

## A Study on a Numerical Modeling of a Friction Pendulum System

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### 1. Introduction

A Friction Pendulum System (FPS) is a well known seismic isolation system. In the case of FPS, the period of a vibration only depends on a radius of a curvature and the gravitational constant, and it does not depend on the mass. For this reason, the FPS is useful for smaller weight equipment and a liquid storage tank which changes its' liquid level. Kim et al. (2004) studied the seismic isolation effect of small equipment by using a natural rubber bearing (NRB), a high damping rubber bearing (HDRB) and a Friction pendulum system (FPS) by an experimental research. In this study, modified Mokha model for a small FPS system was proposed and it was revealed this model matched the experimental results well.

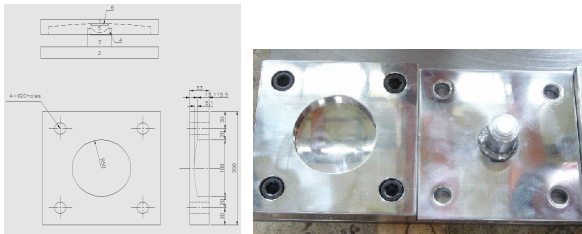
### 2. Friction Pendulum System

The target FPS was used in the research of Kim, et al (2004). The natural frequency of the FPS of the experiment is decided as 1Hz. The period and the stiffness of the FPS can be determined as equations (1) and (2) (Zayas et al. 1987).

$$T = 2\pi \sqrt{\frac{R}{g}} \quad (1)$$

$$K = \frac{W}{R} \quad (2)$$

where, T is the period of the FPS, R is the radius of the curvature, g is the gravity acceleration, K is the effective horizontal stiffness and W is the weight of the upper structure. Using equations (1) and (2), the radius of the curvature is decided as 24.8cm and the effective stiffness is 4.02 kgf/cm<sup>2</sup>. The drawing and the figure of FPS are shown in Fig. 1.



(a) Drawing

(b) Figure

Fig. 1 FPS

Four cases of a characterization test are performed for an evaluation of the mechanical characteristics. The maximum displacements were varied as 10, 20 and 30mm and the exciting velocities were considered as 2, 4, 20, 40 and 80mm/sec.

### 3. Numerical Modeling of FPS System

#### 3.1 Numerical Model of Mokha

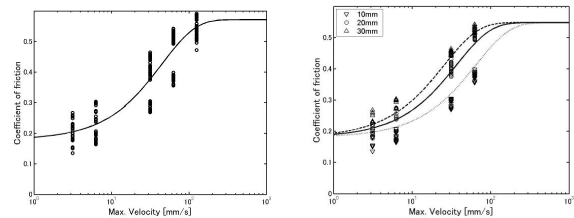
The lateral force at the isolation level follows with a excellent accuracy the following relationship (Mokha et al. 1991)

$$F_b = \left( \frac{W}{R} \right) U_b + \mu(\dot{U}_b) W \cdot \text{sgn}(\dot{U}_b) \quad (3)$$

$$\mu(\dot{U}_b) = f_{\max} - D_f \cdot \exp(-a|\dot{U}_b|) \quad (4)$$

where  $W$  =the weight of the model;  $R$  =the radius of curvature of the bearings;  $\mu$  =the coefficient of friction mobilized during sliding;  $U_b$  =the bearing displacement and  $f_{\max}$  and  $D_f$  are the coefficients of a friction. The period of a vibration of the structure in its rigid body condition is also presented as eq. (1). The regression curve of the mechanical characterization test results by Choun et al. (2004) are shown in Fig. 2(a). As shown in Fig. 2(a), the coefficient of the friction increases with an increasing maximum velocity. The regression results are shown in eq. (5).

$$\mu(\dot{U}_b) = 0.5479 - 0.3680 \cdot \exp(-0.0284|\dot{U}_b|) \quad (5)$$



(a) Mokha Model

(b) Proposed Model

Fig. 2 Maximum Displacement Reliability of Friction Coefficient

### 3.2 modified Numerical Model

Mokha model can't control the hardening effect of the previous experiment results of Choun, a term for considering the hardening effect is added into eq. (3). Moreover for the friction coefficient to consider the excitation displacement reliability, the second term of eq. (2) modified. The proposed model is shown in eq. (6) and (7).

$$F_b = \left(\frac{W}{R}\right)U_b + \mu(\dot{U}_b)W \cdot \text{sgn}(\dot{U}_b) + c \text{sgn}(\dot{U}_b)|\dot{U}_b|^\alpha |U_b|^\beta \quad (6)$$

$$\mu(\dot{U}_b) = f_{\max} - D_f \cdot \exp\left(-(\zeta + \xi|U_b|)|\dot{U}_b|\right) \quad (7)$$

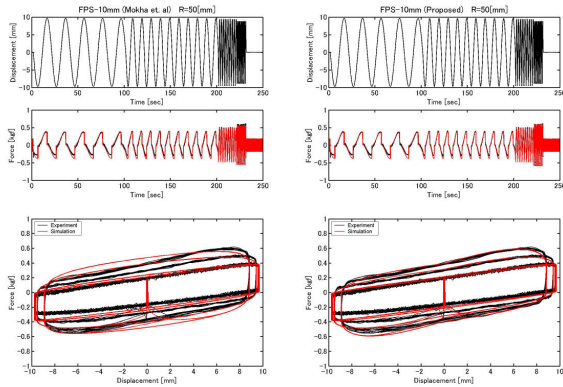
where,  $\alpha$  and  $\beta$  are the coefficients which can determine the regression of the differences between the lateral force.

Fig. 2(b) shows a maximum velocity dependency of the friction coefficient and it also shows the regression results by using the eq. (6). As shown in Fig. 2(b), the friction coefficients are dependent on the maximum velocity and displacement. The final regression results are presented in eq. (8) and the 3<sup>rd</sup> term of eq. (6) is shown as eq. (9).

$$\mu(\dot{U}_b) = 0.5479 - 0.3680 \cdot \exp\left(-\left(0.0012 + 0.0036|U_b|\right)|\dot{U}_b|\right) \quad (8)$$

$$F_3 = 8.217 \times 10^{-4} \cdot \text{sgn}(\dot{U}_b)|\dot{U}_b|^{0.890} |U_b|^{0.944} \quad (9)$$

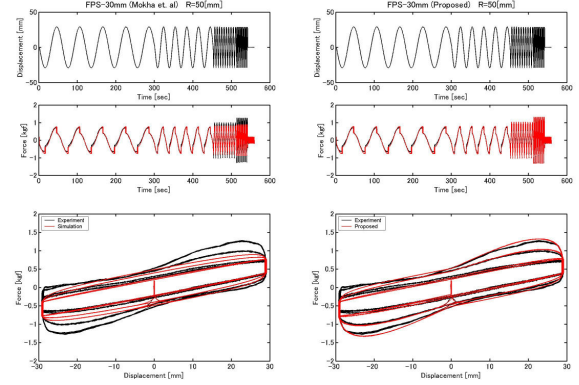
### 3.3 compare of the Experimental Result



(a) Mokha Model (b) Proposed Model  
Fig.3 Compare of the Results  
(Max. Displacement=10mm)

The results of the experiment and proposed model are shown in Figs. 3 and 4. A maximum displacement of 10mm is shown in Fig. 3 and that of 30mm is shown in Fig. 4. As shown in Figs. 3 and 4, where the actuating displacement is small, the results of the Mokha model and proposed model are not that different. Where the actuating displacement is large (Fig. 4), Mokha model can't

simulate the nonlinear behavior, but the proposed model simulates the hardening property well.



(a) Mokha Model (b) Proposed Model

Fig.4 Compare of the Results  
(Max. Displacement=30mm)

## 4. Conclusion

A modified numerical model for a FPS based on the Mokha model was proposed. The proposed model matches the experimental results well. Especially the proposed model can simulate the hardening behavior of a small FPS. As a result, a small FPS whose radius of curvature is small can simulate a numerical model.

## ACKNOWLEDGEMENT

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