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A Comparative Study on Sugeno Integral Approach to Aggregation of Experts' Opinion

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1. Introduction

A multicriteria decision-making (MCDM) problem of preference ranking of various alternatives is common in science and engineering fields. Usually, the MCDM problem is characterized in terms of two factors: relative importance of each evaluation criterion and appropriateness of each alternative. The ranking is determined by a relative degree of appropriateness of decision alternatives. In reality, there are different grades of interaction among decision criteria.

One of well-known approaches to aggregation of those two factors is the weighted arithmetic mean (WAM) approach. Here, importance weights for criteria are viewed as probability measures. The weights are linearly aggregated with appropriateness values.

In the present work, the main objective is to study an aggregation model with various grades of interactions among the decision elements. The successful applications of fuzzy integral aggregation operators to subjective MCDM problems [1] have been motivating this work. On the basis of λ-fuzzy measures and Sugeno integral (SI), the SI aggregation approach is proposed. Here, interaction among criteria is dealt with λ-fuzzy measures. Aggregation of these measures and appropriateness values is implemented, especially, by the Sugeno integral as one of fuzzy integrals. Aggregated values obtained by the SI approach are viewed as decision maker's pessimistic (or conservative) attitude towards information aggregation, compared to the WAM approach.

Firstly, the concepts of the λ-fuzzy measure and the Sugeno fuzzy integral are introduced. Then, as an application of the SI approach, an illustrative example is given.

2. SI approach to information aggregation

The main reasons for the choice of a λ-fuzzy measure (i.e., Sugeno fuzzy measure) are that fuzzy measures for subsets of information sources is easy-to-calculate and the number of fuzzy measures to be known is reduced from 2^n-2 into n due to the λ-rule.

Let a finite set $X = \{x_1, x_2,\ldots, x_n\}$ be a set of information sources and a fuzzy density $g^i = g(\{x_i\})$ describe the degree of importance of each source x_i. Let the power set of X be 2^X. Then, a λ-fuzzy measure is a real-valued nonadditive set function g: $2^X \rightarrow [0,1]$ satisfying the following properties [2]:

$$g(\phi) = 0,\ g(X) = 1. \quad (2.1)$$

$$g(A) \le g(B) \text{ if } A \subseteq B \subseteq X. \quad (2.2)$$

$$\forall A, B \subseteq X \text{ and } A \cap B = \phi,$$
$$g(A \cup B) = g(A) + g(B) + \lambda\, g(A)\, g(B) \text{ for } \lambda \in (-1, \infty). \quad (2.3)$$

The parameter λ in Eq. (2.3) can be determined by solving a polynomial equation Eq. (2.4). The equation is derived by using the second boundedness property in Eq. (2.1) and the λ-rule in Eq. (2.3).

$$\lambda + 1 = \prod_{i=1}^{n} (1 + \lambda\ g^i). \quad (2.4)$$

Let an evaluation function h: $X \rightarrow [0,1]$ be sorted in ascending order such that $h(x_{(1)}) \le h(x_{(2)}) \le\ldots\le h(x_{(n)})$. For partial information source x_i, Sugeno fuzzy measure for a subset $A_{(i)} = \{x_{(i)}, \ldots, x_{(n)}\}$, $i = 1, ., n$, $g(A_{(i)})$, can be recursively characterized by Eq. (2.5). Here, $h(x_{(i)})$ denotes the i-th smallest function:

$$g(A_{(i)}) = g^{(i)} + g(A_{(i+1)}) + \lambda\, g^{(i)}\, g(A_{(i+1)}) \quad (2.5)$$
with $g(A_{(n+1)}) = 0$.

Sugeno integral can be viewed as aggregation operation process between evaluation functions and fuzzy measures representing the importance degrees of partial information. Discrete Sugeno integral, SI, with respect to the Sugeno fuzzy measure $g(A_{(i)})$ over X is formulated by

$$SI(h_1,\ldots,h_n) = \underset{i=1}{\overset{n}{Max}}\{Min[h(x_{(i)}), g(A_{(i)})]\}. \quad (2.6)$$

As a WAM approach uses additive probability measures as weighting factors, the WAM approach does not deal with the interaction among criteria. On the contrary, the SI approach based on λ-fuzzy measures handles various grades of interaction among criteria.

3. Application

To demonstrate a validation of the proposed approach to aggregation of two types of evidence, results of the previous study [3] based on the WAM approach are used and compared with them of the present SI aggregation approach. The comparison of indicators obtained from these two approaches enables us to figure out the effect of different aggregation methods.

Table 1 shows weighting values for subcriterion x_i and evaluation values $h_j(x_i)$ given in form of the 11X7 matrix (subcriterion x_i, system A_j) with i=1, ... , 11 and j=1, ... , 7. Here, decision alternatives consist of the power systems such as nuclear(A_1), coal-fired(A_2), heavy oil-fired(A_3), LNG-fired(A_4), hydropower(A_5), wind power(A_6), and solar photovoltaic (PV) power(A_7)). These seven options are evaluated in terms of eleven conflicting subcriteria. The subcriteria are as follows: generation cost (GC) x_1, land use (LU) x_2, global warming (GW) x_3, acidification (AC) x_4, energy payback (EP) x_5, quality of life (QL) x_6, fuel/energy supply security (SS) x_7, protection of terror (PT) x_8, sustainability degree (SD) x_9, accident mortality (AM) x_{10}, and years of lost life (YOLL) x_{11}.

Table 1. Weighting values and evaluation values.

		A_1	A_2	A_3	A_4	A_5	A_6	A_7
x_1	0.028	0.24934	0.24061	0.13745	0.11237	0.15226	0.09387	0.01411
x_2	0.139	0.71616	0.08952	0.08952	0.08952	0.00236	0.00497	0.00796
x_3	0.029	0.14069	0.00303	0.00427	0.00634	0.22135	0.36892	0.25540
x_4	0.051	0.83492	0.02743	0.01198	0.12341	0.00061	0.00067	0.00097
x_5	0.061	0.04863	0.02128	0.02128	0.01520	0.62310	0.24316	0.02736
x_6	0.083	0.12200	0.12500	0.13990	0.15770	0.13100	0.15180	0.17260
x_7	0.036	0.21430	0.15770	0.18150	0.15770	0.11910	0.08930	0.08040
x_8	0.175	0.13100	0.12200	0.15180	0.15180	0.14580	0.13990	0.15770
x_9	0.064	0.14580	0.10120	0.11900	0.16670	0.17260	0.13990	0.15480
x_{10}	0.218	2.80E-05	9.50E-07	8.10E-07	3.20E-06	1.00E-06	0.49998	0.49998
x_{11}	0.117	0.24034	0.03924	0.01729	0.08360	0.38454	0.19227	0.04273

In Table 2, the results obtained by WAM model and SI model are comparatively listed. The number noted in parenthesis denotes the preference ranking. Here, the λ-fuzzy measure associated with the SI approach was amount to the value of zero. It means the measure reduces into a probability measure.

Table 2. Aggregated preference scores for each system

	SI approach	WAM approach
A_1	0.24034 (2)	0.23410 (1)
A_2	0.12500 (6)	0.07036 (7)
A_3	0.15180 (5)	0.07258 (6)
A_4	0.15770 (4)	0.08865 (5)
A_5	0.20700 (3)	0.14546 (4)
A_6	0.24700 (1)	0.20954 (2)
A_7	0.24700 (1)	0.17932 (3)

In detail, Table 2 listed indicators such as overall scores and preference rankings for seven electricity generation systems. In the WAM approach, objective evidence is aggregated with subjective evidence extracted through a pairwise comparison technique. In the SI approach, these data are aggregated according to the algorithm for the present SI approach.

As shown in Table 2, the wind energy and the solar PV is the most preferred system in the SI approach, whereas the nuclear power is the most preferred one in the WAM approach.

It is found that the aggregation method selected in a modeling stage had an effect on both of ranking and overall score. Furthermore, this Sugeno integral approach can provide more easily interpretable information than the classical WAM does. Thus, it suggests that the proposed approach be one of beneficial tools to aggregate two types of evidence.

4. Conclusion

The SI aggregation approach is proposed to aggregate two types of evidence. As a demonstration, the SI aggregation method is applied to a MCDM problem. In the near future, various grades of interaction will be treated using λ values of non-zero.

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