Measurements of Effective Delayed Neutron Fraction with External Neutron Source at Kyoto University Critical Assembly

Masao Yamanakaa, Cheol Ho Pyeonb, Yasunori Kitamurab and Tsuyoshi Misawab

aDepartment of Fundamental Energy Science, Graduate School of Energy Science, Kyoto University, Yoshida-honmachi, Sakyo-ku, Kyoto 606-8501, Japan
bNuclear Engineering Science Division, Research Reactor Institute, Kyoto University, Asashiro-nishi, Kumatori-cho, Sennan-gun, Osaka 590-0494, Japan

*Corresponding author: m-yamanaka@rri.kyoto-u.ac.jp

1. Introduction

Effective delayed neutron fraction ($\beta_{\text{eff}}$) characterizes the kinetic behavior of the nuclear reactor, and has been used in the subcriticality measurements to convert subcriticality in dollar unit to that in pcm unit. The calculation methodology by the eigenvalue calculations had been developed to evaluate $\beta_{\text{eff}}$ accurately [1-2], and measurements of $\beta_{\text{eff}}$ had been carried out to validate the calculated values by the noise analyses [3-4].

In the conventional methodology, $\beta_{\text{eff}}$ has been obtained under an assumption that neutron source is set at the core center, and the neutron flux in fundamental mode is dominant over the core. Here, these $\beta_{\text{eff}}$ are difficult to apply to the subcriticality evaluation with the use of external neutron source outside the core, because, in fact, the neutron flux distributions in the higher-mode components are considered easily induced, under an existence of external neutron source outside the core, such as the accelerator-driven system (ADS) at the Kyoto University Critical Assembly (KUCA). In the kinetic analyses [5-6], $\beta_{\text{eff}}$ had been evaluated by taking into account the external neutron source defining the adjoint neutron flux in fixed-source calculations, however, no attempts are made to obtain $\beta_{\text{eff}}$ by the measurements.

The purpose of this study was to examine the applicability of conventional measurement methodology of $\beta_{\text{eff}}$ with the use of external neutron source.

2. Experimental Settings

2.1 Core configuration

Experiments on the measurement of $\beta_{\text{eff}}$ were carried out at the KUCA A-core which is solid-moderated and -reflected one as shown in Fig. 1. The core was composed of the assemblies of normal fuel rod “F”, partial fuel rod “4”, and polyethylene reflector rod. Both fuel rods “F” and “4” were composed of 93% highly-enriched uranium and polyethylene moderator, and had a H/U value of 51.6. An americium-beryllium (Am-Be: Fig. 1) neutron source was installed outside the core as an external source with a neutron emission rate of 4.13E+06 n/s.

At a critical state, both excess reactivity and control rod worth (C1, C2 and C3) were measured by the positive period method and the rod drop method, respectively. And measured subcriticality in dollar unit was used in the measurements of $\beta_{\text{eff}}$ by the Nelson number method with the use of a 3He detector installed outside the core. To examine the applicability of the measurement methodology with the variation of the subcriticality, subcriticality was ranging between 0.7 and 2.8 $ by the insertion of the control and the safety rods as shown in Table I.

![Fig. 1 Core configuration.](image)

<table>
<thead>
<tr>
<th>Rod insertion</th>
<th>Subcriticality [$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>C1</td>
</tr>
<tr>
<td>Case II</td>
<td>C1, C2, C3</td>
</tr>
<tr>
<td>Case III</td>
<td>C1, C2, C3, S4</td>
</tr>
<tr>
<td>Case IV</td>
<td>C1, C2, C3, S4, S5, S6</td>
</tr>
</tbody>
</table>
2.2 Measurement methodology

Among \( \beta_{\text{eff}} \) measurement methodologies, the Nelson number method based on the Rossi-\( \alpha \) method can be applied to the measurements without the detector efficiency and the neutron life time. In the Rossi-\( \alpha \) method, the probability that a neutron is detected, after the detection of another one with a time interval \( dt \), is expressed as follows:

\[
P(t)\,dt = C\,dt + A\,e^{\alpha \, dt} \, dt,
\]

where \( \alpha \) is the prompt decay constant, \( C \) average count rate of the detector, and \( A \) correlation amplitude. The relationship between three parameters to \( \beta_{\text{eff}} \) and the subcriticality in dollar unit \( \rho_s \) can be expressed by defining the Nelson number \( N \) as follows:

\[
N = -\left( \frac{2}{g_{\text{p}}^* \, S} \right) \left[ \frac{A}{\alpha \, C} \right] = -\left[ \frac{1 - \beta_{\text{eff}}}{\beta_{\text{eff}}} \right] \left( \frac{\rho_s}{(1 - \rho_s)^2} \right), \tag{2}
\]

where \( v_p \) means the average number of prompt neutrons released per fission, \( \Gamma \) neutron dispersion factor, \( S \) source intensity, \( g \) is a correction factor taking into account the variation in the probability of detecting correlated counts originating from neutrons of different worth, and \( g^* \) is a correction factor for the spatial and energy distribution of the source neutrons as follows:

\[
g = \frac{\left[ P(r) \, dr \right] \left[ P(r) \right] \left[ \Gamma(r) \, dr \right]}{\left[ \int P(r) \, I(r) \, dr \right]^2}, \tag{3}
\]

\[
g^* = \frac{\int s(r) \, I_s(r) \, dr \, \left[ P(r) \right] \, dr}{\left[ P(r) \, I(r) \, dr \right] \left[ s(r) \, dr \right]}, \tag{4}
\]

\[
P(r) = \int \Sigma_f(r, E) \phi(r, E) \, dE, \tag{5}
\]

\[
I(r) = \int \chi(r, E) \phi^*(r, E) \, dE, \tag{6}
\]

\[
I_s(r) = \int \chi_s(r, E) \phi^*(r, E) \, dE, \tag{7}
\]

where \( s \) is neutron source strength per unit volume at position \( r \), \( \phi \) and \( \phi^* \) forward and adjoint fluxes, \( \Sigma_f \) the macroscopic fission cross section, \( \chi \) and \( \chi_s \) are the fission spectrum and the spectrum of external neutron source, respectively. Solving Eq. (2) for \( \beta_{\text{eff}} \), \( \beta_{\text{eff}} \) can be obtained from the Nelson number and subcriticality in dollar units as follows:

\[
\beta_{\text{eff}} = \frac{\rho_s}{N(1 - \rho_s)^2 - \rho_s}. \tag{8}
\]

3. Results and discussion

In the preparation of an estimation of \( \beta_{\text{eff}} \) by the Nelson number method, correction factors \( g \) and \( g^* \) were obtained by the diffusion calculations (SRAC-CITATION [7]) in two dimensional (x-y) and 107 energy groups with JENDL-4.0 [8] as shown in Table II. From the results in Table 2, the correction factor \( g \) was observed to be constant. Conversely, a large correction was needed for neutron source with \( g^* \) indicating much smaller values with a variation, because the external neutron source was located outside the core and source efficiency was differed by the insertion of the control rods. And, the \( g^* \) values were strongly affected by the rod insertion pattern compared with the \( g \) values.

\( \beta_{\text{eff}} \) was obtained by two correction factors in Eq. (2) with the variation of the subcriticality, and measured \( \beta_{\text{eff}} \) were compared with ones obtained by the eigenvalue calculations with the use of MCNP6.1 [9] together with JENDL-4.0, as shown in Table III. Measured \( \beta_{\text{eff}} \), involving the error of measured subcriticality and fitted curve for the estimation of the parameters \( A \), \( C \) and \( \alpha \), showed good agreement with calculated ones around the relative difference of 5%, indicating that the correction factors were accurately estimated even if the external neutron source located outside the core. These results revealed a possibility to apply to a measurement in case of an installation of external neutron source outside the core.

<table>
<thead>
<tr>
<th>Case</th>
<th>( g )</th>
<th>( g^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1.05E+00</td>
<td>1.02E-02</td>
</tr>
<tr>
<td>II</td>
<td>1.05E+00</td>
<td>1.03E-02</td>
</tr>
<tr>
<td>III</td>
<td>1.05E+00</td>
<td>1.06E-02</td>
</tr>
<tr>
<td>IV</td>
<td>1.05E+00</td>
<td>8.63E-03</td>
</tr>
</tbody>
</table>

Table III  Effective delayed neutron fraction \( \beta_{\text{eff}} \)

<table>
<thead>
<tr>
<th>Case</th>
<th>Nelson number method</th>
<th>MCNP6.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>810 ± 18</td>
<td>785 ± 21</td>
</tr>
<tr>
<td>II</td>
<td>781 ± 15</td>
<td>791 ± 21</td>
</tr>
<tr>
<td>III</td>
<td>780 ± 18</td>
<td>836 ± 22</td>
</tr>
<tr>
<td>IV</td>
<td>801 ± 18</td>
<td>781 ± 21</td>
</tr>
</tbody>
</table>

4. Conclusions

Measurements of \( \beta_{\text{eff}} \) by the Nelson number method were conducted to examine its applicability with the use of external neutron source. Results of the measured \( \beta_{\text{eff}} \) were...
showed good agreement with those of calculated ones, indicating the applicability of the measurements to the existence of external neutron source outside the core.

As a future work, the measurements of $\beta_{\text{eff}}$ by the pulsed neutron method could be attempted to be carried out in the ADS experiments to examine the dependence of $\beta_{\text{eff}}$ with the variation of spectrum of external neutron source and subcriticality.

Acknowledgements

This work is supported by Chubu Electric Power Co., Inc. The authors are grateful to all the staff of KUCA for their assistance during the experiments.

References