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Improvement of Subgroup Parameter Generation Procedure for HELIOS Library

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1. Introduction

It is known that the standard HELIOS library has an empirical adjustment of ~ 3.4% in the resonance integral of U^{238}. The scale of adjustment is very large in terms of multiplication factor - 700 PCM for fresh PWR pin cell. As we found, the essential error comes from the mismatch in the background cross section between the generation stage and the usage stage of the subgroup parameters. The mismatch leads to the error in U^{238} resonance integral about 3.5-4.5%. This is exactly the same magnitude as those in the Helios adjustment. To avoid undesirable empirical correction, we suggested a new procedure, which derives the subgroup weights from preservation the shielded cross sections of a few rigorously calculated heterogeneous pin cells. Direct application of this technique would be impossible without some regularization, which secures numerical stability and reasonable smoothness of resulting effective cross sections.

2. Source of inaccuracy.

The simple imaginary test shows that HELIOS can not reproduce effective cross sections if we solve again the same pin cells which have been used for generation subgroup weights. The reason of drawback can be obtained analytically - the drawback comes from usage different type of background cross sections σ_b. The first type σ_b is defined by the RABBLE code to produce resonance integral table for generation subgroup weights w_n. It is important to note that the RABBLE-based σ_b is functions of effective total cross section. The second type σ_b originates from the concept of "enforced equivalence" ([1], page III-14), which has to consider the background cross section of this type as function of subgroup level *n*. Let's use the subscript *n* to distinguish background cross section of the second type.

HELIOS ignores mismatch between σ_b and σ_{bn} for the sake of other benefits. It employs σ_{bn} for predictive calculations, considering the background cross section as a weakly dependent function of absorption. Equivalence theorem provides some theoretical basis to do that. According to classic resonance treatment theory ([2], page 434), the macroscopic background cross section is

$$\Sigma_b = \lambda\Sigma_p + \Sigma_e; \quad \Sigma_e = S/4V \qquad (2.1)$$

where $S/4V$ is the mean chord across fuel region. Formula (2.1) insists trust that background cross section is function of geometry and potential scattering rather than absorption. That is why HELIOS uses different background cross sections during library preparation and library usage stages. Unfortunately formula (2.1) is valid within some assumptions. Therefore approximation $\sigma_b \approx \sigma_{bn}$ can be poor in some cases. Numerical evidences are given below.

3. Calculation consistent subgroup weights

Subgroup weights w_n are derived from the following constrained minimization problem.

$$\begin{cases} F_1 + \beta F_2 \to \min \\ \left\langle \vec{w}, \vec{\sigma} \right\rangle = \sigma^{\infty} \end{cases} \qquad (3.1)$$

Where,

$\vec{w} = Col\{w_1 ..., w_N\}$ -subgroup weights;

$\vec{\sigma} = Col\{\sigma_1 ..., \sigma_N\}$ -subgroup levels;

F_1 - norm of relative error of effective cross section;

F_2 - regularization term, penalty for non-smoothness resonance integral $R(\sigma_b)$ in Lin-Log scale (quadratic norm of the second order derivative).

σ^{∞} -infinite diluted cross section;

$\bar{\sigma}_k^{\mathrm{Ref}}$ -rigorously calculated effective cross section for representative heterogeneous pin cell number k;

R_k^{SG} - resonance integral for pin-cell number k by subgroup formula;

Φ_k^{SG} - neutron flux by subgroup formula;

g_k and β - weighting factors;

$$F_1 = \sum_{k=1}^{K} g_k^2 \cdot \left(\bar{\sigma}_k^{\mathrm{Ref}} \cdot \bar{\Phi}_k^{\mathrm{SG}} - R_k^{\mathrm{SG}} \right)^2 \qquad (3.2)$$

$$R_k^{\mathrm{SG}} = \sum_{n=1}^{N} \frac{w_n \cdot \sigma_n \cdot \sigma_{bnk}}{\sigma_n + \sigma_{bnk}} \qquad (3.3)$$

$$\bar{\Phi}_k^{\mathrm{SG}} = 1 - \sum_{n=1}^{N} \frac{w_n \cdot \sigma_n}{\sigma_n + \sigma_{bnk}} \qquad (3.4)$$

Right hand side expressions at (3.3), (3.4) use standard notations of Reference [1].

Problem (3.1) is reduced to linear system, then it solved for $\vec{w}$ by using standard package.

4. Numerical results

Table 1 shows error in effective cross section $\bar{\sigma}_a^{U238}$ for a PWR pin cell due to replacement $\sigma_b \rightarrow \sigma_{bn}$ in subgroup formula. We can see large error $\bar{\sigma}_a^{U238}$ at the most important resonance groups of 47 group library. Evaluated error of the whole-range resonance integral is 4.6%.

Method (3.1) has been verified for 47 group HELIOS library. The subgroup-based flux volume weighted shielded cross-sections have been examined during PWR pin cell calculations by the DeCART program. Reference solution has been obtained by RMET21, by solving neutron slowing down equation in heterogeneous circular geometry on the ultra-fine mesh by energy.

Subgroup set, calculated by (3.1) gives in 6 times more accurate groupwise effective cross sections by RMS criteria, then conventionally obtained subgroup set. Implementation of (3.1) remarkably improves agreement with MCNP by K-eff and temperature coefficient.

5. Conclusion

We have identified and removed the essential source of the systematic error appearing in the subgroup parameters of the HELIOS neutron cross sections library. The new method based on conserving the effective cross section significantly reduces the magnitiude of the required U-238 RI adjustment.

Table 1. Error in the shielded cross section due to background cross section mismatch.

Group N	σ_b	σ_{b1}	σ_{b2}	σ_{b3}	σ_{b4}	σ_{b5}	σ_{b6}	σ_{b7}	A	B	$\frac{\Delta\bar{\sigma}_a}{\bar{\sigma}_a}$(%)
10	68.95	68.78	67.22	61.70	58.88	57.77	57.26	57.13	5.9908	5.9594	-0.528
11	67.69	67.55	66.01	60.54	57.75	56.64	56.14	56.02	11.328	10.972	-3.245
12	63.77	63.70	62.17	56.76	54.01	52.92	52.43	52.31	11.958	11.368	-5.195
13	62.93	62.82	61.30	55.93	53.19	52.12	51.63	51.50	13.3	12.537	-6.084
14	59.48	59.49	58.02	52.75	50.08	49.03	48.56	48.43	15.48	14.69	-5.372
15	59.56	59.55	58.08	52.84	50.18	49.13	48.66	48.53	17.419	16.352	-6.524
16	59.29	59.20	58.44	53.89	50.56	49.03	48.30	48.10	1.7159	1.7148	-0.063
17	60.51	60.41	59.22	54.39	51.36	49.93	49.45	49.33	2.8258	2.8239	-0.069
18	55.51	55.45	54.05	48.99	46.42	45.42	44.96	44.84	4.5485	4.5461	-0.052
19	54.79	57.27	55.82	50.64	48.02	46.99	46.52	46.40	58.414	53.91	-8.355
20	56.42	56.85	55.39	50.20	47.57	46.54	46.07	45.95	12.742	12.686	-0.446
21	56.53	56.51	55.00	49.82	47.20	46.17	45.70	45.58	3.1012	3.1002	-0.031
22	56.19	56.14	54.93	50.17	47.19	45.95	45.37	45.22	2.3228	2.3213	-0.068
23	58.93	58.86	57.62	52.76	49.73	48.46	47.86	47.71	2.3046	2.3031	-0.064
24	54.77	54.70	53.75	49.51	46.26	44.77	44.06	43.87	1.7883	1.7871	-0.068
25	53.48	53.40	52.67	48.98	45.01	43.54	42.83	42.64	1.743	1.7431	0.004

Here, $\frac{\Delta\bar{\sigma}_a}{\bar{\sigma}_a} = \left(1 - A/B\right)\cdot 100\%$; $A = \sum_{n=1}^{N} \frac{w_n \cdot \sigma_n \cdot \sigma_b}{\sigma_n + \sigma_b} \Big/ \left(1 - \sum_{n=1}^{N} \frac{w_n \cdot \sigma_n}{\sigma_n + \sigma_b}\right)$;

$B = \sum_{n=1}^{N} \frac{w_n \cdot \sigma_n \cdot \sigma_{bn}}{\sigma_n + \sigma_{bn}} \Big/ \left(1 - \sum_{n=1}^{N} \frac{w_n \cdot \sigma_n}{\sigma_n + \sigma_{bn}}\right)$

REFERENCES

[1] R. J. Stamml'er and et al., HELIOS Methods, Studsvik Scandpower, Rev. 5, October 2001.

[2] James J. Duderstadt, Louis J Hamilton, Nuclear Reactor Analysis, John Wiley & Sons, 1976.

[3] Francisco Leszczynski,"Neutron Resonance Spectrum Calculation System," CNEA Report, Argentina, 1999.