A Staggered Semi-implicit Finite Volume Method to Solve 3-dimensional Containment Phenomena Based on Cell and Face Porosity

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1. Introduction

Thermal-hydraulic phenomena inside containment of nuclear power plant include various mass, momentum, and energy transport between gas including steam and non-condensable gases, continuous liquid, dispersed liquid (drop), and solid structures. This paper deals with the development of numerical solution for the above phenomena in containment.

In the development of numerical scheme the porosity method was adopted for handling easily the complex structure inside containment. Such porosity method is very effective in reducing mesh numbers in structured grid system. Staggered grid was used based on finite volume method[1,2]. So volume porosity and surface porosity were defined in scalar grid and vector grid, respectively [3]. Since a semi-implicit scheme for time advancement was utilized, the terms to be treated implicitly were expressed explicitly. Phase change terms were developed according to implicit scheme[4].

2. Governing Equations

3-dimensional three-fluid equations were setup. Gas field includes steam, air and hydrogen. Total 17 major unknowns were identified.

2.1 Continuity Equations

Continuity equations for gas, liquid, and drop are given by:

$$\frac{\partial}{\partial t}\left(a_g\rho_g\right)+\nabla\cdot\left(a_g\rho_g\mathbf{v}_g\right)=\Gamma_{l\to g}+\Gamma_{d\to g}+\Gamma_g \quad (1)$$

$$\frac{\partial}{\partial t}\left(a_l\rho_l\right)+\nabla\cdot\left(a_l\rho_l\mathbf{v}_l\right)=\Gamma_{g\to l}+\Gamma_{d\to l}+\Gamma_l \quad (2)$$

$$\frac{\partial}{\partial t}\left(a_d\rho_d\right)+\nabla\cdot\left(a_d\rho_d\mathbf{v}_d\right)=\Gamma_{g\to d}+\Gamma_{l\to d}+\Gamma_d \quad (3)$$

2.2 Momentum Equations

Momentum equations for gas, liquid, and drop are given by:

$$\begin{aligned}&\alpha_g\rho_g\frac{\partial\mathbf{v}_g}{\partial t}+\alpha_g\rho_g\nabla\cdot\left(\mathbf{v}_g\otimes\mathbf{v}_g\right)-\alpha_g\rho_g\mathbf{v}_g\nabla\cdot\mathbf{v}_g\\&=-\nabla\left(\alpha_g p\right)+\alpha_g\rho_g\mathbf{g}+\nabla\cdot\left(\alpha_g\left(\mu_g+\mu_g^t\right)\nabla\otimes\mathbf{v}_g\right)\\&+\left[\Gamma_{l\to g}\mathbf{v}_l+\Gamma_{d\to g}\mathbf{v}_d-\mathbf{v}_g\Gamma_g\right]\\&+\left[F_{gl}\left(\mathbf{v}_l-\mathbf{v}_g\right)+F_{gd}\left(\mathbf{v}_d-\mathbf{v}_g\right)\right]+\mathbf{M}_g\end{aligned} \quad (4)$$

$$\begin{aligned}&\alpha_l\rho_l\frac{\partial\mathbf{v}_l}{\partial t}+\alpha_l\rho_l\nabla\cdot\left(\mathbf{v}_l\otimes\mathbf{v}_l\right)-\alpha_l\rho_l\mathbf{v}_l\nabla\cdot\mathbf{v}_l\\&=-\nabla\left(\alpha_l p\right)+\alpha_l\rho_l\mathbf{g}+\nabla\cdot\left(\alpha_l\left(\mu_l+\mu_l^t\right)\nabla\otimes\mathbf{v}_l\right)\\&+\left[\Gamma_{g\to l}\mathbf{v}_g+\Gamma_{d\to l}\mathbf{v}_d-\mathbf{v}_l\Gamma_l\right]\\&+\left[F_{lg}\left(\mathbf{v}_g-\mathbf{v}_l\right)+F_{ld}\left(\mathbf{v}_d-\mathbf{v}_l\right)\right]+\mathbf{M}_l\end{aligned} \quad (5)$$

$$\begin{aligned}&\alpha_d\rho_d\frac{\partial\mathbf{v}_d}{\partial t}+\alpha_d\rho_d\nabla\cdot\left(\mathbf{v}_d\otimes\mathbf{v}_d\right)-\alpha_d\rho_d\mathbf{v}_d\nabla\cdot\mathbf{v}_d\\&=-\nabla\left(\alpha_d p\right)+\alpha_d\rho_d\mathbf{g}+\nabla\cdot\left(\alpha_d\left(\mu_d+\mu_d^t\right)\nabla\otimes\mathbf{v}_d\right)\\&+\left[\Gamma_{l\to d}\mathbf{v}_l+\Gamma_{g\to d}\mathbf{v}_g-\mathbf{v}_d\Gamma_d\right]\\&+\left[F_{dg}\left(\mathbf{v}_g-\mathbf{v}_d\right)+F_{dl}\left(\mathbf{v}_l-\mathbf{v}_d\right)\right]+\mathbf{M}_d\end{aligned} \quad (6)$$

2.3 Energy Equations

Energy equations for gas, liquid, and drop are given by:

$$\begin{aligned}&\frac{\partial}{\partial t}\left(\alpha_g\rho_g U_g\right)+\nabla\cdot\left(\alpha_g\rho_g U_g\mathbf{v}_g\right)\\&=-p\frac{\partial\alpha_g}{\partial t}-\alpha_g p\nabla\cdot\mathbf{v}_g-\nabla\cdot\left(\alpha_g\left(\mathbf{q}_g+\mathbf{q}_g^t\right)\right)\\&+\Phi_g+E_{l\to g}+E_{d\to g}+E_g\end{aligned} \quad (7)$$

$$\begin{aligned}&\frac{\partial}{\partial t}\left(\alpha_l\rho_l U_l\right)+\nabla\cdot\left(\alpha_l\rho_l U_l\mathbf{v}_l\right)\\&=-p\frac{\partial\alpha_l}{\partial t}-\alpha_l p\nabla\cdot\mathbf{v}_l-\nabla\cdot\left(\alpha_l\left(\mathbf{q}_l+\mathbf{q}_l^t\right)\right)\\&+\Phi_l+E_{g\to l}+E_{d\to l}+E_l\end{aligned} \quad (8)$$

$$\begin{aligned}&\frac{\partial}{\partial t}\left(\alpha_d\rho_d U_d\right)+\nabla\cdot\left(\alpha_d\rho_d U_d\mathbf{v}_d\right)\\&=-p\frac{\partial\alpha_d}{\partial t}-\alpha_d p\nabla\cdot\mathbf{v}_d-\nabla\cdot\left(\alpha_d\left(\mathbf{q}_d+\mathbf{q}_d^t\right)\right)\\&+\Phi_d+E_{g\to d}+E_{l\to d}+E_d\end{aligned} \quad (9)$$

2.4 Gas Motion Equations

Gas motion equation is given by:

$$\frac{\partial\left(\alpha_g\rho_g Y_i\right)}{\partial t}+\nabla\cdot\left(\alpha_g\rho_g\mathbf{v}_g Y_i\right)=\nabla\cdot\left(\alpha_g D_i\rho_g\nabla Y_i\right)+G_i \quad (10)$$

i =1 for steam, i =2 for air, and i =3 for hydrogen.

2.5 Source Terms

Source terms related with phase change in right hand sides above equations should be explicitly expressed for the implicit scheme. Basic phase change terms are mass phase changes.

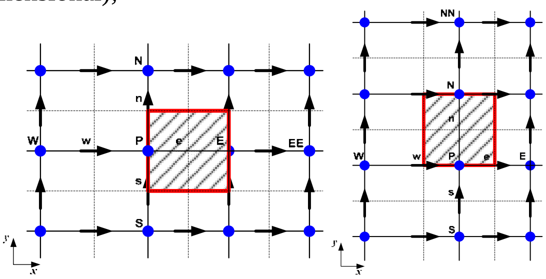
$$\Gamma_{g\to l} = -\frac{Q_{i,l\to g} + Q_{i,g\to l}}{\left(h_g^* - h_l^*\right)} = -\frac{\frac{p_v}{p} H_{ig}\left(T_{igl} - T_g\right) + H_{il}\left(T_{igl} - T_l\right)}{\left(h_g^* - h_l^*\right)} \quad (11)$$

$$\Gamma_{g\to d} = -\frac{Q_{i,d\to g} + Q_{i,g\to d}}{\left(h_g^* - h_d^*\right)} = -\frac{\frac{p_v}{p} H_{ig}\left(T_{igd} - T_g\right) + H_{id}\left(T_{igd} - T_d\right)}{\left(h_g^* - h_d^*\right)} \quad (12)$$

The other terms related with phase change were all developed based on above 2.

3. Numerical Solution Scheme

Conceptual 2-dimensional numerical grids are as like following figures (actual numerical grid is 3-dimensional);



(a) Scalar Cell (b) Vector Cell
Fig. 1 Staggered Grid

For the porosity method, a heavy side function is multiplied to each equation before integration over numerical cell.

$$H(\mathbf{x}) = \begin{cases} 1 & if\ \mathbf{x}\ is\ in\ fluid \\ 0 & else \end{cases} \quad (13)$$

Linearization for non-major unknowns is like

$$\phi^{(n+1)}(\zeta_1, \zeta_2, \cdots, \zeta_I) \quad (14)$$
$$= \phi_g^{(n)} + \left(\frac{\partial \phi_g}{\partial \zeta_1}\right)^{(n)} \left(\zeta_2^{(n+1)} - \zeta_2^{(n)}\right) + \left(\frac{\partial \phi_g}{\partial \zeta_2}\right)^{(n)} \left(\zeta_2^{(n+1)} - \zeta_2^{(n)}\right)$$
$$+ \cdots + \left(\frac{\partial \phi_g}{\partial \zeta_I}\right)^{(n)} \left(\zeta_I^{(n+1)} - \zeta_I^{(n)}\right)$$

For the convective terms, BSOUS (Bounded Second Order Upwind Scheme) was used [5]

$$\psi_e = \psi_P + \frac{\Delta x_P}{\Delta x_P + \Delta x_W}\left(\psi_P - \psi_W\right) \quad (15)$$

Each term in each equation was integrated over pre-defined cell and we got each difference equations. Velocity correction equation was obtained from the difference equation of the momentum equation as like;

$$\begin{bmatrix} u_g^{'} \\ u_l^{'} \\ u_d^{'} \end{bmatrix} \equiv \begin{bmatrix} u_g^{(n+1)} \\ u_l^{(n+1)} \\ u_d^{(n+1)} \end{bmatrix} - \begin{bmatrix} u_g^* \\ u_l^* \\ u_d^* \end{bmatrix} = -\begin{bmatrix} A_{\alpha,g,x} \\ A_{\alpha,l,x} \\ A_{\alpha,d,x} \end{bmatrix} \frac{d\delta p}{dx} \quad (16)$$

Where,

$$\begin{bmatrix} A_{\alpha,g,x} \\ A_{\alpha,l,x} \\ A_{\alpha,d,x} \end{bmatrix} \equiv \begin{bmatrix} m_{gg,x} & m_{gl,x} & m_{gd,x} \\ m_{lg,x} & m_{ll,x} & m_{ld,x} \\ m_{dg,x} & m_{dl,x} & m_{dd,x} \end{bmatrix}^{-1} \begin{bmatrix} V_{\alpha,g,x} \\ V_{\alpha,l,x} \\ V_{\alpha,d,x} \end{bmatrix}$$

From the other difference equation we got the linear matrix equation in order to get the unknowns. In particular from pressure related row in the matrix equation we made pressure correction equation and system pressure equation.

$$\mathbf{A}^{'}\mathbf{p} = \mathbf{r} \quad (17)$$

where,

$$\mathbf{A}^{'} = \begin{bmatrix} \left(1 + \sum_{j=u,v,w} (\mathbf{c}_P)_8\right)_1 & \cdots & \cdots & A_{1,i}^{'} & \cdots & \cdots & A_{1,N}^{'} \\ \vdots & \ddots & & \vdots & & & \vdots \\ \vdots & & \ddots & A_{i-1,i}^{'} & & & \vdots \\ A_{i,1}^{'} & & \cdots\ A_{i,i-1}^{'} & \left(1 + \sum_{j=u,v,w} (\mathbf{c}_P)_8\right)_i & A_{i,i+1}^{'} & \cdots & A_{i,N}^{'} \\ \vdots & & & A_{i+1,i}^{'} & \ddots & & \vdots \\ \vdots & & & \vdots & & \ddots & \vdots \\ A_{N,1}^{'} & \cdots & \cdots & A_{N,i}^{'} & \cdots & \cdots & \left(1 + \sum_{j=u,v,w} (\mathbf{c}_P)_8\right)_N \end{bmatrix}$$

$$\mathbf{p} = \left[\delta p_1 \quad \cdots \quad \cdots \quad \delta p_i \quad \cdots \quad \cdots \quad \delta p_N\right]^T$$

$$\mathbf{r} = \left[\left(\left[\mathbf{A}^{-1}\mathbf{b}\right]_8\right)_1 \quad \cdots \quad \cdots \quad \left(\left[\mathbf{A}^{-1}\mathbf{b}\right]_8\right)_i \quad \cdots \quad \cdots \quad \left(\left[\mathbf{A}^{-1}\mathbf{b}\right]_8\right)_N\right]^T$$

More details are attributed to reference 6

4. Sample Run and Test

Sample run was carried out to verify the developed code. The result showed physically reasonable calculation.

5. Conclusions

The foundation of containment analysis code was setup through a series of numerical development.

Acknowledgment

This study was performed under the project, " Development of safety analysis codes for nuclear power plants" sponsored by the Ministry of Knowledge Economy.

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