find the wear amounts between two materials. The results are compared with the solutions by finite element analyses, which show the utilization of the present method to the fretting wear problems.

2. Cauchy Integral Equation

The relative normal displacement $h(y)$ and the relative tangential displacement $g(y)$ in two contact bodies can be represented as:

$$\frac{E^*}{2}\frac{\partial h(y)}{\partial y} = \frac{1}{\pi}\int \frac{p(\xi)}{y-\xi}d\xi - \beta q(y) \tag{1}$$

$$\frac{E^*}{2}\frac{\partial g(y)}{\partial y} = \frac{1}{\pi}\int \frac{q(\xi)}{y-\xi}d\xi + \beta p(y) \tag{2}$$

where $p(y)$ and $q(y)$ are normal and shear stresses along the contact surface, respectively, and Dundurs' parameter β and E^* are defined as follows.

$$\beta = \frac{1}{2}\left[\frac{\{(1-2\nu_1)/G_1\} - \{(1-2\nu_2)/G_2\}}{\{(1-\nu_1)/G_1\} + \{(1-\nu_2)/G_2\}}\right] \tag{3}$$

$$\frac{1}{E^*} = \frac{(1-\nu_1^2)}{E_1} + \frac{(1-\nu_2^2)}{E_2} \tag{4}$$

If two contact bodies are similar materials (Goodman approximation), then we can find $\beta = 0$. In this case, two-dimensional contact problem could be uncoupled in terms of the Cauchy integral equations as follows.

$$\frac{E^*}{2}\frac{\partial h(y)}{\partial y} = \frac{1}{\pi}\int \frac{p(\xi)}{y-\xi}d\xi \tag{5}$$

$$\frac{E^*}{2}\frac{\partial g(y)}{\partial y} = \frac{1}{\pi}\int \frac{q(\xi)}{y-\xi}d\xi \tag{6}$$

3. Cattaneo-Mindlin Problem

For verification of this approach, two-dimensional cylinder-plate contact model, which is subjected to a constant normal loading and cyclic tangential displacement, is considered in this study as shown in Fig. 1. In order to solve the integral equations, contact domain is discretized in such a way that the contact length $2a$ is split into $2N$ elements of width $d = a/N$ as shown in Fig. 2. Normal displacement at element j influenced by the normal pressure at element i could be written as:

$$h_j(y) = c_{ij}p_i \tag{7}$$

where c_{ij} is known as the influence function.

Resulting normal displacement at element j is given by

$$h_j(y) = \sum_{i=1}^{N} c_{ij}p_i \tag{8}$$

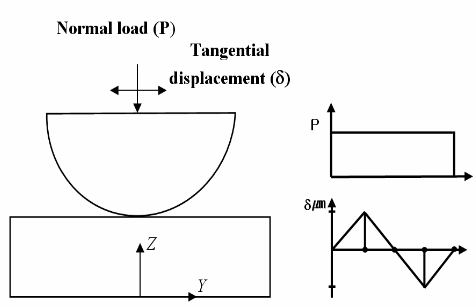

Fig. 1 Two-Dimensional Modeling and Loading History

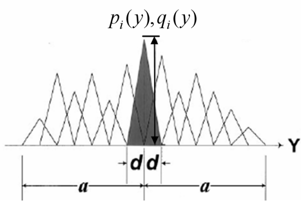

Fig. 2 Principle of Influence Function Method in Contact Region

In fretting wear, general stick-slip regions could exist in contact region, and the following conditions could be imposed in each region.

In stick zone,

$$|q| \le f|p| \tag{9}$$

$$h = 0 \tag{10}$$

In slip zone,

$$|q| = f|p| \tag{11}$$

$$q \cdot h \le 0 \tag{12}$$

General normal and shear displacements at element j could be found by superposition:

$$h_j = A\left(\frac{a}{N}\right)\sum_{i=-N+1}^{N-1} p_i c(i,j) \tag{13}$$

$$g_j = A\left(\frac{a}{N}\right)\sum_{i=-N+1}^{N-1} q_i c(i,j) \tag{14}$$

where

$$c(i,j) = (i-j+1)^2 \ln(i-j+1)^2 + (i-j-1)^2 \ln(i-j-1)^2 - 2(i-j)^2 \ln(i-j)^2 + C \tag{15}$$

$$A = \frac{-(1-\nu^2)}{\pi E} \quad \text{for plane strain} \tag{16}$$

p_i and q_i are the normal and shear traction presented at the ith triangle in direction y ,and C is a constant depending on the chosen datum point in the geometry.

The contact and shear pressures calculated by influence function method (IFM) are compared with the solutions by finite element method under different values of shear force. In Fig. 3, shear pressure distribution along contact length is illustrated according to various normal-shear force ratio $r = Q/fP$ where Q and P are the total shear and normal forces respectively. Comparison of the solutions showed very good agreement both qualitatively and quantitatively.

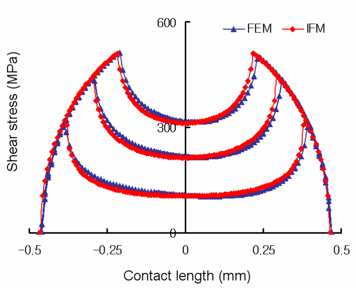

Fig. 3 Comparison between IFM and FEM

4. Fretting Wear Problem

The previous numerical method is applied to a fretting problem where a cylinder is in contact with a plate under constant normal force and oscillating shear displacement. The radius of the cylinder is $6mm$, and elastic modulus and Poisson's ratio of both the cylinder and plate are taken as $200GPa$ and 0.3 , respectively. As loading conditions, vertical normal force, $120N$, is applied to the top surface of the cylinder and a periodic horizontal displacement $\delta(t)$ with the amplitude of $1.28\mu m$ is applied to its bottom.

To simulate the fretting wear, the work rate model is used which can be defined as follows;

$$\text{wear depth} \equiv K \cdot |h| \cdot p \tag{17}$$

where K is a wear constant, h is the amount of relative slip, and p is the normal contact pressure. A numerical algorithm based on the previous method is developed and applied to this problem. The contact pressures and the amount of wear are calculated along the number of loading cycles.

The results of fretting wear analyses after the number of cycles $N = 18000$ are plotted in Fig. 4. Distributions of the contact pressure along the contact surface with increasing number of cycles are depicted. The evolutions of contact surfaces of cylinder and plate due to fretting wear are also sketched in Fig. 4.

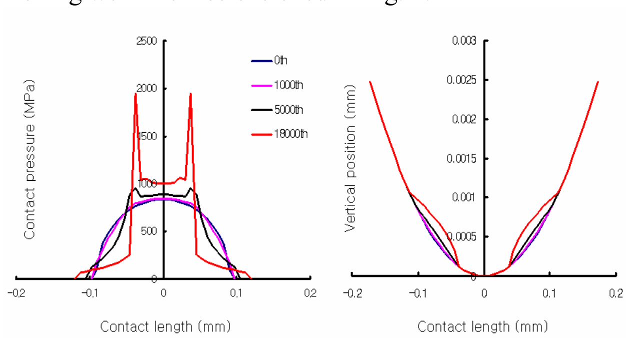

Fig. 4 Contact Pressures and Evolutions of Contact Surfaces until $N=18000$ cycles.

5. Conclusions

A numerical algorithm is developed based on Cauchy integral equation. First, the Cattaneo-Mindlin problem is selected for verification. The contact and shear pressures are compared with the solutions by finite element method, which shows good agreement. Secondly, this numerical method is applied to a fretting problem where a cylinder is in contact with a plate under constant normal force and oscillating shear displacement. The work rate model is used to simulate the fretting wear amount.

The most important advantage of this method is that it can reduce the computing time. For example, using this method, the fretting wear analysis in Fig. 4 takes about 6 and 1/2 hours while it takes more than three days by the finite element method.

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