An Analysis of Reactivity Feedback Coefficients for a Small Research Reactor

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1. Introduction

A small research reactor has been proposed in order to provide better neutron distribution on one side for the purpose of neutron beam utilization.[1] The fuel is a typical plate type U_3Si_2 metal and the Al-alloy cladding. The core is composed of 3X3 fuel assemblies and surrounded by beryllium blocks.

In this study, the reactivity coefficients of the small core are evaluated including the moderator temperature coefficient, the reflector temperature coefficient, and the void coefficient. The analysis tool is the MCNP6[2] code and TMP card is used to assign different temperatures. To obtain reactivity coefficient efficiently, the generalized least square fitting method is applied and the reactivity is expressed a quadratic and a cubic polynomials.

2. Configuration of the Small Core

The basic design of the 3x3 small core as shown in Fig. 1 comes from the reference 1. The core is composed of the 5 standard fuel assemblies (SFA) and 4 control fuel assemblies (CFA). The typical fuel plate is used and the control plates are also considered as shown in Fig. 2. The fuel is U_3Si_2-Al metal fuel and the enrichment of U-235 is 19.75 wt%. The cladding is Al6061 alloy and the control rod is composed of Hf. The core is surrounded by the beryllium reflector of which size is the same as the fuel assembly. Table I shows the basic data of the small core. The core is shifted to the right side in order to increase leakage on the right side for the neutron beam utilization.

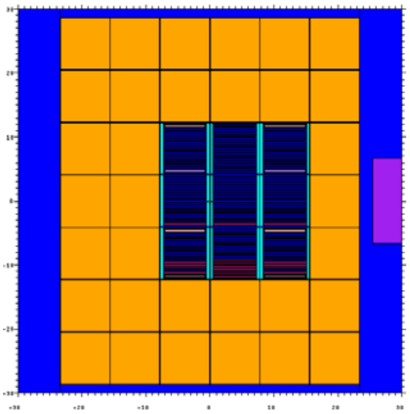
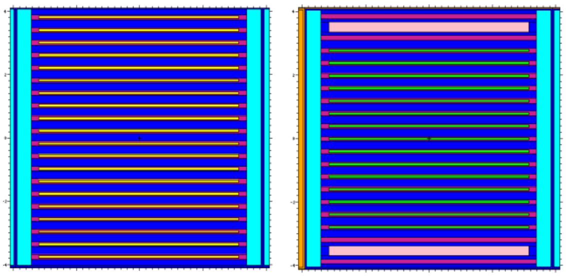


Fig. 1. Core configuration of 3x3 small research reactor.



(a)standard fuel assembly (b)control fuel assembly
Fig. 2. Core configuration of plate fuel assemblies

Table I. Characteristics of the 3x3 Small Core

Parameter	Value
Thermal power(MW)	2
# of standard fuel assembly(SFA)	5
# of control fuel assembly(CFA)	4
Fuel plate number in SFA	19
Fuel plate number in CFA	15
Thickness of fuel plate (cm)	0.076
Width of fuel plate (cm)	6.32
Height of fuel plate (cm)	60
Fuel assembly size (cm)	7.8x8.2x65
Plate water gap (cm)	0.246
Clad thickness (cm)	0.038
Control rod thickness (cm)	0.31

3. Generalized Least Square Fitting

The reactivities are expressed as a function of temperature or void fraction, the general least square fitting method[3] of the third order polynomial, for example, is expressed in the matrix form as follows

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ \vdots & \vdots & \vdots & \vdots \\ x_{n1} & x_{n2} & x_{n3} & x_{n4} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_n \end{pmatrix} \qquad (1)$$

where x_{ij} is a temperature (or a void fraction), a_j is a coefficient of x^{j-1}-th polynomial to be determined, and ρ_i is a reactivity. In the case of the polynomial fitting method, x_{ij} is expressed as $x_{ij} = x_i^{j-1}$ and n is a simulation or test number.

The matrix equation with weight matrix (W) is written as in a simplified form such as

$$WXA = WY \qquad (2)$$

where

$$W = \begin{pmatrix} \sigma_1^{-2} & 0 & \cdots & 0 \\ 0 & \sigma_2^{-2} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \sigma_n^{-2} \end{pmatrix}, X = \begin{pmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ \vdots & \vdots & \vdots & \vdots \\ x_{n1} & x_{n2} & x_{n3} & x_{n4} \end{pmatrix}$$

$$A = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}, \quad Y = \begin{pmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_n \end{pmatrix} \tag{3}$$

and σ_i is a uncertainty of the i-th reactivity data.

Equation (2) is solved easily with the generalized least square fitting by multiplying the transpose matrix on both terms. Then the coefficients are obtained as

$$A = \left(X^T W X\right)^{-1} X^T W Y \tag{4}$$

and their variances are also obtained as

$$Var(a_j) \approx \frac{R}{n-m}\left(X^T W X\right)^{-1}_{jj} \tag{5}$$

Where R is the total residue, $R = \sum_i (y_i - \hat{y}_i)^2$, $\hat{y}$ is an estimate of the given reactivity of y, and m, n are the fitting polynomial order and number of data sets, respectively. In general, when evaluating the fitting variance, it is assumed that the variable of X has no errors. Finally, the reactivity and its coefficient is expressed as follows

$$\rho(x) = a_1 + a_2 x + a_3 x^2 + a_4 x^3 \tag{6}$$

$$\frac{\partial \rho}{\partial x} = a_2 + 2a_3 x + 3a_4 x^2 \tag{7}$$

Linear and quadratic least square fittings are applied by following the above procedures. This approach is an indirect approach for the reactivity feedback coefficient when the direct approach is heavily burdened in the case of small reactivity coefficients. It is especially applicable for the Monte Carlo approach, which requires huge computational time for the direct estimation.

4. Analysis Results

The reactivities with different conditions such as moderator temperatures, reflector temperatures, and void fractions are obtained from the MCNP6 calculation. As a cross section, the ENDF/B-VII.0 library is used by default, and total 150 active cycles and 50 inactive cycles are used. 1E+5 particles per cycle is given to obtain the standard deviation of the multiplication factor of around 0.0003. Table II provides the calculated k-eff values for various conditions. After applying the generalized least square fitting methods, the coefficients of the fitted polynomial are obtained and are tabulated in Table III. For comparison, quadratic and linear fitting results are also provided for the void coefficient. The standard deviations of the fitting coefficients are also estimated using Eq.(4). Figs. 3, and 4 show the moderator temperature coefficient(MTC) and reflector temperature coefficient(RTC) by using cubic polynomial fitting approach, respectively. In the case of the MTC, the values distributed in the range of -1.7E-1 mk/^{o}C and -1.82E-02 mk/^{o}C. Around 30 ^{o}C, the MTC lies -6.45E-02 mk/^{o}C, which is a typical behavior in the research reactor. From Fig. 4, the reflector temperature coefficients exhibit slightly positive due to the decrease in absorption in the reflector as the temperature increases. However, the contribution of reflector temperature contribution is almost negligible. The total temperature coefficients become negative because of large contribution of the MTC. The void coefficient(VC) is of importance when the loss of coolant accident or unexpected void insertion into the core. It assures the negative values for various densities. Figs. 5, 6, and 7 depict the three different fitting orders such as cubic, quadratic, and linear fittings, respectively. The linear fitted VC is -1.19E+1 mk/%void. Depending on the fitting method, the values of VC changes between -52mk/%void to -3.5 mk/%void. In the case of void coefficient, any fitting order is possible because a large reactivity change is expected. However, the quadratic fitting is generally recommended in order to get meaningful temperature coefficient. If a linear fitting is used instead, a constant value of temperature coefficient is obtained, which is bad to explain the temperature dependency. And the cubic fitting results exhibit the unfamiliar curved trend in the low void fraction.

Table II. k-eff with various conditions of small reactor

Temp (^{o}C)	MTC k-eff*	RTC k-eff	Void Fraction	VC k-eff
10 ^{o}C	1.14915	1.14691	10 %	1.11626
20 ^{o}C	1.14864	1.14693	20 %	1.08152
30 ^{o}C	1.14791	1.14697	50 %	0.94104
50 ^{o}C	1.14579	1.14704	60 %	0.87723
60 ^{o}C	1.14431	1.14700	70 %	0.80212
70 ^{o}C	1.14283	1.14710	80 %	0.71233
80 ^{o}C	1.14074	1.14713	90 %	0.60133
90 ^{o}C	1.13893	1.14714	100 %	0.45605

*standard deviation = 0.0003

Table III. Coefficients and their standard deviation from least square fitting methods

Coef	MTC cubic	RTC cubic	VC cubic	VC quadratic	VC linear
a1	1.175e-6	2.562e-8	-4.468e-3	-2.338e-1	-1.191e1
a2	-1.025e-3	1.334e-6	5.239e-1	1.323e1	2.705e2
a3	-6.223e-3	1.912e-3	-2.236e1	-1.855e2	-
a4	2.148e0	2.772e-1	1.785e2	-	-
σ(a1)	1.950e-6	5.757e-7	9.383e-4	3.992e-2	2.462e0
σ(a2)	3.006e-4	8.872e-5	1.602e-1	4.413e0	1.651e2
σ(a3)	1.348e-2	3.979e-3	7.750e0	1.039e2	-
σ(a4)	1.599e-1	4.720e-2	9.045e1	-	-

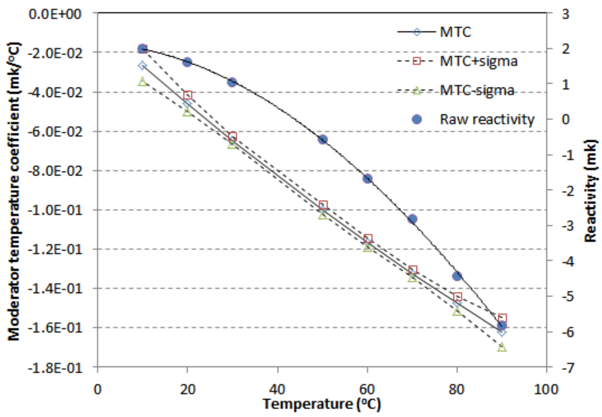


Fig. 3. Moderator temperature coefficient through the cubic polynomial fitted function.

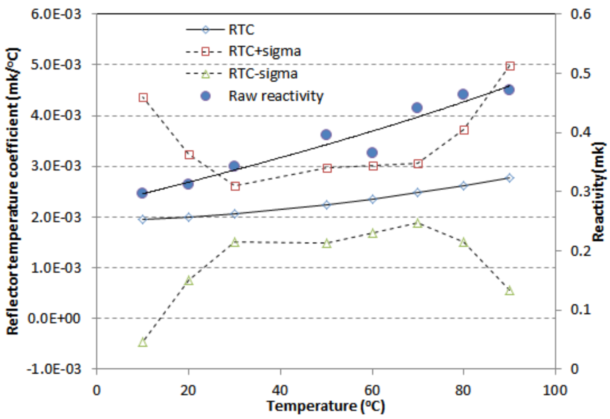


Fig. 4. Reflector temperature coefficient through the cubic polynomial fitted function.

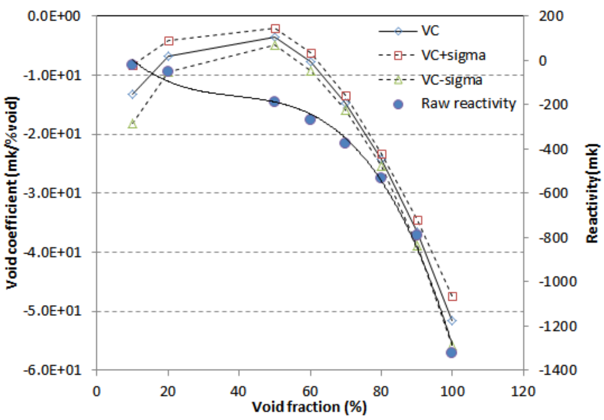


Fig. 5. Void coefficient through the cubic polynomial fitted function.

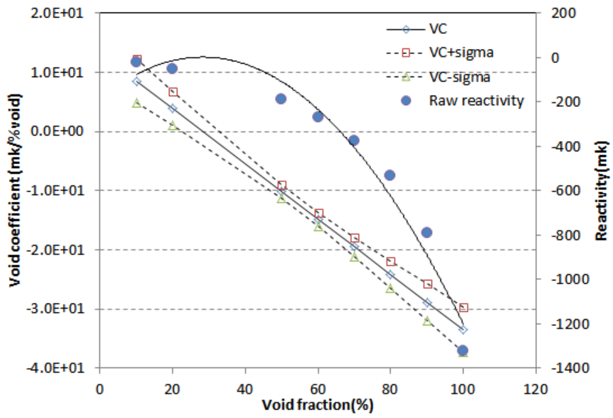


Fig. 6. Void coefficient through the quadratic polynomial fitted function.

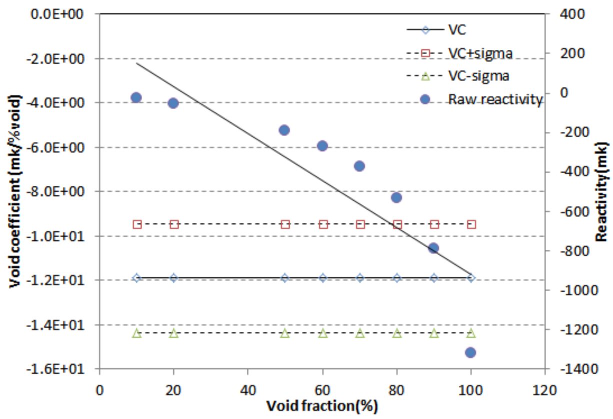


Fig. 7. Void coefficient through the linear polynomial fitted function.

5. Conclusions

The reactivity feedback coefficients for the small 3X3 research reactor are evaluated based on the generalized least square fitting method. Depending on the fitting order, the reactivity coefficients show slightly different distributions. In order to get temperature dependent reactivity coefficient, the higher order polynomial least square fitting is recommended such as quadratic or cubic polynomial fitting. It is noted that this indirect method of reactivity feedback coefficient is effectively applicable for the long computing analysis schemes such as the Monte Carlo core analysis. Furthermore, we should try to obtain accurate temperature dependent cross section libraries to get more reliable reactivity coefficients using the least square fitting method, too.

References

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