Proceedings of the Reactor Physics Asia 2015 (RPHA15)Conference
Jeju,Korea,Sept. 16-18, 2015

Uncertainty analysis of core power distribution monitoring system SMROMS

Xingjie Peng[a,b,*], Yun Cai[a,b], Qing Li[b], Kan Wang[a]

[a]*Department of Engineering Physics, Tsinghua University, Beijing, China*
[b] *Science and Technology on Reactor System Design Technology Laboratory, Nuclear Power Institute of China, Chengdu, China*
[*]Corresponding author: pxj11@mails.tsinghua.edu.cn

1. Introduction

Core power distribution monitoring in operating power reactors is very important in core surveillance, the 3-D power distribution is one of the basic operation parameters which can determine many other important parameters used to evaluate the operation condition of reactor and the safe margin. Many kinds of on-line monitoring systems, such as BEACON [1] and SCOMS [2], have been developed to estimate in-core power distributions using fixed in-core detectors. In order to get the reliability of the estimated power distribution, the uncertainty analysis procedure of the monitoring system is needed.

The uncertainty analysis method [1] of BEACON system includes a statistical simulation of various core state conditions, power distributions and detector behavior based on the measurement variability. A number of 'true' and 'predicted' core model pairs are selected to provide the uncertainty analysis with a bounding set of differences that could be encountered between the BEACON core monitor model and plant conditions under normal and off-normal conditions. The uncertainty analysis method used in SCOMS system [2] get the overall uncertainty from the uncertainties of the input parameters. Individual uncertainties of input variables are randomly sampled within the range and then added to the original value.

In this study, a new uncertainty analysis method for core power distribution monitoring system SMROMS (Small Modular Reactor On-line Monitoring System) was discussed. SMROMS uses in-core self powered neutron detectors (SPNDs) to monitor the power distribution continuously. This method is different from two methods mentioned above, because the calculation uncertainty and measurement uncertainty are considered separately. A rod controlled small modular reactor developed by Nuclear Power Institute of China, i.e. ACP100-K, was selected as a study case. The 3DCC [3] (3D Coupling Coefficients) power distribution reconstruction method was implemented in SMROMS.

2. Theory

The purpose of this paper is to discuss the uncertainty analysis method of the SMROMS. The 3DCC (3D Coupling Coefficients) power distribution reconstruction method was implemented in SMROMS. Firstly, the 3DCC method used to reconstruct the power distribution was introduced briefly. Secondly, the overall uncertainty analysis procedure was elaborated.

2.1 3DCC Method

The detector results at certain locations reflect the actual reactor flux or power can be applied to improve the results of the only diffusion calculations. In-core detector signals are converted into the power of a detector node power by using the signal-to-power conversion factor. In 3DCC method, each node power is determined from the power coupling coefficients. The 3DCC is defined as the ratio of the power of a node (l, k) to the sum power of the neighboring nodes as Eq. (1):

$$C_{l,k} = \frac{1}{P_{l,k}}\left(\sum_{j=1}^{N_l} P_{j,k} + \sum_{j=1}^{N_k} P_{l,j}\right) \qquad (1)$$

where $C_{l,k}$ = power coupling coefficient at node (l, k), N_l = number of radial neighboring nodes (including the east, west, north and south nodes) to node (l, k), and N_k = number of axial neighboring nodes (including the top and bottom nodes) to node (l, k). The approximated power coupling coefficient can be determined using 3D neutronics calculation as Eq. (2):

$$C_{l,k} \approx C^C_{l,k} = \frac{1}{P^C_{l,k}}\left(\sum_{j=1}^{N_l} P^C_{j,k} + \sum_{j=1}^{N_k} P^C_{l,j}\right) \qquad (2)$$

Because the calculated 3DCCs can be provided by the neutronics calculation beforehand, the power of the undetected node can be solved by Eq. (3):

$$C^C_{l,k} P_{l,k} - \sum_{j\in U(l,k)} P_{j,k} - \sum_{j\in U(l,k)} P_{l,j} = \sum_{j\in I(l,k)} P^M_{j,k} + \sum_{j\in I(l,k)} P^M_{l,j} \qquad (3)$$

where the superscript "M" means the measured power, groups $U(l,k)$ and $I(l,k)$ mean the undetected and detected neighboring node groups of node (l, k), respectively. Eq. (3) is applied to all the nodes and can be expressed as the following matrix-vector form:

$$AP^U = S \qquad (4)$$

where A = coupling coefficient matrix, P^U = vector of undetected node powers, and S = source vector from detected node powers. All the node powers throughout the whole core can be obtained by solving Eq. (3).

2.2 Uncertainty Analysis Method

The uncertainty analysis method utilized in this paper regards the uncertainty of reconstructed parameter as a weighted average of calculation uncertainties and measurement uncertainties. It is assumed that calculation uncertainty and measurement uncertainty are independent of each other. In order to explain the procedure of the uncertainty analysis clearly, the following definitions are introduced:

$$\varepsilon_m = \frac{P_m - P_t}{P_t} \quad (5)$$

$$\varepsilon_c = \frac{P_c - P_t}{P_t} \quad (6)$$

$$\varepsilon_{mc} = \frac{P_m - P_c}{P_c} \quad (7)$$

where P_t = true power, P_m = measured power, P_c = calculated power, ε_m = measurement error, ε_c = calculation error, ε_{mc} = observed difference. The observed difference can be approximated by:

$$\varepsilon_{mc} = \frac{P_m - P_c}{P_c} \approx \frac{P_m - P_c}{P_t} = \varepsilon_m - \varepsilon_c \quad (8)$$

As calculation uncertainty and measurement uncertainty are assumed to be independent of each other, the following relationship exists when uncertainties have normal distributions:

$$\sigma_{mc}^2 = \sigma_m^2 + \sigma_c^2 \quad (9)$$

where σ_{mc} = variance of observed difference, σ_m = variance of measurement error, σ_c = variance of calculation error.

Here an assumption is made that the reconstructed power of whole core can be regarded as a weighted combination of measured power and calculated power as Eq. (10):

$$P_r = aP_c + bP_m \quad (10)$$

where P_r = reconstructed power, a, b = core-wide average weights to be determined by theoretical analysis. Eq. (10) can't be applied directly because it is more likely a semi-qualitative equation. The following relationship exists according to Eq. (10):

$$\sigma_r^2 = a^2\sigma_c^2 + b^2\sigma_m^2 \quad (11)$$

where σ_r = variance of reconstruction error. If we can know the values of a, b, σ_m, σ_c, then the variance of reconstruction error, i.e. the uncertainty of reconstructed parameter, is obtained.

σ_m can be estimated by analyzing the time series of in-core neutron detector signals with an assumption that there is no error in signal-to-power conversion factor:

$$\sigma_m^2 = \frac{\sum_{i=1}^{N} (I_m^i / \overline{I}_m - 1)^2}{N-1} \quad (12)$$

where N is the sampling number of the corresponding time series, I_m^i is the detector measurement of ith time point, $\overline{I}_m$ is the average value of detector measurement.

σ_{mc} can be estimated from the calculation power and measurement power considering all independent detector node, and then σ_c can be obtained from Eq. (9).

a, b are two unknown weights, and both should be determined by theoretical analysis, since we don't have direct access to the actual measured power.

When there isn't measurement error, a can be evaluated by the following equation:

$$a = \frac{\sigma_r}{\sigma_c} \quad (13)$$

where a represents the error reduction through 3DCC method. A number of 'true' and 'calculated' core condition cases have been simulated to estimate the distribution of a, and the estimation procedure has the following steps:

The "baseline" core conditions are selected at different burnup points, and the corresponding power distributions are regarded as calculated power;

The "perturbed" core conditions are generated by perturbing the core parameters, such as burnup, rod position and power level, of the "baseline" core condition. In this study, Latin Hypercube Sampling (LHS) method [4] is utilized to generate different "perturbed" core conditions. The corresponding power distributions are regarded as true power, and the detector measurements can be generated from true power without noise;

Using 3DCC method to get the reconstructed power from detector measurements and calculated power;

Calculating σ_r and σ_c from true power, reconstructed power and calculated power, and then a can be obtained.

Because we can set different "baseline" conditions and "perturbed" conditions, the distribution of a can be obtained.

b can be estimated by the following equation:

$$b = \sqrt{\frac{\sigma_r^2 - a^2\sigma_c^2}{\sigma_m^2}} \quad (14)$$

and the estimation procedure has the following steps:

For a certain pair of "baseline" core condition and "perturbed" core condition, a can be obtained from its estimation procedure;

The detector measurements can be generated from true power with noise perturbation according to the value of σ_m;

Using 3DCC method to get the reconstructed power from detector measurements and calculated power;

Calculating σ_r from true power and reconstructed power, and then b can be obtained.

3. APPLICATION IN ACP100-K

A rod controlled small modular reactor developed by Nuclear Power Institute of China, i.e. ACP100-K, was selected as a study case to describe the uncertainty analysis method introduced above. The core of ACP100-K has 57 assemblies, and 9 of them are instrumented with SPNDs. SMART neutronics calculation code in SCIENCE code package is used to simulate power distribution and detector signals.

As mentioned before, a number of 'true-calculated' cases should be simulated to emulate uncertainty in the calculation part to obtain the knowledge of error reduction factor *a*. For ACP100-K study, the "baseline" core conditions are selected at 16 burnup points of the equilibrium cycle, and 100 "perturbed" core conditions are generated for each "baseline" core conditions. Generally, 4 core parameters should be perturbed: core burnup, core power level, rod positions and xenon distribution. But in this study, the perturbation of xenon distribution was ignored for two reasons:

Xenon distribution is hard to be quantized and sampled;

The reconstructed errors are very small for xenon distribution perturbation cases compared to calculated errors according to the analysis of the previous study [5], so as of these cases are very small. The statistical result will be conservative when the perturbation of xenon distribution has been ignored.

LHS method is used to sample core burnup, power level and 4 different rod positions within a reasonable range established according to the "baseline" core parameters, and this method allows a much better coverage of the input parameter uncertainties than simple random sampling (SRS) because it densely stratifies across the range of each input probability distribution.

In this study, core burnup, power level and 4 different rod positions are assumed to follow the uniform distribution. The sample ranges of each variable perturbation are: Perturbation of burnup is in the range of [-40EPPD,40EPPD]; Perturbation of power level is in the range of [-40%FP,40%FP]; Perturbation of rod bank position is in the range of [-5steps, 5steps].

These perturbations are considered to be conservative and bounding for the uncertainty analysis. Two different sets of random numbers are generated by LHS method which has been implemented in MATLAB code package to show the distribution of a separately. Fig.1.a shows the distribution of *a* using random set No.1, and Fig.1.b shows the distribution of *a* using random set No.2. We can see that the distributions of two sets are almost the same, and we can get the conclusion that the distribution of *a* isn't related to the implementation of LHS.

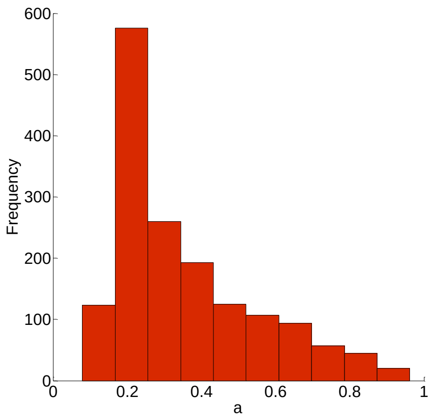

Fig.1.a. Distribution of *a* of random set No.1

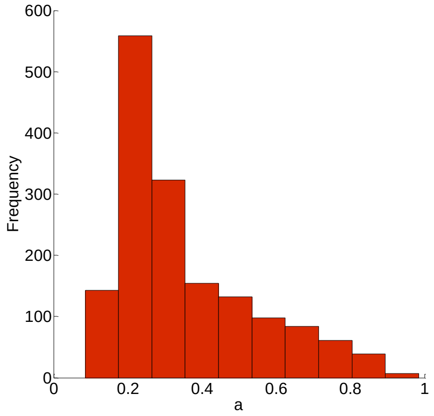

Fig.1.b. Distribution of *a* of random set No.2

The scatter diagrams that show the relationships between calculation root-mean-square (RMS) errors and *a*s are plotted in Fig.2.

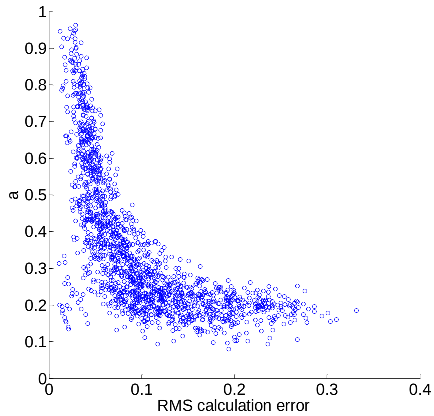

Fig.2.a.
Scatter diagram of random set No.1

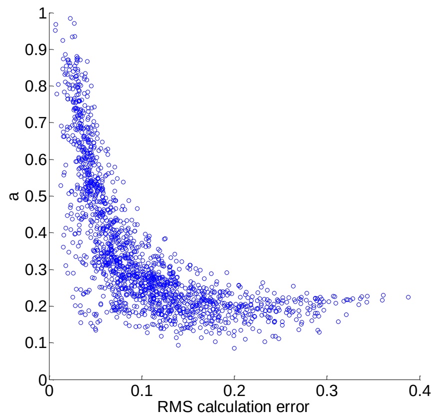

Fig.2.b. Scatter diagram of random set No.2

From Fig.2, we can get the same conclusion as shown in Fig.1 that the distribution of error reduction factor *a* isn't related to the implementation of LHS. And we can find that there exists a bounding curve that bounds almost all scatter points as shown in Fig.3.

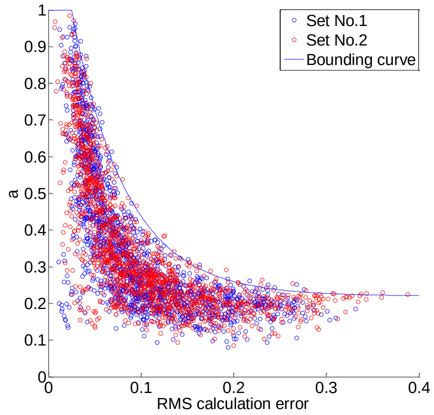

Fig.3. Bounding curve of error reduction factor *a*

As a preliminary study, the bounding curve in this paper can be expressed as the following function form according to the distribution of scatter points:

$$\begin{aligned}&\mathrm{RMS}_{\mathrm{cal}} < 0.025: \\ &a = 1; \\ &\mathrm{RMS}_{\mathrm{cal}} >= 0.025: \\ &a = 0.78 * \exp(-17 * (\mathrm{RMS}_{\mathrm{cal}} - 0.025)) + 0.22\end{aligned} \tag{15}$$

From Fig.3, we can know that the importance of power reconstruction becomes big when the RMS calculation error becomes big.

In order to show the influence of measurement error on the reconstruction error, the estimation procedure of *b* was implemented and 5 “baseline” core conditions of different burnup points of the equilibrium cycle of ACP100-K were used where “perturbed” core conditions are assumed to be as same as “baseline” core conditions. By assuming the normal distribution of measurement uncertainties about the “baseline” detector measurement, the detector signals were sampled 100 times for each “baseline” core condition. Fig.4 shows the relationship between measurement uncertainties and reconstruction uncertainties of 5 “baseline” core conditions.

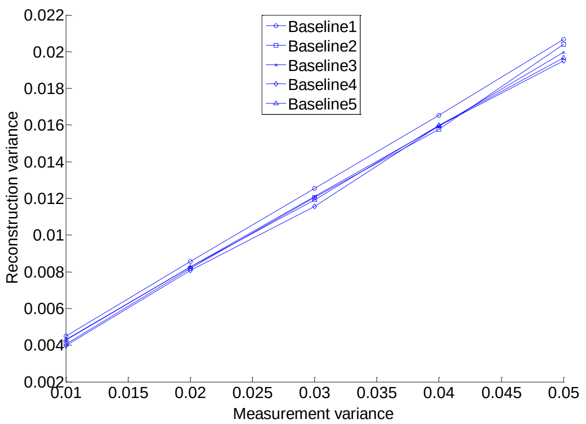

Fig.4. Relationship between measurement variance and reconstruction variance

From Fig.4, we can see that the reconstruction uncertainty does have a linear relationship with measurement uncertainty. The bounding curve has a slope of about 0.4 for these 5 cases. In order to obtain a conservative estimation of the reconstruction uncertainty, *b* is set as 1 to ensure the sum of *a* and *b* is always bigger than 1.

For the real power distribution monitoring of ACP100-K, σ_m can be estimated by analyzing the time series of in-core neutron detector signals, and σ_c can be estimated by σ_{mc} and σ_m. RMS calculation error is supposed to have the same value of σ_c, and a can be determined from σ_c using the function of bounding curve. As mentioned before, *b* is set as 0.78. Then the reconstruction uncertainty of 3D power distribution can be determined by Eq. (11).

4. Conclusions

This paper presents the methodology of uncertainty analysis used in SMROMS. The ACP100-K small modular reactor is taken as an example to illustrate this method. The following conclusions are drawn from the study:

The uncertainties of reconstructed core power parameters, such as 3D power distribution and power peaking factor, are analyzed to be a weighted average of calculation uncertainties and measurement uncertainties of in-core neutron detectors.

The weights of calculation uncertainties and measurement uncertainties can be determined through theoretical simulation analysis.

References

1. Seong K, “Uncertainty analysis for the BEACON-COLSS core monitoring system application,” ICONE13, Beijing, China, May 16-20, 2005.

2. Koo B, “Design features and thermal margin assessment of core protection and monitoring systems of an integral reactor, SMART,” *Journal of Nuclear Science and Technology*, 51, 390-404 (2014).

3. Jang J, “On-line generation of three-dimensional core power distribution using incore detector signals to monitor safety limits,” *Journal of Nuclear Science and Technology*, **36**, 528-539 (2004).

4. Helton J, “Latin hypercube sampling and the propagation of uncertainty in analyses of complex systems,” *Reliability Engineering and System Safety*, **81**, 23-69 (2003).

5. Gong H， Research on the Physical Model of On-Line Power Distribution Monitoring System, Master thesis, Nuclear Power Institute of China, (2013).