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# Simulation of a Natural Convection by the Hybrid Thermal Lattice Boltzmann Equation

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## 1. Introduction

Recently, the lattice Boltzmann method(LBM) has gained much attention for its ability to simulate fluid flows, and for its potential advantages over conventional CFD method. The key advantages of LBM are, (1) suitability for parallel computations, (2) absence of the need to solve the time-consuming Poisson equation for a pressure, and (3) an ease with multiphase flows, complex geometries and interfacial dynamics may be treated[1]. In spite of its success in solving various challenging problems involving athermal fluids, the LBM has not been able to handle realistic thermal fluids with a satisfaction. The difficulty encountered in the thermal LBM seems to be the numerical instabilities.

The existing thermal lattice Boltzmann models may be classified into three categories based on their approach in solving the Boltzmann equation, namely, the multispeed, the passive scalar and the thermal energy distribution approach. For more details see Ref. [2].

In the present work, the hybrid thermal lattice Boltzmann scheme proposed by Lallemand and Luo[2] is used for simulating a natural convection in a square cavity. They proposed a hybrid thermal lattice Boltzmann equation(HTLBE) in which the mass and momentum conservation equations are solved by using the multiple-relaxation-time(MRT) model, whereas the diffusion-advection equations for the temperature are solved separately by using finite-difference technique. The main objective of the present work is to establish the lattice Boltzmann method as a viable tool for the simulation of temperature fields at high Rayleigh numbers.

## 2. Methods and Results

### *2.1 LBM with MRT model*

Lallemand and Luo[3] have defined a new column vector of macroscopic variables and $\vec{R}$ can be related $\vec{R} = (\rho, e, \varepsilon, j_x, q_x, j_y, q_{y,} p_{xx}, p_{xy})^T$ to the column vector of $\vec{F} = (f_0, f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8)^T$ as follows :

$$\vec{R} = \overline{\overline{M}}\vec{F} \qquad (1)$$

where $\overline{\overline{M}}$ is a 9x9 matrix transformation $\overline{F}$ to $\overline{R}$. In the column vector $\overline{R}$, $\rho$ is the fluid density, $\varepsilon$ is related to the square of the energy $e$, $j_x$ and $j_y$ are the mass flux in two directions, and $p_{xx}$ and $p_{xy}$ correspond to the diagonal and off-diagonal component of the viscous stress tensor respectively. One immediate advantage of the MRT model is that macroscopic variables of interest can be obtained readily by simply performing the matrix multiplication $\overline{\overline{M}}\overline{F}$ if $\overline{F}$ is known. In addition, due to the conservation of the mass and momentum before and after a particle collision, the total mass and momentum should not relax at all. Physically speaking, different physical modes should have different relaxation rates. By taking this into account in the MRT model, the collision procedure for $\overline{R^*}$ is performed as follows :

$$\overline{R^*} = \overline{R} - \overline{\overline{S}}(\overline{R} - \overline{R}^{eq}) \qquad (2)$$

where * denotes the post-collision state, $\overline{\overline{S}}$ is the 9x9 diagonal matrix, which will be shown later. In $\overline{\overline{S}}$, $s_1 = s_4 = s_6 = 0$ enforces the mass and momentum concervation before and after a collision. Before the stream step, Eq. (2), is performed, one needs to transform the post-collision values, $\overline{R^*}$, back to $\overline{F^*}$ by using Eq. (3) as

$$\overline{F^*} = \overline{\overline{M}}^{-1}\overline{R^*} = \overline{F} - \overline{\overline{M}}^{-1}\overline{\overline{S}}(\overline{R} - \overline{R}^{eq}) \quad (3)$$

where $\overline{\overline{S}}$ is the diagonal matrix.

$$\overline{\overline{S}} = diag(0, s_2, s_3, 0, s_5, 0, s_7, s_8, s_9)$$

Finally, the streaming step for all the $f_i$'s in the MRT model is performed exactly the same as in the standard LBGK model .

Lallemand and Luo[3] have shown that the MRT model can reproduce the same viscosity as that by a single-relaxation-time(SRT) model if we set $s_8 = s_9 = 1/\tau$. Once this is decided, the rest of the relaxation parameters( $s_2, s_3, s_5$ and $s_7$ ) for different physical modes can then be chosen more flexibly. It is worthy to note that the MRT model reduces to the SRT model by simply setting $s_2 = s_3 = s_5 = s_7 = s_8 = s_9 = 1/\tau$

The lattice Boltzmann equation can include external fields, such as gravity. For a forcing F, one can simply add it to the momentum, by $j + F\delta t \rightarrow j$ [6].

### *2.2 Hybrid thermal model*

The spurious mode coupling and numerical instability in the energy-conserving LBE models cannot be overcome by increasing the number of discrete velocities or including higher order terms in the equilibria. However, the athermal LBE models do not have such problems. Therefore, the best approach to formulate a TLBE model is to treat the energy-

conservation equation separately from the mass and momentum conservation equations. This means that the lattice Boltzmann equation is used to simulate the mass and momentum conservation laws, and a finite-difference scheme is used to solve the diffusion-advection equation for the temperature.

The temperature $T$ evolves according to the standard diffusion-advection equation,

$$\partial_t T + u \cdot \nabla T = \kappa \Delta T \qquad (4)$$

where $\kappa$ is the thermal diffusivity.

*2.3 Simulations*

The D2Q9 HTLBE model is used to simulate the convective flows in a square cavity with two opposite vertical walls at different temperatures. For the lattice Boltzmann part, the bounce-back boundary conditions are applied for the four walls. As for the temperature, two opposite vertical walls are maintained with constant temperatures $-T_0$ and $+T_0$, respectively. And the other two walls are adiabatic. The gravitation is pointing downward. The Rayleigh number Ra is defined as

$$Ra = \frac{2T_0 g \beta N^3}{\nu \kappa} \qquad (5)$$

where $g$ is the gravitational acceleration, $\beta$ is the coefficient of a thermal expansion, and $N$ is the lattice units. Two effective Nusselt numbers are defined for the flow. The effective local Nusselt number $Nu_w$ is defined by the temperature gradient at the wall maintained at a constant temperature :

$$\overline{Nu_w} = \frac{1}{2T_0} \sum_y \partial_x T)_{x=0} \qquad (6)$$

The effective(area) averaged Nusselt number $Nu_v$ is defined as

$$\overline{Nu_v} = \frac{1}{2T_0 \kappa N} \sum_{xy} u_x T - 1 \qquad (7)$$

where $\kappa (= \nu / \mathrm{Pr})$ is thermal diffusivity.

*2.4 Results*

Results from the HTLBE models are compared with those of the N-S solvers by de Vahl Davis[4](Figure 1). It is clearly shown that the difference of the average Nusselt numbers between the current study and de Vahl Davis[4] is very small. Generally, the overall flow structures (streamlines, temperatures) predicted by the HTLBE models are very similar to those predicted by de Vahl Davis[4]. As the Rayleigh number increases, the solution becomes time dependent. We have found that for $Pr$=0.71, and a square cavity of size 450x450, transition to time dependency takes place at $Ra$=1.8x10$^8$[5].

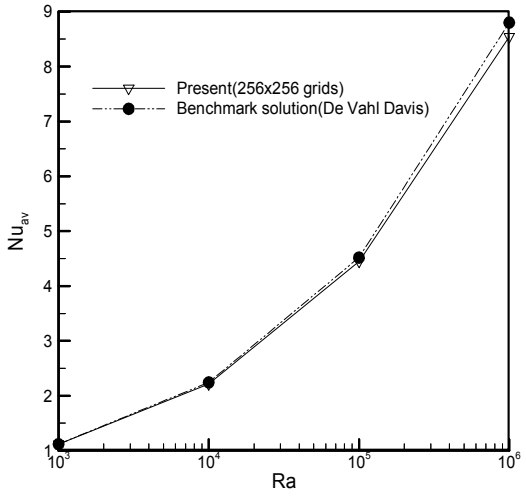


Figure 1. Comparison with averaged Nusselt numbers.

## 3. Conclusion

Simulations of flow and temperature fields that arise due to a natural convection in a square cavity have been carried out by using the hybrid thermal LBE methods. This numerical scheme is able to capture the velocity and temperature gradients accurately. Results for both are in good agreement with those using the N-S solver for $Ra$=10$^3$~10$^6$[4]. Also, we have found that the transition to turbulence takes place at $Ra$~1.8x10$^8$.

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