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Monte Carlo Thermal Hydraulic Feedback Calculation for KALIMER Analysis

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1. Introduction

The thermal hydraulic (T/H) conditions - regional temperatures and corresponding densities - are required to calculate the nuclear parameters such as *k* and power densities of the nuclear reactor core. Because these T/H conditions are priori-unknown and dependent on power distribution, the T/H feedback calculation should be conducted.

In the Monte Carlo (MC) T/H Feedback algorithm [1], the updates of power densities and T/H conditions are repeated until the temperature distribution converges. Because the iterative MC calculations are very time-consuming, it is important to use an efficient T/H feedback algorithm.

The objective of this paper is to develop an efficient temperature convergence criterion. And we apply the new criterion to the KALIMER [2] pin and fuel assembly problems.

2. MC T/H Feedback Algorithm and Convergence Criteria

2.1 MC T/H Feedback Algorithms

The MC iteration scheme consists of two calculation stages: I and II. The stage I is designed to determine the T/H condition that matches with the power distribution of the system as quickly as possible by simulating the smaller number of particle histories than the stage II conducted with the target number of particle histories.

The power distribution calculated at *i*-th iteration is used to update the regional temperatures for the next iteration. When the temperature convergence criterion is satisfied in the stage I, the stage II MC iteration is switched on for more refined MC power distribution calculation and the T/H calculation in turn. And the stage II calculations end when the convergence criterion is satisfied. Figure 1 shows this MC T/H feedback calculation algorithm.

2.2 χ^2 Convergence Criteria

The convergence criterion used in the MC T/H feedback calculations [1] is based on the difference of two consecutive temperatures, $T_m^i - T_m^{i-1}$ where T_m^i is the temperature of *m*-th region updated at iteration *i*. When $T_m^i - T_m^{i-1}$ is less than a limit value proportional to its standard deviation, the iteration terminates. The original criterion can be expressed as

$$\left| T_m^i - T_m^{i-1} \right| \le \alpha\sigma[T_m^i]\,. \tag{1}$$

In order to statistically check the convergence of the temperature distribution more precisely, we apply the chi-square (χ^2) criterion for the fission source convergence [3] to checking the temperature convergence. The χ^2 criterion is derived from the assumption that that the relative differences of two consecutive temperatures, $\left(T_m^i - T_m^{i-1}\right)/T_m^i$ $(m = 1, \cdots, N_m)$ follow the normal distributions and they are independent from each another. Let us designate the square sum of the relative differences by Z as

$$Z = \sum_{m=1}^{N_m} \left(\frac{\left(\dfrac{T_m^i - T_m^{i-1}}{T_m^i}\right) - \left(\dfrac{\bar{T}_m^i - \bar{T}_m^{i-1}}{\bar{T}_m^i}\right)}{\sigma\left[\dfrac{T_m^i - T_m^{i-1}}{T_m^i}\right]} \right)^2 . \tag{2}$$

N_m is the total number of regions in MC calculations and $\bar{T}_m^i$ is the expected value of T_m^i.

Because of the assumptions made on the relative differences of temperatures, *Z* can be regarded as the random variable that follows the chi-square distribution with N_m degrees of freedom, $\chi^2(N_m)$.

And the variance of $\left(T_m^i - T_m^{i-1}\right)/T_m^i$, $\sigma^2\left[\left(T_m^i - T_m^{i-1}\right)/T_m^i\right]$ can be approximated by

$$\sigma^2\left[\frac{T_m^i - T_m^{i-1}}{T_m^i}\right] \cong \left(\frac{\bar{T}_m^{i-1}}{\bar{T}_m^i}\right)^2 \left(\frac{\sigma^2\left[T_m^i\right]}{\left(\bar{T}_m^i\right)^2} + \frac{\sigma^2\left[T_m^{i-1}\right]}{\left(\bar{T}_m^{i-1}\right)^2} - 2\frac{\operatorname{cov}\left[T_m^{i-1}, T_m^i\right]}{\bar{T}_m^{i-1}\cdot\bar{T}_m^i} \right). \tag{3}$$

The substitution of Eq. (3) into Eq. (2) leads to

$$Z = \sum_{m=1}^{N_m} \frac{\left(T_m^i - T_m^{i-1}\right)^2}{2\left(\sigma^2\left[T_m^i\right] - \operatorname{cov}\left[T_m^i, T_m^{i-1}\right]\right)} . \tag{4}$$

When a random variable *V* follows the chi-square distribution with N_m degrees of freedom, $\chi^2(N_m,\alpha)$ denotes the value v satisfying $P\{V \ge v\} = \alpha$. Then, it can be used convergence criterion that the value of Eq. (4) falls below $\chi^2(N_m,\alpha)$ as

$$\sum_{m=1}^{N_m} \frac{\left(T_m^i - T_m^{i-1}\right)^2}{2\left(\sigma^2\left[T_m^i\right] - \operatorname{cov}\left[T_m^i, T_m^{i-1}\right]\right)} < \chi^2(N_m,\alpha)\,. \tag{5}$$

Assuming that the two consecutive temperatures, T_m^i and T_m^{i-1}, are independent each other, Eq. (5) can be written as

$$\sum_{m=1}^{N_m} \frac{\left(T_m^i - T_m^{i-1}\right)^2}{2\sigma^2\left[T_m^i\right]} < \chi^2(N_m, \alpha). \qquad (6)$$

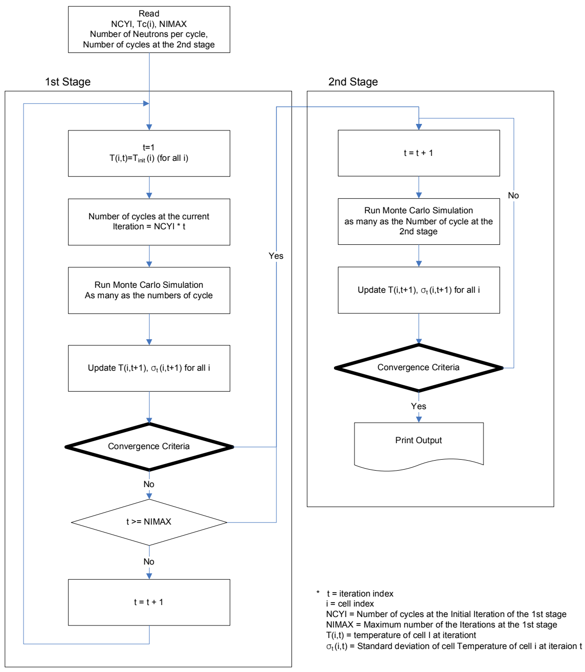

Figure1. Flowchart of iterative MC neutronics calculation with thermal hydraulic feedback.

3. Numerical Results and Discussion

In order to examine effectiveness of the χ^2 criterion, the MC T/H feedback calculations are performed for the KALIMER pin cell and fuel assembly problems [2]. The calculations were performed with 20 inactive cycles and the maximum number of active cycles of 100 on 1,000 histories per cycle.

The pin and assembly are axially divided into 5 segments with equal heights. Because each axial segment consists of fuel, cladding and coolant regions in the pin problem, the total number of regions which temperatures should be updated is 15. When α=0.9 and N_m=15 in the pin problem, $\chi^2(N_m,\alpha)$ of Eq. (6) becomes 8.547. When α=0.9 and N_m=305 in the fuel assembly problem, $\chi^2(N_m,\alpha)$ of Eq. (6) becomes 273.807.

Table 1 shows the temperature convergence characteristics of the MC T/H feedback calculations for the KALIMER pin problem. From Table 1, one can observe that the T/H feedback iterations end with 5 iterations with or without the stage I calculations. 450 cycles are required to converge with stage I calculations and 500 cycles without stage I calculations. This means that the computation time is reduced by applying the stage I calculations.

Table 2 shows the temperature convergence characteristics for the KALIMER fuel assembly problem. From Table 2, one can observe that 175 cycles are required to converge with the stage I calculations but 600 cycles without without stage I.

Table 1. Temperature convergence for the KALIMER pin problem

Case / Iteration	With Stage I			Without Stage I		
	Number of cycles	*Z*	Max ΔT	Number of cycles	*Z*	Max ΔT
1	50	91243	123.7	100	182718	122.9
2	100	156	2.8	100	76	2.0
3	100	11	0.9	100	34	1.4
4	100	44	2.1	100	29	1.8
5	100	5	0.6	100	6	0.9
Total	450			500		

Table 2. Temperature convergence for the KALIMER fuel assembly problem

Case / Iteration	With Stage I			Without Stage I		
	Number of cycles	*Z*	Max ΔT	Number of cycles	*Z*	Max ΔT
1	25	804963	317.8	100	3231983	314.3
2	50	127	4.7	100	1423	7.8
3	100	96	3.4	100	591	5.1
4	-	-	-	100	3717	11.3
5	-	-	-	100	1276	7.4
6	-	-	-	100	62	1.9
Total	175			600		

4. Conclusion

The χ^2 criterion has been developed to diagnose the temperature convergence for the MC T/H feedback calculations. From the application results to the KALIMER pin and fuel assembly problems, it is observed that the new criterion works well.

REFERENCES

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[3] Hyung Jin Shim and Chang Hyo Kim, "Convergence Criteria of Inactive Cycle Monte Carlo Calculations," *Trans. Am. Nucl. Soc.*, 2005