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CMFD Formulation for a Hexagonal MOC Transport Calculation

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1. Introduction

Recently, the DeCART (Deterministic Core Analysis based on Ray Tracing)[1] code equips the hexagonal transport kernel which solves the heterogeneous hexagonal core problem by the MOC transport equation. Also, the multi-group CMFD module is equipped to obtain the transport solution efficiently. In this paper, some features included in the MOC and CMFD modules are described first, and then the acceleration performance of the CMFD module is examined.

2. Methods and Results

2.1 Ray Tracing Features

The hexagonal transport kernel implemented into the DeCART code solves the MOC equation by the modular ray tracing scheme based on a hexagonal assembly and by a path linking scheme between the modular rays. Therefore, the requirements for the modular ray include a complete construction capability for a core ray by a path linking and a complete reflection capability to another ray at the core boundary. The path linking requirement is achieved by adjusting the ray angle and the ray spacing from the input values. If $\tilde{\alpha}$ and $\Delta\tilde{A}$ are given for the ray angle and the ray spacing for $\tilde{\alpha}$ <60° from the input file, then the number of modular rays passing through surface 1 and surface 2 of Fig. 1 are calculated as:

$$n1 = \mathrm{int}\left[\sin(\tilde{\alpha}) \frac{P}{\Delta\tilde{A}} \right] \text{ and}$$

$$n2 = \mathrm{int}\left[\sin(60° - \tilde{\alpha}) \frac{P}{\Delta\tilde{A}} \right]. \quad (1)$$

Here, P means the side length of the hexagonal assembly. By using the determined ray numbers of *n1* and *n2*, the adjusted angle and the ray spacing meeting the path linking requirements are given as:

$$\tan\alpha = \frac{\sqrt{3}}{2\left(n2/n1\right)+1} \text{ and } \quad \Delta A = \frac{P\sin\alpha}{n1}. \quad (2)$$

With the above adjusted ray angle and the ray spacing, the number of rays passing through surface 3 can be obtained easily from the *n1* and *n2* as:

$$n3 = n1 + n2 \quad (3)$$

DeCART uses Eq. (1) and (2) to determine the ray angles in the angle range of [0,30°]. The other reflective angles to meet the complete reflection requirements are determined as:

$$\alpha_{2,3} = \frac{\pi}{3} \pm \alpha_1, \ \ \alpha_{4,5} = \frac{2\pi}{3} \pm \alpha_1, \ \alpha_6 = \pi - \alpha_1. \quad (4)$$

Where α_1 is an angle determined from Eq. (2) in the angle range of [0,30°].

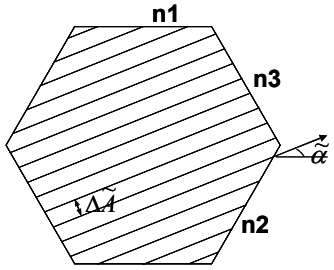


Figure 1. Modular Rays.

2.2 CMFD Acceleration

The CMFD module solves the multi-group problem which keeps the same energy group structure as the transport problem. This CMFD module is designed to solve an unstructured cell to treat an irregular geometry for the gap region of an assembly boundary.

The net current in the CMFD formulation can be written as:

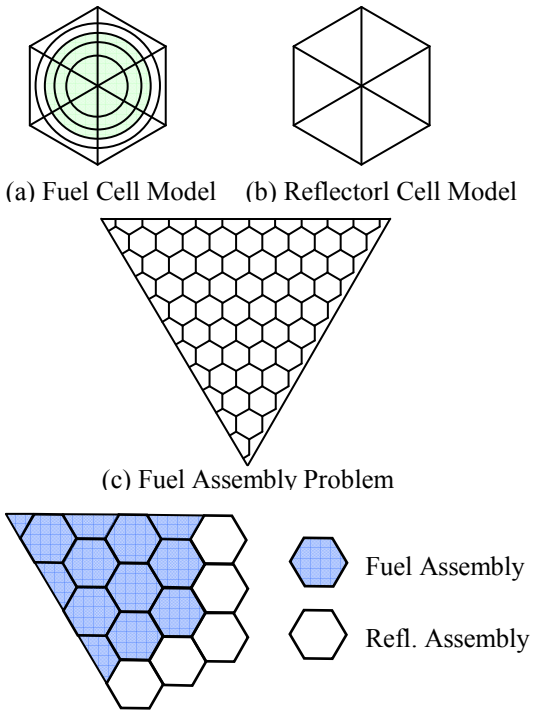
$$J_s = -\tilde{D}_s\left(\bar{\phi}^{i+1} - \bar{\phi}^{i}\right) - \hat{D}_s\left(\bar{\phi}^{i+1} + \bar{\phi}^{i}\right). \quad (4)$$

Where $\hat{D}$ is a current corrective coefficient which is determined from the transport solution, and $\tilde{D}$ is the coupling coefficient determined from the conventional finite difference method (FDM). When solving the hexagonal core by applying FDM only, how to define $\tilde{D}$ wholly governs the solution accuracy. Therefore, in this case, $\tilde{D}$ should be defined carefully for the accuracy of the computational results. However, in the CMFD formulation, the definition of $\tilde{D}$ is not that important because the net current is finally adjusted to the transport solution by $\hat{D}$. Therefore, in the DeCART code, $\tilde{D}$ is approximately defined by using the length of the perpendicular line from a cell's center point.

2.3 Benchmark Results

The acceleration performance of the CMFD module is examined for the single pin, the single assembly and the core problems. The reflective boundary condition is used for the first two problems and the vacuum boundary condition is applied to the last problem. The cross sections are taken from the C5G7 benchmark problem which was proposed for testing the ability of a modern deterministic code for a rectangular core. Fig. 2

shows the problems solved in this paper. For the ray option, 0.05 cm for the ray spacing and 2 azimuthal and 2 polar angles in the 30° and 90° sectors are used. In the performance examination, the residual error which is defined by the difference between the neutron source and the loss terms based on a homogenized cell is used. The absolute eigenvalue error is also used to determine the convergence of the calculation.



(a) Fuel Cell Model (b) Reflectorl Cell Model

(c) Fuel Assembly Problem

(d) 2D-Core Problem

Figure 2. Benchmark Problems and Modeling

Fig. 3 shows the CMFD performance in the aspect of the iteration number. The two values in the parenthesis of Fig. 3 mean the number of outer iterations and the eigenvalue error. The number of iterations when applying the MOC ray tracing alone is dependent on the problem size, showing 53 iterations for the pin cell problem, 78 iterations for the assembly problem and more than 500 iterations for the core problem. However, when accelerated by the CMFD module, the dependency on the problem size is considerably reduced, showing less than 8 iterations for all the problems. The speedup for the number of iterations turned out to be about 10 for the pin and the assembly problems, and about 200 for the core problem.

Fig.4 shows the computational time breakup. Though applying the CMFD acceleration, most of the computing time is required for the MOC transport calculation, and the CMFD module uses less than 10 % of the total computing time. The computing time for the MOC transport calculation is proportional to the number of outer iterations. The total computing time required for the core problem in the case of the CMFD acceleration is less than about 8 minutes, which indicates the applicability of the DeCART hexagonal version to a realistic core problem by using the multi-group cross section library.

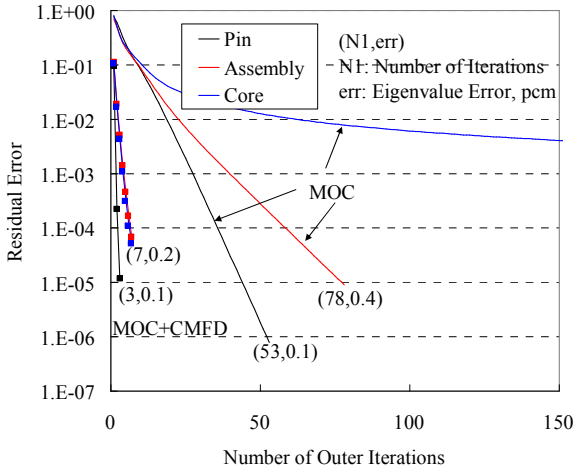


Figure 3. Residual Errors with the Outer Iterations

Table 1. Computing Time Breakup
(PENTIUM-IV M 733 1.6 GHz, sec)

Problem	Cal.	MOC	CMFD	Etc.	Total
Pin	MOC	-	-	-	0.89
	MOC/CMFD	0.05	0.00	0.14	0.19
Assy.	MOC	-	-	-	61.0
	MOC/CMFD	5.3	0.1	1.7	7.1
Core	MOC	-	-	-	>90000
	MOC/CMFD	417	24	13	454

3. Conclusion

In this paper, the hexagonal modules including the MOC transport calculation module and the CMFD acceleration module were implemented into the DeCART code. The CMFD accelerated the MOC transport calculation reduced about 10 times for the pin and assembly problems and about 200 times for the core problem in both the total computing time and the number of outer iterations. Therefore, it is concluded that a whole core transport calculation for a hexagonal core is achievable with an affordable computing time by using the CMFD acceleration.

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