LWR Homogenization Analysis in both Pin-Cell and Assembly Levels

Bin Zhang, Yunzhao[*] Li, Hongchun Wu, Liangzhi Cao
Xi'an Jiaotong University, 28 West Xianning Road, Xi'an, Shaanxi 710049, China
[*] *yunzhao@mail.xjtu.edu.cn*

1. Introduction

Traditionally, Light Water Reactors (LWRs) are calculated based on the so-called two-step approach [1], which consists of lattice calculation, few-group constant parameterization and reactor core calculation. It is very efficient and requires only a small amount of memory. But it needs two homogenizations respectively in pin-cell and assembly levels to obtain single or quarter assembly homogenized few-group constants[2], and requires pin-power reconstruction techniques to obtain the pin-power distribution which is important for safety analysis. With the development of computing technology, an alternative scheme named pin-by-pin calculation becomes popular in recent years. It eliminates the assembly homogenization and the pin-power reconstruction. However, the change of calculation scheme requires the improvement of homogenization technology.

Focused on the homogenization technology, this paper analyzed the generalized equivalence theory (GET) applied to both assembly and pin-cell levels [3], named to obtain both the assembly discontinuity factor (ADF) and the pin cell discontinuity factor (CDF), and also applied the super homogenization (SPH) method [4] for the pin cell homogenization [5]. In order to analyze those three homogenization schemes, based on a typical PWR problem [6], four kinds of calculation schemes are compared in this summary. The reference solution will be obtained by the multi-group heterogeneous core calculation with transport code. Three homogeneous calculations will be carried out in both pin-cell and assembly levels with different homogenization methods. The influence affected by the different energy group-structures and the different boundary conditions will also be analyzed in this summary.

2. Theory

In GET, discontinuity factor is defined as the ratio of the heterogeneous surface flux over the homogeneous one:

$$f_{s,i,g} = \frac{\phi_{i,g}^{s,het}}{\phi_{i,g}^{s,hom}} \qquad (1)$$

where, subscript s, i and g stand for the surface, the node and the energy group. $\phi_{i,g}^{s,het}$ (cm^{-2}·s^{-1}) is the nodal surface flux obtained from heterogeneous neutron transport calculation together with the nodal surface net current $J_{i,g}^{s,het}$ (cm^{-2}·s^{-1}) and the nodal volumetric averaged flux $\bar{\phi}_{i,g}^{het}$ (cm^{-2}·s^{-1}); in contrast, $\phi_{i,g}^{s,hom}$ (cm^{-2}·s^{-1}) is the nodal surface flux which is supposed to be obtained from the homogeneous neutron diffusion calculation. Consequently, the point is how to estimate the homogeneous surface flux before homogeneous calculation.

For assembly homogenization, a flat flux distribution approximation is adopted considering that the assembly is optically thick. Thus, the homogeneous surface flux can be estimated as following by preserving the volumetric flux:

$$\phi_{i,g}^{s,hom} = \bar{\phi}_{i,g}^{het} \qquad (2)$$

For pin-by-pin homogenization, the flux distribution approximation has to be exactly the same as the active following homogeneous calculation. In this summary, the Exponential Function Expansion Nodal method (EFEN) is employed for the homogeneous diffusion calculation [7]. In EFEN, we can get the partial current response relation via the undetermined coefficient method with the constraints for net currents surface fluxes and node-average flux:

$$J_{x+,i,g}^{hom,out} = \mu_{x,i,g}^{hom} J_{x+,i,g}^{hom,in} + \eta_{x,i,g}^{hom} \bar{\phi}_{i,g}^{hom} + \xi_{x,i,g}^{hom} S'_{i,g} \qquad (3)$$

where, subscript x stands for the coordinate sides of the node, the coefficients $\mu_{x,i,g}^{hom}, \eta_{x,i,g}^{hom}, \xi_{x,i,g}^{hom}$ are determined by the cross sections and the dimension of the node, the $S'_{i,g}$ depends on the outer-scattering source, fission source and the net currents.

The net current and the surface flux have the relationship with the partial current:

$$\begin{cases} \phi_{x\pm,i,g}^{s,hom} = 2\left(J_{x\pm,i,g}^{hom,out} + J_{x\pm,i,g}^{hom,in} \right) \\ J_{x\pm,i,g}^{hom} = J_{x\pm,i,g}^{hom,out} - J_{x\pm,i,g}^{hom,in} \end{cases} \qquad (4)$$

Thus, the homogeneous nodal surface flux can be expressed as the following by preserving both the volumetric flux and the surface net current:

$$\phi_{x\pm,i,g}^{s,hom} = \frac{2}{\left(\mu_{x,i,g}^{hom}-1\right)}\Big[\left(\mu_{x,i,g}^{hom}+1\right)J_{x\pm,i,g}^{het} - $$
$$2\eta_{x,i,g}^{hom}\bar{\phi}_{i,g}^{het} - 2\xi_{x,i,g}^{hom}S'_{i,g}\Big] \qquad (5)$$

where, h is the nodal dimension (cm).

In SPH method, a correction factor (SPH factor) is to keep the cell-average reaction rate preserved before and after the homogenization via the Eq. (6) and the SPH factors are calculated by Eq. (7) as below:

$$\bar{\Sigma}_{x,i,g}^{SPH} = \mu_{i,g}\bar{\Sigma}_{x,i,g} \qquad (6)$$

$$\mu_{i,g} = \frac{\bar{\phi}_{i,g}^{het}}{\bar{\phi}_{i,g}^{hom}} \qquad (7)$$

where, $\mu_{i,g}$ is the SPH correction factors of region i. $\overline{\Sigma}_{x,i,g}$ is the average cross section via the flux volume weight method. $\overline{\phi}_{i,g}^{het}$ is the average neutron flux from the cell heterogeneous calculation. $\overline{\phi}_{i,g}^{hom}$ is the average neutron flux obtained by the cell homogenization calculation using the SPH corrected cross section $\overline{\Sigma}_{x,i,g}^{SPH}$.

In the SPH method, the following normalization of the neutron fluxes is performed:

$$\overline{\phi}_{i,g}^{hom,norm} = \overline{\phi}_{i,g}^{hom} \frac{\sum_{g' \in g} \sum_k \phi_{k,g}^{het} V_k}{\sum_i \overline{\phi}_{i,g}^{hom} V_i} \tag{8}$$

Where $V_i = \sum_{k \in i} V_k$: Volume of the homogenized node.

It is an iterative calculation to obtain the SPH factors. The cell homogenization calculation method (EFEN) used in the iterative calculation has to be exactly the same with the active following core calculation.

The calculation strategy is split into four steps:

1. Calculate the cell-heterogeneous problem to get the average flux and the flux volume weighted (FVW) cross section.
2. Set the SPH factors to the initial values.
3. Calculate the cell-homogenization problem to update the SPH factors and correct the cross sections.
4. Check if the SPH factors are convergent. If not then go back to step 3.

In order to analysis the homogenization techniques in pin cell and assembly levels, four kinds of calculation procedures are supposed to be compared and analyzed. (1) It is the multi-group heterogeneous core calculation with transport code to provide the reference solution and the heterogeneous flux spectrum. (2) Assembly homogenizations based on GET with and without reflective boundary condition. 2-group structure is employed to be consistent with the legacy scheme. (3) Pin-by-pin homogenizations based on GET with and without reflective boundary condition. (4) Pin-by-pin homogenizations based on SPH method. Both 2- and 7-group structures have been used respectively to compare with the legacy scheme and to satisfy the pin-by-pin calculation requirement.

3. Numerical Results

Two typical heterogeneous PWR problems, with control rods in and out respectively, are defined in slab geometry as shown in Fig. 1. Each number represents a type of assembly. "3" and "4" represent the fuel assembly with and without control rods respectively. "5" represents the water. The length of each assembly is 21.42cm. The configuration of the assemblies is in Fig.2. "1" and "2" represent the fuel pin and the control rod pin respectively. The length of each pin cell is 1.26cm. The configuration of the pin cell is in Fig.3.

Vacuum | 5 | 3 | 4 | 3 | 4 | 3 | 4 | 3 | 4 | 3 | 4 | 3 | 4 | 3 | 4 | 3 | 5 | Vacuum
Fig. 1 Configuration of the core problem

Fuel Assembly without CR | 1 | 1 | 5 | 1 | 1 | 5 | 1 | 1 | 5 | 1 | 1 | 5 | 1 | 1 | 5 | 1 | 1 |

Fuel Assembly with CR | 1 | 1 | 2 | 1 | 1 | 2 | 1 | 1 | 5 | 1 | 1 | 2 | 1 | 1 | 2 | 1 | 1 |
Fig. 2 Configuration of assembly

Fig. 3 Configuration of the pin cell

The eigenvalues are summarized in Table I. The "Two-step" and the "Pinbypin" in Table I represent the two-step calculation scheme and the pin-by-pin calculation scheme respectively. The "EC-ADF" and the "EC-CDF" represent that the cross sections and the discontinuity factors are generated by the reference spectrum. It can be found that 2-energy group structure for pin-by-pin calculation is not proper no matter whether the GET or the SPH method is utilized for the pin cell homogenization. Similar to the 2-group for the assembly homogenization, the coarse energy group (7-group) is much more accurate for pin-by-pin calculation.

Table I. Results of k_{inf}

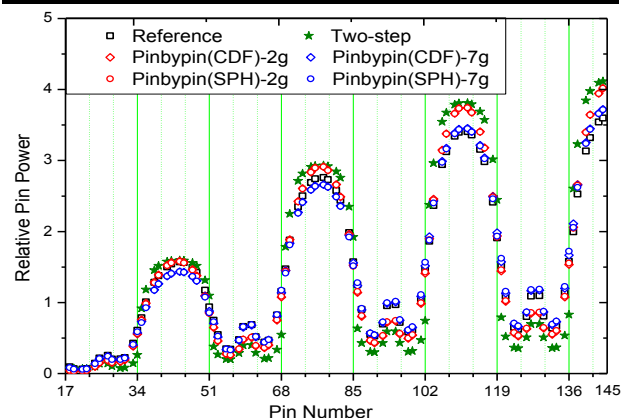| CASE	Number of Groups	k_{inf}	Error (*pcm*)
Reference	69	0.99904	-
Two-step(ADF)	2	0.99345	-559
Pinbypin(CDF)	2	1.02877	2973
Pinbypin(SPH)	2	1.02701	2797
Pinbypin(CDF)	7	0.99575	-329
Pinbypin(SPH)	7	0.99338	-566
Two-step (EC-ADF)	2	0.99910	6
Pinbypin (EC-CDF)	7	0.99933	29

Fig. 4 Pin-power distributions of different calculation

The pin power of the two-step calculation scheme is obtained via a pin-power reconstruction technique. The results of the pin-power and pin-power error showed in

Fig. 4 and Fig. 5 further prove that the 7-group is appropriate for the pin-by-pin calculation. The differences between the pin-power of the two-step calculation and of the pin-by-pin calculation show the potential of the pin-by-pin calculation to improve the accuracy of core calculation. The pin-power errors of the calculation used with the GET and with the SPH method are similar with each other. Both of them are effective.
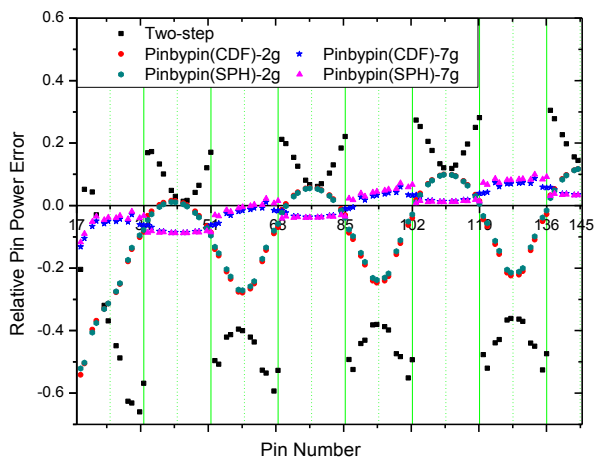

Fig. 5 Comparison of pin-power for different calculation

Since the discontinuity factors and the SPH factors are generated with a specific boundary conditions (zero current), they cannot exactly correct the homogenization error when they come to the core. So there exists a need to improve these methods to make them less environment dependent.
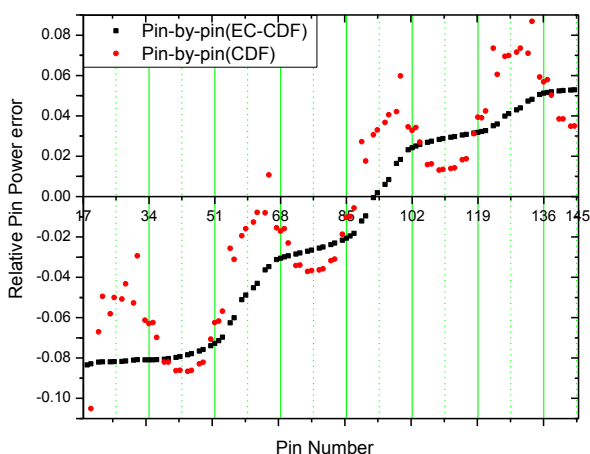

Fig. 6 Pin power error with/without the reflective BC

Fig. 6 and Fig. 7 show the differences between the calculations with cross sections and discontinuity factors generated in the single assembly calculation with reflective boundary condition and with exact boundary condition. The error is reduced when the exact boundary condition is utilized. When the cross sections and the discontinuity factors are generated under the exact boundary condition, there still exist errors. It is because that the homogenization methods cannot keep the leakage preserved in a vacuum boundary. The next research is about to improve the homogenization to

make the cross section and discontinuity factor more accurate under the reflective boundary condition and the vacuum boundary condition.
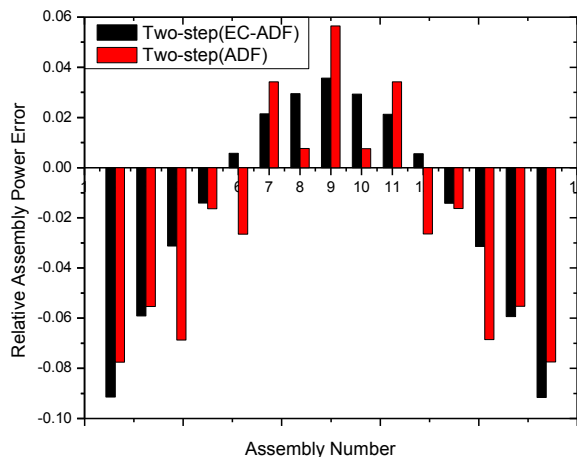

Fig. 7 Assembly power error distribution

4. Conclusions

The GET and the SPH method work effective in pin-by-pin calculation scheme. The CDFs must be calculated based on the flux distribution approximation used for the active following homogeneous calculation. And in the SPH method, the cell homogenization calculation method used in the iterative calculation also has to be exactly the same with the active following core calculation.

Compared with the two-step calculation scheme, the pin-by-pin calculation scheme skips the pin power reconstruction and it is in more details for the core simulator. The pin-by-pin calculation scheme is more accurate for LWR simulators.

In pin-by-pin calculation, it is no longer appropriate to use the 2-group structure. The 7-group structure would be more accurate.

The error brought in by the infinite environment in unit assembly calculation is measured. What's more, it is a challenge to do a good performance for the recent homogenization method in the vacuum boundary condition. Further research will focus on these challenges.

Acknowledgements

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