# Benchmark Calculation for the VHTR 2-D Core by Using the DeCART Code 

Jin-Young Cho, a Kang-Seog Kim, a Chung-Chan Lee a a Korea Atomic Energy Research Institute, 150 Deokjin-dong, Yuseong-gu, Daejon, 305-353, jyoung@kaeri.re.kr

## 1. Introduction

Recently, a hexagonal module has been equipped to the DeCART (Deterministic Core Analysis based on Ray Tracing) whole core code for a hexagonal core analysis [1]. The equipment includes a ray tracing module to solve the 2-D whole-core transport problem and a multi-group CMFD module to perform an efficient transport calculation. In this paper, the capability of the DeCART hexagonal module is examined by solving VHTR core problems.

## 2. Methods and Results

The VHTR core uses helium as a coolant which is realized as a void hole in a neutronics calculation. This void hole has no influence in the MOC transport calculation, but it brings about a near singular matrix for a CMFD formulation. Therefore, how to manipulate this problem is described first in this section and then the computational results of the DeCART code for the VHTR core are compared with the MCNP code.

### 2.1 Lumped CMR scheme

CMFD module solves the following equation for the acceleration of the ray tracing transport calculation.

$$
\begin{equation*}
-\frac{1}{h} \sum_{s=1}^{N_{n a}^{i}}\left(\widetilde{D}_{s}^{i}+\hat{D}_{s}^{i} \phi_{s}^{i}+\left\{\Sigma_{a}^{i}+\frac{1}{h} \sum_{s=1}^{N_{n a}^{i}}\left(\widetilde{D}_{s}^{i}-\hat{D}_{s}^{i}\right)\right\} \bar{\phi}^{i}=\bar{S}^{i}\right. \tag{1}
\end{equation*}
$$

Where $\hat{D}$ is a current corrective coefficient which is determined from the transport solution, and $\widetilde{D}$ is the coupling coefficient of the FDM which is defined as:

$$
\widetilde{D}_{s}=\frac{\beta_{s}^{i} \beta_{s}^{i+1}}{\beta_{s}^{i}+\beta_{s}^{i+1}}, \beta_{s}^{i}=\frac{D_{i}}{h_{s}^{i}} .
$$

The void problem occurs when two or more than two void cells are grouped. In a void cell, the diffusion constant and $\beta$ are nearly infinite. Therefore, if a void cell is adjacent to another void cell, $\widetilde{D}_{s}$ at the interface between the void cells is also nearly infinite so that a nearly singular matrix is produced. However, if a void cell exists alone, $\widetilde{D}_{s}$ is close to the $\beta$ value of the adjacent non-void cell, which produces a normal matrix. The nearly singular matrix converges very slowly and degrades the calculation performance of the CMFD module.

For an efficient calculation of the nearly singular matrix, a lumped coarse mesh rebalance (LCMR) scheme is applied to the void cell group. The LCMR equation can be derived by summing Eq. (1) over all the
cells of a void group. For a simplicity, Eq. (1) can be rewritten as:

$$
\begin{equation*}
\sum_{s=1}^{N_{n d}^{i}} A_{s, i} \bar{\phi}_{s}^{i}+A_{i, i} \bar{\phi}^{i}=\bar{S}^{i} . \tag{2}
\end{equation*}
$$

In the LCMR scheme, the shape of the cell averaged flux for a void cell group is assumed to be invariant during the CMFD calculation and determined only by the transport calculation. Then, the cell averaged flux at the ( $n+1$ )-th iteration can be written by the lumped rebalancing factor and the cell averaged flux at the $n$-th iteration as:

$$
\begin{equation*}
\bar{\phi}^{i, n+1}=f^{n+1} \bar{\phi}^{i, n} . \tag{3}
\end{equation*}
$$

By inserting Eq. (3) to Eq. (2) and summing all the cells of a void group after moving the non-infinite terms of the left hand side (LHR) to the right hand side (RHS), the following equation can be obtained.

$$
\begin{align*}
& f^{n+1} \sum_{i=1}^{N_{g}}\left(\sum_{s=1}^{N_{\text {rad, soid }}^{i}} A_{s, i} \bar{\phi}_{s}^{i, n}+A_{i, i} \bar{\phi}^{i, n}\right)= \\
& \sum_{i=1}^{N_{g}}\left(\bar{S}^{i, n+1}+\sum_{s=1}^{N_{\text {mad, non-v }}^{i}} A_{s, i} \bar{\phi}_{s}^{i, n+1}\right) . \tag{4}
\end{align*}
$$

The summation term of the LHS in Eq. (4) is same as the RHS at the $n$-th iteration. Therefore, the lumped rebalancing factor at the $(n+1)$-th iteration can be determined simply by dividing the RHS at the $(n+1)$-th iteration by that at the $n$-th iteration.

The linear system of the CMFD formulation does not converge if the LCMR scheme is applied because Eqs. (3) and (4) for the void cells do not solve the CMFD equation of Eq. (1) and only produce an approximated solution. Therefore, when applying the LCMR to the CMFD linear system, the CMFD calculation should be stopped if the residual error does not reduce any more, and then moved to the next MOC calculation to update the flux shape of a void group.

### 2.2 Benchmark Problems

The hexagonal module of the DeCART code is examined for three types of assembly block problems and one 2-D whole core problem for the VHTR core (Fig. 1). The void hole exists in most of the problems except for BLOCK-1 and is modeled by the regular hexagonal cell. Therefore, in the case of a void hole, there exist not only fully voided cells but also partly voided cells in the graphite matrix. The partly voided cell is modeled to use the homogenized material which is indicated by GRPH2 in Fig. 1. In the core problem, DeCART models the core boundary by surrounding the graphite assembly while MCNP models the core barrel
explicitly. The double heterogeneity in the fuel compact is simplified by the RPT [2] theory and by using the HELIOS code for the BLOCK-1 300 K problem. The solutions of the DeCART code are obtained by using the 190 G library for the assembly problems and the 47 G library for the core problems, and compared with those of the MCNP and HELIOS codes. For the ray option, 0.05 cm for the ray spacing and 2 azimuthal and 2 polar angles in the $30^{\circ}$ and $90^{\circ}$ sectors are used.

(a) BLOCK-1

(b) BLOCK-3

(c) BLOCK-3

(d) 2-D CORE

Figure 1. Benchmark Problems for VHTR Core.
Table 1 and Fig. 2 show the computational results. DeCART shows less than 100 pcm errors for the assembly problems and less than 600 pcm errors for the core problems. The reason why the core problem shows a large eigenvalue error is mainly due to the cross section library, which is examined in an in-house comparison. In the radial power distribution, DeCART shows a very good agreement with the MCNP showing less than a $0.5 \%$ error.

For the computing time, DeCART takes about 3 minutes for the BLOCK-1 and BLOCK-3 sextant
assembly problems and about 15 minutes for the BLOCK-2 full assembly problem. In the 2-D core problem, DeCART takes less than 5 hours on a PENTIUM-IV 3.0 GHz personal computer.

Table 1. Eigenvalue Comparison

| Problem | Codes | Param | 300 K | 600 K | 900 K |
| :---: | :---: | :---: | :---: | :---: | :---: |
| BLOCK-1 | MCNP | K-inf | 1.53112 | 1.48272 | 1.44922 |
|  | HELIOS | $\Delta \rho, \mathrm{pcm}$ | 0 | 124 | 111 |
|  | DeCART | $\Delta \rho, \mathrm{pcm}$ | -98 | -8 | -43 |
|  | MCNP | K-inf | 1.53382 | 1.48853 | 1.45417 |
|  | HELIOS | $\Delta \rho, \mathrm{pcm}$ | 87 | 91 | 127 |
|  | DeCART | $\Delta \rho, \mathrm{pcm}$ | -8 | -27 | -13 |
| BLOCK-2 | MCNP | K-inf | 1.54717 | 1.50096 | 1.46770 |
|  | HELIOS | $\Delta \rho, \mathrm{pcm}$ | 35 | 128 | 149 |
|  | eCCART | $\Delta \rho, \mathrm{pcm}$ | -51 | 20 | 20 |
| 2-D | MCNP | K-eff | 1.43657 | 1.41004 | 1.38729 |
| CORE | DeCART | $\Delta \rho, \mathrm{pcm}$ | 327 | 542 | 599 |

* Standard Deviation ( $\sigma$ ) is less than 0.00051


Figure 2. Radial Power Comparison at 300 K

## 3. Conclusion

The hexagonal module equipped in the DeCART code was examined for VHTR problems. During the examination, a nearly singular problem due to a void cell group occurred and was resolved by applying the LCMR scheme. The computation results indicated that the hexagonal module of the DeCART code worked very well by showing a good solution within an affordable computing time.

## REFERENCES

[1] J. Y. Cho, et al., "CMFD Formulation for Hexagonal MOC Transport Calculation," 2006 KNS Autumn Mtg, Kyungju, Nov. 2-3, 2006.
[2] Y. Kim and M. Baek, "Elimination of DoubleHeterogeneity through a Reactivity-Equivalent Physical Transformation," GLOBAL2005, Tskuba, Japan, Oct. 9-13, 2005.

