MGL Model for Heterogeneous Redundant System with Non-staggered Test and Staggered Test

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1. Introduction

In a typical redundant system shown Fig. 1, general CCF methods separately handle group 1 or group 2. However, in reality there is the possibility that all components affect each other or share coupling factors, which may cause simultaneous failures so that the methodology is needed to deal a heterogeneous redundant system. Besides, either a non-staggered test or a staggered test is normally applied to a redundant system.

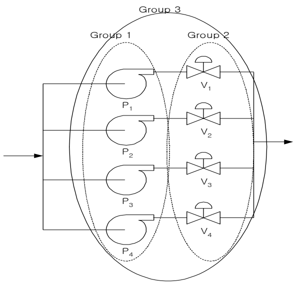

Fig. 1 Typical redundant system

This paper presents the MGL(Multiple Greek Letters) model and its parameter estimator for a redundant system are presented both with staggered testing and non-staggered testing. A non-staggered test is that all components are tested at the same time (or at least the same shift) and a staggered test is that if there is a failure in the first component, all the other components are tested immediately, and if it succeeds, no more is done until the next scheduled testing time.

2. MGL Model for dependent system

The MGL model consists of the total component failure probability Q_t and the associated β, γ, δ parameters. The equation below is the dependent failure model for both non-staggered testing strategy and a staggered testing strategy.

$$
\begin{aligned}
Q_t &= \sum_{i=1}^{m} {}_{m-1}C_{i-1} Q_{i0} + \sum_{j=1}^{m} {}_{m-1}C_{j-1} Q_{0j} \\
&\quad + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} ({}_{m-1}C_{j-1}\,{}_{m}C_{i} + {}_{m}C_{j}\,{}_{m-1}C_{i-1}) Q_{ij} \\
&= Q_{0t} + Q_{t0} + Q_{tt}
\end{aligned}
$$

$$Q_{p0} = \frac{Q_{t0}}{{}_{m-1}C_{p-1}} (1-\lambda_{p+1})\left(\prod_{i=1}^{p} \lambda_i\right), \ where\ \ p = 1,2,\dots,m$$

$$\lambda_1 = 1, \quad \lambda_2 = \beta_p, \quad \lambda_3 = \gamma_p, \quad \dots\dots \quad \lambda_{m+1} = 0$$

$$Q_{0v} = \frac{Q_{0t}}{{}_{m-1}C_{v-1}} (1-\sigma_{v+1})\left(\prod_{i=1}^{v} \sigma_i\right), \ where\ \ v = 1,2,\dots,m$$

$$\sigma_1 = 1, \quad \sigma_2 = \beta_v, \quad \sigma_3 = \gamma_v, \quad \dots\dots \quad \sigma_{m+1} = 0$$

$$Q_{pv} = \frac{Q_{tt}}{{}_{m-1}C_{p-1}\ {}_{m-1}C_{v-1}} (1-\rho_{p+1})\left(\prod_{i=1}^{p} \rho_i\right)(1-\omega_{p,v+1})\left(\prod_{j=1}^{v} \omega_{p,j}\right)$$

$$where, \quad p = 1,2,\dots,m, \quad v = 1,2,\dots,m$$

$$\rho_1 = 1, \quad \rho_2 = \beta, \quad \rho_3 = \gamma, \quad \dots\dots \quad \rho_{m+1} = 0$$

$$\lambda_p = \frac{\sum_{i=p}^{m} {}_{m-1}C_{i-1} Q_{t0}}{\sum_{i=p-1}^{m} {}_{m-1}C_{i-1} Q_{t0}}, \quad \sigma_v = \frac{\sum_{j=v}^{m} {}_{m-1}C_{j-1} Q_{0t}}{\sum_{j=v-1}^{m} {}_{m-1}C_{j-1} Q_{0t}},$$

$$\rho_p = \frac{\sum_{i=p}^{m}\sum_{j=1}^{m} ({}_{m-1}C_{i-1}\,{}_{m}C_{i} + {}_{m}C_{j}\,{}_{m-1}C_{j-1}) Q_{ij}}{\sum_{i=p-1}^{m}\sum_{j=1}^{m} ({}_{m-1}C_{i-1}\,{}_{m}C_{i} + {}_{m}C_{j}\,{}_{m-1}C_{j-1}) Q_{ij}},$$

$$\omega_{p,v} = \frac{\sum_{j=v}^{m} ({}_{m-1}C_{p-1}\,{}_{m}C_{j} + {}_{m}C_{p}\,{}_{m-1}C_{j-1}) Q_{ij}}{\sum_{j=v-1}^{m} ({}_{m-1}C_{p-1}\,{}_{m}C_{j} + {}_{m}C_{p}\,{}_{m-1}C_{j-1}) Q_{ij}}$$

3. Estimation of MGL parameters

Consider a system of *m* identical components, e.g., which is the system of group 1 or group 2 in Fig. 1. The total failure probability Q_t of a component in a common cause group of *m* components is

$$Q_t = \sum_{k=1}^{m} {}_{m-1}C_{k-1} Q_k$$

The likelihood estimator for Q_k which is the probability of a basic event involving k specific components, is given as

$$Q_k = \frac{n_k}{N_k}$$

n_k is the number of events involving *k* components in a failed state and N_k is the number of demands on any k components in the common cause group. In the testing strategy a number of testing works in a certain period is N_D.

For non-staggered testing, all components are tested in a time, N_k is described as follows.

$$N_k = {}_{m-1}C_{k-1} N_D$$

For staggered testing, at each testing work one component is tested. If the test of the first component succeeds, no more test work is done until the next scheduled testing work. It means that there is no common cause failure for ${}_{m-1}C_{k-1}$ groups of *k* components. If, however, the first test fails, all other components are tested. Therefore, the total number of tests on any group of k components N_k is given as follows.

$$N_k = \left(N_D - \sum_{j=1}^{m} n_j\right) {}_{m-1}C_{k-1} + \left(\sum_{j=1}^{m} n_j\right) {}_{m-1}C_{k-1} + \left(\sum_{j=1}^{m} n_j\right) {}_{m-1}C_k$$
$$= {}_{m-1}C_{k-1} N_D + {}_{m-1}C_k n_t,$$
$$where,\quad k = 1, ..., m-1,\ and\ N_m = N_D,\ n_t = \sum_{j=1}^{m} n_j$$

Estimation of parameters with non-staggered test

The estimation of parameters for non-staggered testing strategy is described as follows:

$$Q_t = Q_{0t} + Q_{t0} + Q_{tt}$$
$$Q_{t0} = \frac{1}{mN_D}\sum_{i=1}^{m} i \bullet n_{i,0},\quad Q_{0t} = \frac{1}{mN_D}\sum_{j=1}^{m} j \bullet n_{0,j}$$
$$Q_{tt} = \frac{1}{2mN_D}\sum_{i=1}^{m}\sum_{j=1}^{m}(i+j) \bullet n_{i,j}$$
$$\lambda_2 = \frac{\sum_{i=2}^{m} i \bullet n_{i,0}}{\sum_{i=1}^{m} i \bullet n_{i,0}},\quad \lambda_3 = \frac{\sum_{i=3}^{m} i \bullet n_{i,0}}{\sum_{i=2}^{m} i \bullet n_{i,0}},\quad \lambda_4 = \frac{\sum_{i=4}^{m} i \bullet n_{i,0}}{\sum_{i=3}^{m} i \bullet n_{i,0}},\quad ...$$
$$\sigma_2 = \frac{\sum_{j=2}^{m} j \bullet n_{0,j}}{\sum_{j=1}^{m} j \bullet n_{0,j}},\quad \sigma_3 = \frac{\sum_{j=3}^{m} j \bullet n_{0,j}}{\sum_{j=2}^{m} j \bullet n_{0,j}}, \sigma_4 = \frac{\sum_{j=4}^{m} j \bullet n_{0,j}}{\sum_{j=3}^{m} j \bullet n_{0,j}},\quad ...$$
$$\rho_2 = \frac{\sum_{i=2}^{m}\sum_{j=1}^{m}(i+j) \bullet n_{i,j}}{\sum_{i=1}^{m}\sum_{j=1}^{m}(i+j) \bullet n_{i,j}},\quad \rho_3 = \frac{\sum_{i=3}^{m}\sum_{j=1}^{m}(i+j) \bullet n_{i,j}}{\sum_{i=2}^{m}\sum_{j=1}^{m}(i+j) \bullet n_{i,j}},$$
$$\rho_4 = \frac{\sum_{i=4}^{m}\sum_{j=1}^{m}(i+j) \bullet n_{i,j}}{\sum_{i=3}^{m}\sum_{j=1}^{m}(i+j) \bullet n_{i,j}},\quad ...$$
$$\omega_{p,1} = 1,\quad ...,\quad \omega_{p,v} = \frac{\sum_{j=v}^{m}(j+p) \bullet n_{p,j}}{\sum_{j=v-1}^{m}(j+p) \bullet n_{p,j}},\quad ...\quad \omega_{p,m+1} = 0$$
$$where,\quad p = 1,2,...,m,\quad v = 1,2,...,m$$

Estimation of parameters with staggered test

The estimation of parameters for staggered testing strategy is described as follows:

$$Q_t = \sum_{i=1}^{m} {}_{m-1}C_{i-1}Q_{i0} + \sum_{j=1}^{m} {}_{m-1}C_{j-1}Q_{0j}$$
$$+ \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}({}_{m-1}C_{j-1}\,{}_mC_i + {}_mC_j\,{}_{m-1}C_{i-1})Q_{ij}$$
$$= Q_{0t} + Q_{t0} + Q_{tt}$$
$$Q_{t0} = \sum_{i=1}^{m} {}_{m-1}C_{i-1}Q_{i0} = \sum_{i=1}^{m}\frac{n_{i,0}}{N_D + s_i n_{to}},$$
$$where\ n_{to} = \sum_{i=0}^{m} n_{i,0},\quad s_i = \frac{m}{i} - 1$$
$$Q_{0t} = \sum_{j=1}^{m} {}_{m-1}C_{j-1}Q_{0j} = \sum_{j=1}^{m}\frac{n_{0,j}}{N_D + s_j n_{ot}},$$
$$where\ n_{ot} = \sum_{j=1}^{m} n_{0,j},\quad s_j = \frac{m}{j} - 1$$
$$Q_{tt} = \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}({}_{m-1}C_{i-1}\,{}_mC_i + {}_mC_j\,{}_{m-1}C_{j-1})Q$$
$$= \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\frac{(s_i+1)+(s_j+1)}{N_s + s_i s_j n_{tt}} \bullet n_{i,j}$$
$$where,\ n_{tt} = \sum_{i=1}^{m}\sum_{j=1}^{m} n_{i,j}$$
$$\lambda_p = \frac{\sum_{i=p}^{m} {}_{m-1}C_{i-1}Q_{t0}}{\sum_{i=p-1}^{m} {}_{m-1}C_{i-1}Q_{t0}} = \frac{\sum_{i=p}^{m}\frac{n_{i,0}}{N_D + s_i n_{t0}}}{\sum_{i=p-1}^{m}\frac{n_{i,0}}{N_D + s_i n_{t0}}},\quad p = 2..m$$
$$\sigma_v = \frac{\sum_{j=v}^{m} {}_{m-1}C_{j-1}Q_{0t}}{\sum_{j=v-1}^{m} {}_{m-1}C_{j-1}Q_{0t}} = \frac{\sum_{j=v}^{m}\frac{n_{0,j}}{N_D + s_j n_{0t}}}{\sum_{j=v-1}^{m}\frac{n_{0,j}}{N_D + s_j n_{0t}}},\quad v = 2..m$$
$$\rho_p = \frac{\sum_{i=p}^{m}\sum_{j=1}^{m}({}_{m-1}C_{i-1}\,{}_mC_i + {}_mC_j\,{}_{m-1}C_{j-1})Q_{ij}}{\sum_{i=p-1}^{m}\sum_{j=1}^{m}({}_{m-1}C_{i-1}\,{}_mC_i + {}_mC_j\,{}_{m-1}C_{j-1})Q_{ij}}$$
$$= \frac{\sum_{i=p}^{m}\sum_{j=1}^{m}\frac{(s_i+1)+(s_j+1)}{N_s + s_i s_j n_{tt}} \bullet n_{i,j}}{\sum_{i=p-1}^{m}\sum_{j=1}^{m}\frac{(s_i+1)+(s_j+1)}{N_s + s_i s_j n_{tt}} \bullet n_{i,j}},\quad \rho = 2..m$$
$$\omega_{p,v} = \frac{\sum_{j=v}^{m}({}_{m-1}C_{p-1}\,{}_mC_j + {}_mC_p\,{}_{m-1}C_{j-1})Q_{ij}}{\sum_{j=v-1}^{m}({}_{m-1}C_{p-1}\,{}_mC_j + {}_mC_p\,{}_{m-1}C_{j-1})Q_{ij}}$$
$$= \frac{\sum_{j=v}^{m}\frac{(s_p+1)+(s_j+1)}{N_s + s_p s_j n_{tt}} \bullet n_{p,j}}{\sum_{j=v-1}^{m}\frac{(s_p+1)+(s_j+1)}{N_s + s_p s_j n_{tt}} \bullet n_{p,j}},$$
$$\omega_{p,1} = 1,\quad ...,...\quad \omega_{p,m+1} = 0$$
$$where,\quad p = 1,2,...,m,\quad v = 1,2,...,m$$

6. Conclusions

MGL dependent failure model and estimation of its parameters is proposed for dealing with a redundant system with homogeneous and heterogeneous components with non-staggered testing and staggered testing.

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