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Plastic Limit and Collapse Loads of Pipe Bends under Combined Pressure and In-Plane Bending

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1. Introduction

For design and defect assessment of pipe bends, estimation of maximum load-carrying capacities is required, and thus information on plastic limit, collapse loads is important. Miller[1] summarized existing limit load solution for pipe bends, but also noted these solutions are lower bounds and should be used with caution. Although numerical and experimental works on plastic limit analyses of pipe bends can be found recently in the literature[2-4], it is believed that systematic investigation is still missing, probably due to complexity associated with plastic limit analysis of pipe bends. Based on detailed three-dimensional finite element (FE) limit analyses, this paper provides plastic limit and collapse load solution of pipe bends subject to combined pressure and in-plane bending.

2. Finite Element Limit Analyses

A 90° pipe bend is considered in the present work. The mean radius and thickness of the pipe are denoted by r and t, respectively, and the bend radius by *R*. The following bend characteristic variable should be noted :

$$\lambda = \frac{Rt}{r^2} = \frac{(R/r)}{(r/t)} \tag{1}$$

Elastic-perfectly plastic analyses of the pipe bend were performed using ABAQUS[7]. Material was assumed to be elastic-perfectly plastic. Symmetry conditions were fully utilized in FE models, and the reduced integration element (C3D20R in ABAQUS) was used.

3. Plastic Limit Loads Using Small Geometry Change

3.1 Internal Pressure

A theoretical limit pressure of a pipe bend, P_0, (based on the von Mises yield condition) is given by [1]

$$P_o = \left(\frac{2}{\sqrt{3}}\sigma_o \frac{t}{r}\right)\left[\frac{1-r/R}{1-r/(2R)}\right] = P_0^s\left[\frac{1-r/R}{1-r/(2R)}\right] \tag{2}$$

Figure 2(a) shows that Eq. (2) agrees well with the FE results.

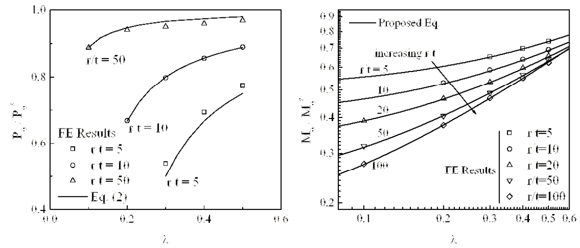
3.2 In-Plane Bending

The exiting theoretical solutions[5,6] are linear in log-log scale, and much lower than FE results. Based on the FE results, the following limit moment is suggested for pipe bends:

$$\frac{M_o}{M_o^s} = A(\lambda + k)^n \tag{3}$$

$$A = 0.6453\left(\frac{r}{t}\right)^{0.0772} \; ; \; k = 1.5398\left(\frac{r}{t}\right)^{-0.6755} \; ; \; \text{n} = 0.5157\left(\frac{r}{t}\right)^{0.0601}$$

Figure 2(b) shows that Eq. (3) agrees well with the FE results.



(a) (b)

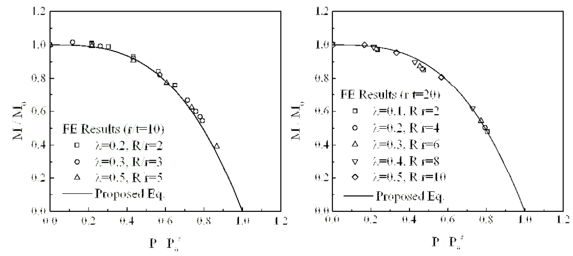
Figure 2. Comparison of the limit load solutions with the FE results under (a) internal pressure and (b) in-plane bending.

3.3 Combined Pressure and Bending

Based on the FE results and the closed-form limit load solution for a single loading, the following yield locus for pipe bends under pressure and bending is proposed:

$$\frac{M}{M_o} = 1 - \left(\frac{1-r/R}{1-r/(2R)} \cdot \frac{P}{P_o}\right)^3 = 1 - \left(\frac{P}{P_0^s}\right)^3 \tag{4}$$

Figure 3 shows that Eq. (4) agrees well with the FE results.



(a) (b)

Figure 3. Comparison of the proposed solution with the FE results under combined pressure and bending; (a) r/t=10, (b) r/t=20

4. Plastic Collapse Loads Using Large Geometry Change

4.1 Pure Bending

In the literature, several collapse load solutions[1-3] can be found for plane pipe bends under closing. These solutions underestimate significantly the FE results. Modifying relevant coefficients in Eq. (3), this paper lead to the following approximation:

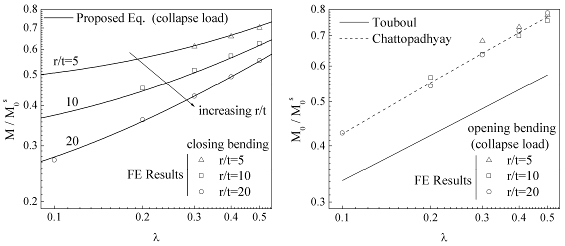
$$\frac{M_o}{M_o^s} = A_c(\lambda + k_c)^{n_c} \qquad (5)$$

$$A_c = 0.800\left(\frac{r}{t}\right)^{-0.017} \;;\; k_c = 1.460\left(\frac{r}{t}\right)^{-0.911} \;;\; n_c = 0.423\left(\frac{r}{t}\right)^{0.127}$$

Recently Chattopadhyay[3] proposed collapse moment solution for plane pipe bends under opening bending as

$$\frac{M}{M_o} = 1.048\lambda^{1/3} - 0.0617 \qquad (6)$$

Comparison with the FE results in Fig. 4 shows that Eq. (5) and (6) are good approximations.



(a) (b)

Figure 4. Comparison of the bending collapse moment solutions with the FE results (a) under closing bending and (b) under opening bending

4.2 Internal Pressure Effects

For compact notation, let us introduce normalized moment and pressure:

$$m \equiv \frac{M}{M_o(P=0)} \;\;;\;\; p \equiv \frac{P}{\sigma_o}\frac{t}{r} \qquad (7)$$

The FE results for the effect of internal pressure on plastic collapse loads of pipe bends with r/t=20 under closing bending are summarized in Fig. 5(a), and the following approximation for closing bending is proposed.

$$m = 1 + \alpha p + \beta p^2 + \gamma p^3 \qquad (8)$$

$$\alpha = \begin{cases} 135\lambda^3 - 202.5\lambda^2 + 101.25\lambda - 14.575 & \text{for} \quad 0.3 \le \lambda \le 0.5 \\ 1.22 & \text{for} \quad 0.1 \le \lambda \le 0.3 \end{cases}$$

$$\beta = \begin{cases} 105\lambda^2 - 105\lambda + 23.75 & \text{for} \quad 0.3 \le \lambda \le 0.5 \\ 1.70 & \text{for} \quad 0.1 \le \lambda \le 0.3 \end{cases}$$

$$\gamma = \begin{cases} -75\lambda^2 + 75\lambda - 18.83 & \text{for} \quad 0.3 \le \lambda \le 0.5 \\ -3.08 & \text{for} \quad 0.1 \le \lambda \le 0.3 \end{cases}$$

The FE results for the effect of internal pressure on plastic collapse loads of pipe bends with r/t=20 under opening bending are summarized in Fig. 5(b), and the following approximation for opening bending is proposed.

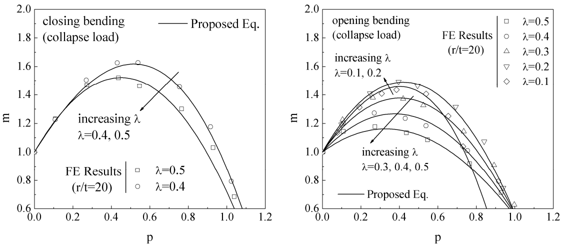
$$m = 1 + \alpha p + \beta p^2 + \gamma p^3 \qquad (9)$$

$$\alpha = \begin{cases} -4.4\lambda + 3.2 & \text{for} \quad 0.2 \le \lambda \le 0.5 \\ 3.0\lambda + 1.72 & \text{for} \quad 0.1 \le \lambda \le 0.2 \end{cases}$$

$$\beta = \begin{cases} 3.865\lambda - 3.523 & \text{for} \quad 0.2 \le \lambda \le 0.5 \\ -12.0\lambda - 0.35 & \text{for} \quad 0.1 \le \lambda \le 0.2 \end{cases}$$

$$\gamma = \begin{cases} 0.47\lambda - 0.094 & \text{for} \quad 0.2 \le \lambda \le 0.5 \\ 16.0\lambda - 3.2 & \text{for} \quad 0.1 \le \lambda \le 0.2 \end{cases}$$

Figure 5 shows that Eq. (8) and (9) are good approximations.



(a) (b)

Figure 5. Effect of internal pressure on FE collapse loads for pipe bends with r/t=20 (a) under closing bending and (b) opening bending

5. Conclusion

Based on detailed three-dimensional (3-D) FE limit analyses, this paper provides closed-form plastic limit, collapse load solutions for pipe bends under combined pressure and bending. FE limit analyses are performed based on elastic-perfectly plastic materials using both the small geometry change and the large geometry change option.

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