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Splitting/Russian Roulette Technique for Monte Carlo Pin Power Peaking Estimation

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1. Introduction

The Monte Carlo (MC) method for particle transport is a special numerical method to calculate the expected values of various nuclear parameters statistically from the results of stochastic simulations of particle kinetics. The MC calculation is capable of estimating the real nuclear parameters with precise three-dimensional geometry input and continuous energy cross-section libraries. However a large number of particles should be simulated to obtain the average values with small statistical uncertainties and it takes lots of computing time proportional to the number of the particles.

In spite of growth in computing power, the drastic increase in number of sources to achieve 1% statistics on local pin powers makes it hard to apply the MC method to the nuclear core analysis.

The objective of this paper is to efficiently estimate the pin power peaking factor (PPPF) of the nuclear core by the variance reduction techniques of splitting/Russian roulette [1].

2. Methods and Results

2.1 PPPF Estimation by Splitting/Russian Roulette

The maximum pin power is usually found in the one of several fuel assemblies (FA's) having larger assembly powers than the others. The assembly powers can be estimated by the MC calculations on the small number of sources to achieve 1% statistics.

Therefore it is efficient for the PPPF prediction to estimate the precise pin powers for only the several high-power assemblies that can be determined by the preceding MC calculations on the small number of histories. And the splitting/Russian roulette technique can be used to control the particle population in the important FA's that have high assembly powers.

In the MC track length estimator, the pin power is estimated by summing the power responses to the particle tracks generated at the pin. And these tracks must be induced from the particles moving into the FA including the pin and the source particle generated at the FA.

The particle population is controlled for the high-power assembly of importance I as below.

(1) When a particle of weight W moves into the FA of importance I from the FA of importance I' ($I' < I$), the particle is split into $\left[I/I' + \xi \right]$ particles which weights are $W \cdot I'/I$. ξ is a random number on [0,1) and $[x]$ means the largest integer less than or equal to x.

(2) When a particle of weight W leaves the FA of importance I to the FA of importance I' ($I' < I$), Russian roulette is played and the particle is killed with probability of $1-\left(I'/I\right)$, or followed further with probability I'/I and weight $W \cdot I/I'$.

(3) When a source particle of weight W is generated at the FA of importance I, the source particle is split into $\left[I + \xi \right]$ particles which weights are W/I .

(1) and (2) schemes are the same as the geometry splitting/Russian roulette technique [2].

2.2 Variance of Estimated Pin Power

In the normal Monte Carlo eigenvalue calculations, the variance of the pin power denoted by Q, $\sigma^2\left[Q\right]$ can be expressed as

$$\sigma^2\left[Q\right] = \sigma^2\left[Q_i\right] = E\left[\left(Q_i - E\left[Q\right]\right)^2\right]. \quad (1)$$

Q_i is the pin power estimated from i-th fission source.

Q_i can be expressed by the summation of the pin power contributions of the particles moving into or generated at the FA including the pin.

$$Q_i = \sum_j q_{ij} \quad (2)$$

q_{ij} is the pin power calculated from j-th particle moving into or generated at the FA while simulating i-th fission source.

q_{ij} has the statistical error, δ_{ij} from the expected value, q_{0ij}.

$$q_{ji} = q_{0ij} + \delta_{ij} \quad (3)$$

Using equations (2) and (3), Eq. (1) can be written as

$$\sigma^2\left[Q_i\right] = E\left[\left(\sum_j q_{0ij} - E\left[Q\right]\right)^2\right] + E\left[\left(\sum_j \delta_{ij}\right)^2\right] + 2\sum_j E\left[\delta_{ij} \cdot \left(\sum_j q_{0ij} - E\left[Q\right]\right)\right]. \quad (4)$$

Because the statistical error, δ_{ij} is independent of the other statistical errors and $\sum_j q_{0ij} - E\left[Q\right]$, Eq. (4) can be written as

$$\sigma^2\left[Q_i\right] = \sigma^2\left[Q_{0i}\right] + \sum_j \sigma^2\left[\delta_{ij}\right]. \quad (5)$$

$Q_{0i} = \sum_j q_{0ij}$.

For the MC eigenvalue calculations conducted with the active cycle number of N on the fission source number per cycle of M, the mean value of the pin power, $\bar{Q}$ can be calculated by

$$\bar{Q} = \frac{1}{NM}\sum_{i=1}^{NM} Q_i \, . \qquad (6)$$

From Eq. (5), the sample variance of $\bar{Q}$ in the normal MC calculations, $\sigma_S^2\left[\bar{Q}\right]$ can be written as

$$\sigma_S^2\left[\bar{Q}\right] = \frac{1}{NM}\sigma^2\left[Q_{0i}\right] + \frac{1}{NM}\sum_j \sigma^2\left[\delta_{ij}\right]. \qquad (7)$$

When I is set to the importance of the FA including the target pin in the splitting/Russian roulette calculations, j-th particle moving into or generated at the FA from i-th fission source is simulated I times changing the sequence of random numbers. And these repeated simulations reduces δ_{ij} of Eq. (3) as

$$\delta_{ij} = \frac{1}{I}\sum_{k=1}^{I}\left(q_{ij}\right)_k - q_{0ij} \, . \qquad (8)$$

$\left(q_{ij}\right)_k$ is the pin power from k-th replica of j-th particle moving into or generated at the FA from i-th fission source.

From the substitution of Eq. (8) into Eq. (5), the variance of Q_i in the splitting/Russian roulette calculations, $\sigma_{VR}^2\left[Q_i\right]$ can be expressed as

$$\sigma_{VR}^2\left[Q_i\right] = \sigma^2\left[Q_{0i}\right] + \frac{1}{I}\sum_j \sigma^2\left[\delta_{ij}\right]. \qquad (9)$$

From Eq. (9), $\sigma_{S,VR}^2\left[\bar{Q}\right]$ becomes

$$\sigma_{S,VR}^2\left[\bar{Q}\right] = \frac{1}{NM}\sigma^2\left[Q_{0i}\right] + \frac{1}{NM}\cdot\frac{1}{I}\sum_j \sigma^2\left[\delta_{ij}\right]. \qquad (10)$$

Comparing equations (7) and (10), one can find that the $\sum \sigma^2\left[\delta_{ij}\right]/NM$ term is reduced by $1/I$ in the splitting/Russian roulette calculations.

2.3 Application Results

The effectiveness of the splitting/Russian roulette techniques for the radial PPPF estimation is examined by the MCCARD [3] analysis for the SMART core [4].

The normal MC eigenvalue calculations were conducted with 1,000 active cycles changing the number of fission sources per cycle. Table 1 shows the relative error, R of the maximum pin power and the average Figure of Merit (FOM) of pin powers in the center FA. R is defined to be one estimated standard deviation divided by the estimated mean value and FOM is defined by

$$\text{FOM} = \frac{1}{R^2 T} \, . \qquad (11)$$

T is the computing time. For a fixed computing time, the smaller the variance the larger the FOM.

The splitting/Russian roulette calculations were conducted with 1,000 active cycles on 1,000 sources per cycle changing the importance of the center FA. Table 2 shows the FOM's for the splitting/Russian roulette calculations. From the table, one can observe that the average FOM with the importance of 100 is about 7 times as large as the one of the normal MC calculations.

From Eq. (10), R^2 can be expressed as a function of I,

$$R^2 = C_1 + C_2 I^{-1} \, . \qquad (12)$$

C_1 and C_2 are constant values for the SMART core problem. And the computing time, T can be expressed as the sum of time to simulate particles at FA's with the importance of 1 and time for the center FA with the importance of I that must be proportional to I.

$$T = C_3 + C_4 I \qquad (13)$$

From equations (12) and (13), FOM can be written as

$$\text{FOM} = \frac{1}{\left(C_1 + C_2 I^{-1}\right)\cdot\left(C_3 + C_4 I\right)} \, . \qquad (14)$$

Figure 1 shows the FOM trend according to the importance values in the splitting/Russian roulette calculations. We can observe that the measured FOM's perfectly match with the fitting curve by Eq. (14).

Table 1 Relative Error of the maximum pin power without splitting/Russian roulette

Number of sources per cycle, M	Rel. Err. of max. pin power	FOM of max. pin power	Avg. FOM*	CPU time (min.)
1,000	0.03978	32.6	59.8	19.4
10,000	0.01261	32.9	60.5	190.9
100,000	0.00392	33.8	60.6	1927.3

* Average over FOM's of pin powers in the center assembly

Table 2 Relative Error of the maximum pin power with splitting/Russian roulette

Importance of the center FA , I	Rel. Err. of max. pin power	FOM of max. pin power	Avg. FOM*	CPU time (min.)
10	0.01372	249.4	332.9	21.3
100	0.00710	460.2	423.0	43.1
1000	0.00682	83.8	86.3	256.5

* Average over FOM's of pin powers in the center assembly

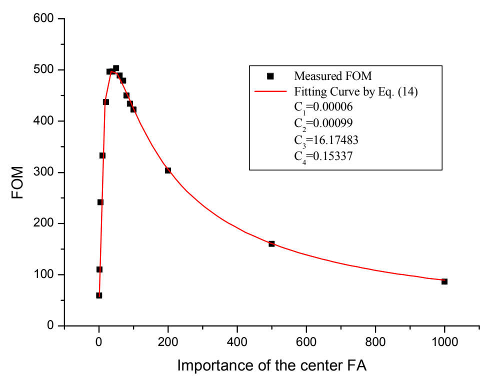

Figure 1. FOM according to the FA importance

3. Conclusion

We have applied the splitting/Russian roulette techniques to estimate the PPPF of the SMART core problem. From the numerical results, we observed their effectiveness in FOM. In particular, we derived the variance equation about the importance value for the splitting/Russian roulette calculations and it was helpful to analyze the FOM behavior according to the importance value.

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