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Interface Capturing method for a Two-phase flow using the Lattice Boltzmann method

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1. Introduction

Recently, the lattice Boltzmann method(LBM) has gained much attention for its ability to simulate fluid flows, and for its potential advantages over a conventional CFD method. The key advantages of LBM are, (1) suitability for parallel computations, (2) absence of the need to solve the time-consuming Poisson equation for a pressure, and (3) an ease with multiphase flows, complex geometries and interfacial dynamics may be treated[1]. An efficient scheme to predict the interface accurately is essential for multiphase flows.

To track or capture the interface, the popular methods in a traditional CFD are the Volume of Fluid (VOF) [2] and the level set method [3]. Both of them solve a scalar transport equation in the Eulerian frame. The VOF method does not track the interface itself. Instead, it tracks the volume fraction of each phase and component. The interface is reconstructed from the values of volume fraction. In this sense, it is often called the volume tracking method. The VOF approach cannot be extended easily to three dimensional applications. Besides, most of VOF interface reconstruction schemes only have the first order accuracy. In contrast, the level set method utilizes a level set function to indicate the interface. One advantage is that the level set function varies smoothly across the interface while the volume fraction is discontinuous. However, the level set method requires a re-initialization procedure to maintain the distance property when large topological changes occur around the interface. This may violate the mass conservation for each phase.

In the lattice Boltzmann method, three main models have been developed for multiphase and multi-component flows during the past twenty years. They are the color method, the potential method, and the free energy method [4]. The color and potential methods do not track the interface explicitly. In these two methods, the interface is considered as the region where the gradient of the density difference is not zero. The free energy method naturally captures the interface by explicitly solving a convection-diffusion equation. A distribution function is designed to solve this equation. Based on the Chapman Enskog expansion, it can be shown that the distribution function will recover the modified Cahn-Hilliard equation which is an evolution equation for the interface. This equation evolves the order parameter (for example, density difference) which is used to distinguish the different phases. In this sense, the order parameter is quite similar to the indicator of the traditional tracking methods such as the VOF and level set method. However, the original free energy method does not completely recover the convective Cahn-Hilliard equation but with two additional terms. Another limitation is that the density difference of each phase must not be too large. To remove these difficulties, a new method is proposed by Zheng et al. [5]. They solve the convective Cahn-Hilliard equation instead of the scalar transport equation. They remove the additional terms by applying a modified lattice Boltzmann method, replace the pressure tensor with the chemical potential, and use another free energy functional which has a good property such that the interface profile and coexistence curve can be given analytically.

In the present work, the interface capturing method for multiphase flows proposed by Zheng et al. [5] is used for simulating simple advection benchmark tests. The main objective of the present work is to establish the lattice Boltzmann method as a viable tool for the simulation of the multiphase or multi-component flows.

2. Methods and Results

2.1 The interface capturing equation

The interface capturing is modeled by an evolution equation of the order parameter ϕ, which serves as a counterpart of the volume fraction in the VOF method and the level set function in the level set method,

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi u) = \theta_M \nabla^2 \mu_\phi \quad (1)$$

where θ_M is called mobility and μ_ϕ is the chemical potential. Eq. (1) is a convective Cahn-Hilliard equation. The chemical potential is approximately a constant when the system approaches an equilibrium state. Thus Eq. (1) is similar to the interface transport equation of the VOF or level set method where the corresponding right-hand side is zero. The chemical potential is related to the free energy density functional. To distinguish different phases, the free energy functional is chosen as

$$F = \int \mathrm{Im}(\phi, \nabla \phi) dV = \int dV \left\{ \psi(\phi) + \frac{\kappa}{2} (\nabla \phi)^2 \right\} \quad (2)$$

where κ is related to the surface energy, and the bulk free energy density

$$\psi(\phi) = a(\phi^2 - \phi^{*2})^2 \quad (3)$$

where ϕ^* is a constant which is related to the equilibrium state. Eq. (2) ensures smooth local deviations near the interface. The energy density (3) will give two stable states of the system. The coexistence curve can be obtained as

$$\phi = \pm\phi^* \quad (4)$$

Thus the positions, where the condition $\phi = 0$ is satisfied, are defining the interface; while the regions where $\phi < 0$ and $\phi > 0$ represent the two phases respectively.

The chemical potential can be derived easily as

$$\mu_\phi = \frac{\partial \mathrm{Im}}{\partial \phi} - \nabla \cdot \frac{\partial \mathrm{Im}}{\partial \nabla \phi} = a(4\phi^3 - 4\phi^{*2}\phi) - \kappa \nabla^2 \phi \quad (5)$$

2.2 The implementation of lattice Boltzmann method

Under the lattice Boltzmann framework, Eq. (1) can be solved by iterating the evolution equation for a set of distribution functions. These distribution functions evolve with a modified lattice Boltzmann equation and BGK approximation,

$$f_i(x + e_i\delta, t + \delta) = f_i(x,t) + \Omega_i + (1-q)\delta f_i \quad (6)$$

with

$$\Omega_i = \frac{f_i^0(x,t) - f_i(x,t)}{\tau_\phi}$$

$$\delta f_i = f_i(x + e_i\delta, t) - f_i(x,t)$$

where f_i is the distribution function, Ω_i is the collision term, τ_ϕ is the dimensionless single relaxation time, e_i is the lattice velocity, and q is a constant coefficient. The details are Ref. [5].

2.3 Results

The above method is verified and applied to some test cases (simple translation, solid body rotation, shear flow). The simple translation, solid body rotation, shear flow are basic test cases for many interface capturing or tracking methods. To validate our method, we compared our results with those of Hirt-Nichol's VOF and Young's VOF[6]. The results of the present method are much better than Hirt-Nichol's VOF. From Fig. 1, we can clearly see that the Hirt-Nichol's VOF shows quantities of a jetsam and sawteeth at the interface as mentioned by many investigators. The present method generates a relatively sharp solution although it did not completely recover the initial configurations.

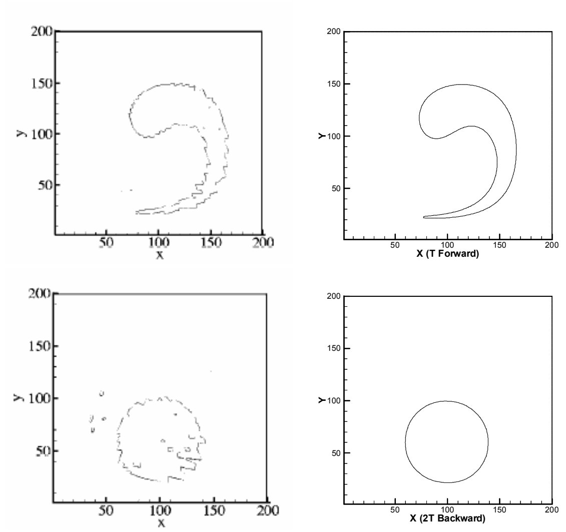

Figure 1. Comparison with VOF(left) and LBM(right) for a shear flow.

3. Conclusion

A lattice Boltzmann interface capturing method is applied in this paper. It does not require an interface reconstruction as needed by most of the traditional methods. Numerical results show that the present method can capture an accurate position of the interface and maintain its sharpness. It generates relatively sharp interfaces and shows a good performance under a shear flow with a stretching and tearing. Furthermore, the LBM method can be extended easily to the three-dimensional case.

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